INTERMEDIATE ARITHMETIC

FOR USE IN THE

SCHOOLS AND ACADEMIES

OF THE

UNITED STATES.

BY

D. W. CRONAN, Jr.,

PRINCIPAL OF THE BIENVILLE BOYS' SCHOOL, NEW ORLEANS, LA.

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PREFACE.

In submitting my INTERMEDIATE ARITHMETIC to the consideration of my fellow-teachers, I take this opportunity to make a few remarks in regard to the several characteristics of my work. It has been my aim to lead the pupil gradually into the mysteries of numbers; from the beginning I have tried to advance the pupil slowly, step by step, until he at last arrives at those intricate problems in Denominate Numbers. He is now taught to reason with them. The problems are numerous, and cover every rule in Denominate Numbers, from the most simple to the most difficult. In the perusal of this work, no one will fail to note that I have written this volume in the same language in which it is taught, using no terms beyond the comprehension of any pupil fit to study this difficult branch. My explanations are plain and simple, and will be readily understood by any pupil. I would respectfully call the attention of my fellow-

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teachers to the system of teaching Fractions, which I think will answer all purposes for which it was intended.

Practice in U. S. Money, Denominate Numbers, and Duodecimals, are treated to their fullest extent, and I am satisfied that there are enough of examples in this little volume to satisfy any teacher. I have taken great care to demonstrate every principle and to give a plain analysis of the methods employed from the most simple to the most intricate, and to explain the reasons for every rule. Satisfied that a trial of this volume will meet with success, I leave it to the jury of my fellow-teachers, who alone are to judge of its merits, and if it meets with their approval my labors will have their reward.

D. CRONAN, Jr.,
Principal Bienville School, New Orleans.

N. Orleans, July, 1869.

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ARITHMETIC.

CHAPTER I.

LESSON I.

DEFINITIONS.

1. ARITHMETIC is the science of numbers.

2. A figure is a character used to represent a certain number of objects.

3. An object is anything which can be seen, heard, or imagined.

4. Numbers are either concrete or abstract.

5. Concrete numbers are used to designate particular objects: as 3 feet, 4 inches.

6. Abstract numbers do not refer to any object in particular, and are merely numbers, having no reference to anything: as 1, 2, 3, 4, 5, 6.

7. A unit is the first figure of the nine digits, and represents but one object.

8. A number is a collection of units arranged in order, according to their values.

9. The fundamental principles, or the four ground rules of arithmetic are, Addition, Subtraction, Multiplication, and Division.

10. They are called the four ground rules, because all problems, however difficult, depend upon one or several of them for their solution.

Quantity is applied to anything which can be measured.

11. A fractional number is a part of a unit. An integral number is a whole number.

12. A decimal fraction represents the object as divided into tenths, hundredths, etc., etc.

NOTATION.

13. Notation is the art of expressing numbers by means of figures, and in some instances by means of letters.

14. The latter method is called the Roman method, and is used in numbering the chapters, lessons, etc., of a book. It is also used to mark the value of bank-notes, etc.

15. The letters on the dial of a clock are of the Roman method. It is used for a great many other purposes.

16. The first method is called the Arabic method, from its having been employed by the Arabians.

17. The ten Roman digits are formed thus:

$$\underset{1}{\text{I}},\ \underset{2}{\text{II}},\ \underset{3}{\text{III}},\ \underset{4}{\text{IV}},\ \underset{5}{\text{V}},\ \underset{6}{\text{VI}},\ \underset{7}{\text{VII}},\ \underset{8}{\text{VIII}},\ \underset{9}{\text{IX}},\ \underset{10}{\text{X}}.$$

18. The Arabic digits are supposed to have been derived from the following symbols, each one having

tho necessary number of *straight* lines to indicate its value, thus :
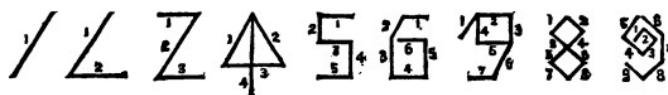
and the ◯ which represented nothing, having no straight line, or because it represented something which had no beginning or end.

LESSON II.

ARABIC NOTATION.

The figure 1 one   represents 1 one   unit.
The figure 2 two   represents 2 two   units.
The figure 3 three represents 3 three units.
The figure 4 four   represents 4 four   units.
The figure 5 five   represents 5 five   units.
The figure 6 six   represents 6 six   units.
The figure 7 seven represents 7 seven units.
The figure 8 eight represents 8 eight units.
The figure 9 nine   represents 9 nine   units.
The figure 10 ten   represents 10 ten   units.
The figure 0 represents nothing.

LESSON III.

19. Write in figures, one, three, five, seven, nine, two, four, six, eight, ten, naught.

LESSON IV.

20. To represent any number between ten and twenty, place the figure representing the number of units to the right of the figure, 1 (one.)

Thus, to represent 12 (twelve), place the figure 2 (two) to the right of the figure 1 (one)—12.

Write in figures, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen.

LESSON V.

21. To write in figures any number between twenty and twenty-nine, inclusive, write the figure representing the units to the right of the figure 2 (two.)

Thus, to write the number twenty-one, place the the figure one to the right of the figure two—21.

Twenty-one represents two tens and one unit.

Write in figures, twenty-one, twenty-two, twenty-three, twenty-four, twenty-five, twenty-six, twenty-seven, twenty-eight, twenty-nine.

LESSON VI.

22. To write in figures any number from thirty to thirty-nine, inclusive, write the figure representing the units to the right of the figure three.

To write the number thirty-four, write the figure four to the right of the figure three : thus, 34.

Thirty-four represents three tens and four units.

Write in figures, thirty, thirty-one, thirty-two, thirty-three, thirty-four, thirty-five, thirty-six, thirty-seven, thirty-eight, thirty-nine.

LESSON VII.

23. To write in figures any number from forty to forty-nine, inclusive, write the figure representing the units to the right of the figure four.

24. To write forty-five, write the figure five to the right of the figure four : thus, 45.

Forty-five represents four tens and five units.

Write in figures, forty, forty-one, forty-two, forty-three, forty-four, forty-five, forty-six, forty-seven, forty-eight, forty-nine.

LESSON VIII.

25. To write in figures any number from fifty to fifty-nine, inclusive, place the figure representing the units to the right of the figure five.

To write fifty-four, place the figure four to the right of the figure five, thus : 54.

54 represents five tens and four units.

Write in figures, fifty, fifty-one, fifty-two, fifty-three, fifty-four, fifty-five, fifty-six, fifty-seven, fifty-eight, fifty-nine.

LESSON IX.

26. To write in figures any number from sixty to sixty-nine, inclusive, place the figure representing the units to the right of the figure six.

To write sixty-eight, place the figure eight to the right of the figure six : thus, 68.

Sixty-eight represents six tens and eight units.

Write in figures, sixty, sixty-one, sixty-two, sixty-three, sixty-four, sixty-five, sixty-six, sixty-seven, sixty-eight, sixty-nine.

LESSON X.

27. To write in figures any number from seventy to seventy-nine, inclusive, place the figure representing the units to the right of the figure seven.

To write seventy-nine, place the figure nine to the right of the figure seven : thus, 79.

Seventy-nine represents seven tens and nine units.

Write in figures, seventy, seventy-one, seventy-two, seventy-three, seventy-four, seventy-five, seventy-six, seventy-seven, seventy-eight, seventy-nine.

LESSON XI.

28. To write in figures any number from eighty to eighty-nine, inclusive, place the figure representing the units to the right of the figure eight.

To write the number eighty-four, place the four to the right of the figure eight: thus, 84.

Eighty-four represents eight tens and four units.

Write in figures, eighty, eighty-one, eighty-two, eighty-three, eighty-four, eighty-five, eighty-six, eighty-seven, eighty-eight, eighty-nine.

LESSON XII.

29. To write in figures any number from ninety to ninety-nine, inclusive, place the figure representing the units to the right of the figure nine.

To write the number ninety-eight, place the figure eight to the right of the figure nine: thus, 98.

Ninety-eight (98) represents 9 tens and 8 units.

---

30. Units of the third order, or hundreds.

LESSON XIII.

354 is read three hundred and fifty-four, because it is composed of 3 hundreds, 5 tens, and 4 units.

To write in figures any number from one hundred to nine hundred and ninety-nine: first, write the figure representing the hundreds; to the right of it place the figure representing the tens; to the right of the tens place the figure representing the units.

To write eight-hundred and ninety-seven: first,

write the figure 8, which represents the hundreds; to the right of it place the figure 9, which represents the tens, followed by the figure 7, which represents the units, and the number formed will be 897.

Write in figures (and give the reason) the following:

> Eight hundred and twenty-five.
> Two hundred and forty-six.
> Six hundred and thirty-four.
> One hundred and thirty-six.
> Seven hundred and forty-two.
> Five hundred and twenty-two.
> Three hundred and sixteen.
> Nine hundred and forty-eight.
> Four hundred and ninety-two.

LESSON XIV.

31. The figure 0 has no value, and is only used to fill omissions in counting.

If we wish to write in figures seven hundred and four, we see immediately that the figure representing the tens is wanting to fill the space, but since we do not require tens, we place the figure 0 in the place which should be occupied by the tens; as it has no value, it serves to show that there are no tens, and also to form one more space.

The figure written would be 704, which is read seven hundreds, no tens, and four units.

Write in figures:

> One hundred and sixty.   *Ans.* 160.

In the preceding we have one hundred, six tens, and no units.

Write in figures:

>   Two hundred and forty.
>   Six hundred and eight.
>   Nine hundred and sixty.
>   Three hundred and three.
>   Four hundred and five.
>   Five hundred and fifty.
>   Seven hundred and eighty.
>   Eight hundred and ten.
>   One hundred and one.

LESSON XV.

32. A figure is valued according to the position it occupies in connection with other figures.

Thus, the figure 7, in its simple value, means nothing more than seven whole units. By placing the figure five (5) to the right of it, its value is increased tenfold, that is, it is made to occupy the second place or order, and consequently it comes under the denomination of tens. By the placing of the figure five to the right of the figure 7, we make a number ten times as great as the original number. If an 0 was placed in the position occupied by the figure five in the above example, the same result would be obtained, as regards the figure 7. If, after the number 75, the figure nine is placed, the seven is removed from the tens to the hundreds, the five from the units to the tens. The number thus

2*

formed is 759, and is read, seven hundred and fifty-nine. By placing the figure 0 after the 5 in the above example, the same results are obtained as regards the figures 7 and 5. From the above, we have the following:

For every period from right to left, the figure or figures increase tenfold.

For every period from left to right, the figure or figures decrease tenfold.

33. For convenience in counting, figures are divided into groups of three, and each group has its particular name and value.

The groups are named as follows:

Hundreds, Thousands, Millions, Billions, Trillions, Quadrillions, Quintillions, Sextillion, Septillions, Octillions, etc., etc.

Each group is subdivided into units, tens, and hundreds.

34. To represent in figures any number,

1. Place the figures in the groups to which they belong.

2. If in the number to be written there should be some vacant spaces, fill those places with 0's. Thus:

Write one hundred trillions, two hundred and forty millions, nine hundred and one thousand, three hundred and four.

Hundreds. Tens. Units.	Hundreds. Tens. Units.	Hundreds. Tens. Units.	Hundreds. Tens. Units.	Hundreds. Tens. Units.
1		2 4	9 1	3 4
Trillions.	Billions.	Millions.	Thousands.	Hundreds, or Units.
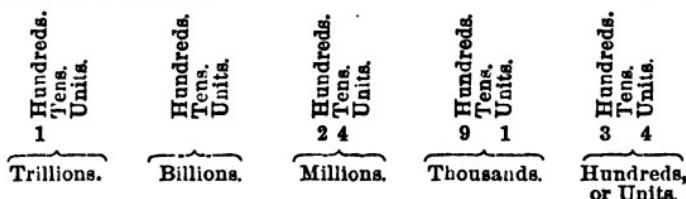
By filling the blank spaces in the preceding example with 0's, the required number would be represented as follows :

Hundreds. Tens. Units.	Hundreds. Tens. Units.	Hundreds. Tens. Units.	Hundreds. Tens. Units.	Hundreds. Tens. Units.
1 0 0	0 0 0	2 4 0	9 0 1	3 0 4
Trillions.	Billions.	Millions.	Thousands.	Hundreds, or Units.

and read, one hundred trillions, no billions, two hundred and forty millions, nine hundred and one thousand, three hundred and four.

Write in figures :

One million and ten thousand.   *Ans.* 1,010,000.

Ten thousand four hundred and one.

One hundred trillions, one billion, one hundred millions, one thousand, one hundred and one.
        Ans. 100,001,100,001,101.

Three hundred and thirty-four thousand, six hundred and fifty-three.   *Ans.* 334,653.

Three millions, three hundred and forty-six thousand, five hundred and thirty-six.
        Ans. 3,346,536.

Nine hundred and three thousand, one hundred and forty-five.   *Ans.* 903,145.

Fifty-three billions, four hundred millions, six hundred and seventy-eight thousand, seven hundred and forty-two.

Twenty-nine millions, three hundred and sixty-seven thousand and ninety-three.

Twelve thousand and thirteen.   *Ans.* 12,013.

One hundred and twenty thousand, one hundred and thirty.   *Ans.* 120,130.

One hundred and one sextillions, one hundred quintillions, two hundred and forty quadrillions, forty-nine trillions, one billion, twenty-one millions, two thousand and ten.

Ans. 101,100,240,049,001,021,002,010.

Two hundred thousand, one hundred and one.

Ans. 200,101.

Ninety-eight thousand, seven hundred and six.

Ans. 98,706.

Two trillions, twenty billions, two hundred millions, two thousand and two.   *Ans.*

NOTE.—As notation is so closely allied to numeration, the one being the reverse of the other, it is not necessary to dwell further on the subject of notation.

CHAPTER II.

LESSON I.

NUMERATION.

35. Numeration is the art of reading numbers.

There are two methods, the English and the French. The French method is most commonly used in the United States and Continental Europe.

36. As stated in treating notation, to form numbers, figures are separated into groups of three

figures, each group having its value, and each figure, according to the position it occupies in the group, is classed under the order or head of units, tens, and hundreds.

37. NOTE.—To read numbers, a knowledge of the art of numerating is necessary, and to form numbers, a knowledge of the art of notation is necessary. The one is so closely connected with the other that, to be able to read numbers, it is necessary to be able to write numbers, and *vice versa.*

38. To read or numerate, begin with the right-hand figure, or units, and numerate from right to left, thus, units, tens, hundreds, thousands, etc. Place a comma after every third figure to distinguish the group or set to which the figures may belong; then begin and read from left to right, after having carefully observed the order of the groups.

39. In numerating, no notice is to be taken of the 0, further than to see that it occupies its proper place among the figures.

LESSON II.

Read the following numbers:

1	102030	40503026070
12	123400	12030405060
102	10203040	700800900200120
120	1234567	900080007000600
123	1023045067	80000700006000303
1023	2034056070	123004560007890
10203	30250406070	2460462026404

9876432286071
18097642876543
2876432154087621
8800770066005500440033
67008900101100121308643
628754321865932416
123456789123456789
9876543210123456789
28675432108800246189
135792468358742651
101001000100001000010

LESSON III.

PROPERTIES OF THE 9's.

40. In any number with but one significant figure, such as 5, 50, 500, etc., the excess of exact 9's is equal to the significant figure.

If we write any number, as 87643, it may be read in this way, 80 thousands, 7 thousands, 6 hundreds, 4 tens, or 40, and 3 units.   Now, the excess of

    9's in 80 thousands is 8
       in 7 thousands is 7
       in 6 hundreds is 6
       in 4 tens is 4
       in 3 units is 3

                Total, . . 28, which is 3 nines and 1 over, therefore, 1 is the excess of the exact number of 9's in the above figure.

41. To find the excess of 9's in any number,

add the figures comprising the number together. Divide the sum by 9. The remainder, if any, is the excess of 9's in the number.   Or—

II. Add the figures, and drop 9 from the sum as soon as it occurs.   Thus, 8 and 7 are 15, 9 in 15, 1 time and 6 over; 6 and 6 are 12, 9 in 12, 1 and 3 over; 3 and 4 are 7 and 3 are 10, 9 in 10, 1 time and 1 over, which is the excess over the exact 9's.

The same rule will apply to all similar operations.

  1. What is the excess of 9's in 183462 ?   *Ans.* 6.
  2. What is the excess of 9's in 10203 ?   *Ans.* 6.
  3. What is the excess of 9's in 86401 ?   *Ans.* 1.
  4. What is the excess of 9's in 102030 ?   *Ans.* 6.
  5. What is the excess of 9's in 2345 ?   *Ans.* 5.
  6. What is the excess of 9's in 3456 ?   *Ans.* 0.
  7. What is the excess of 9's in 87643 ?   *Ans.* 1.
  8. What is the excess of 9's in 2468 ?   *Ans.* 2.
  9. What is the excess of 9's in 46810 ?   *Ans.* 1.
10. What is the excess of 9's in 6789 ?   *Ans.* 3.
11. What is the excess of 9's in 78910 ?   *Ans.* 7.
12. What is the excess of 9's in 89123 ?   *Ans.* 5.
13. What is the excess of 9's in 91234 ?   *Ans.* 1.
14. What is the excess of 9's in 123456 ?   *Ans.* 3.
15. What is the excess of 9's in 234567 ?   *Ans.* 0.
16. What is the excess of 9's in 345678 ?   *Ans.* 6.
17. What is the excess of 9's in 456789 ?   *Ans.* 3.

CHAPTER III.

ADDITION.

LESSON I.

42. SIGNS.—The sign plus is the sign of addition, and when placed between two quantities, signifies that they are to be added together.

43. This sign (=) is the sign of equality, and signifies that the quantity or quantities on each side of it are equal to each other.

$$5+3+4=6+4+2 \text{ is read :}$$

5 plus 3 plus 4 are equal to 6 plus 4 plus 2 ; or, it may be read in this way, by substituting the word and for the word plus, 5 and 3 and 4 are equal to 6 and 3 and 2.

ADDITION TABLE.

2+ 1= 3	3+ 1= 4	4+ 1= 5	5+ 1= 6	6+ 1= 7	7+ 1= 8
2+ 2= 4	3+ 2= 5	4+ 2= 6	5+ 2= 7	6+ 2= 8	7+ 2= 9
2+ 3= 5	3+ 3= 6	4+ 3= 7	5+ 3= 8	6+ 3= 9	7+ 3=10
2+ 4= 6	3+ 4= 7	4+ 4= 8	5+ 4= 9	6+ 4=10	7+ 4=11
2+ 5= 7	3+ 5= 8	4+ 5= 9	5+ 5=10	6+ 5=11	7+ 5=12
2+ 6= 8	3+ 6= 9	4+ 6=10	5+ 6=11	6+ 6=12	7+ 6=13
2+ 7= 9	3+ 7=10	4+ 7=11	5+ 7=12	6+ 7=13	7+ 7=14
2+ 8=10	3+ 8=11	4+ 8=12	5+ 8=13	6+ 8=14	7+ 8=15
2+ 9=11	3+ 9=12	4+ 9=13	5+ 9=14	6+ 9=15	7+ 9=16
2+10=12	3+10=13	4+10=14	5+10=15	6+10=16	7+10=17
2+11=13	3+11=14	4+11=15	5+11=16	6+11=17	7+11=18
2+12=14	3+12=15	4+12=16	5+12=17	6+12=18	7+12=19

8+ 1= 9	9+ 1=10	10+ 1=11	11+ 1=12	12+ 1=13
8+ 2=10	9+ 2=11	10+ 2=12	11+ 2=13	12+ 2=14
8+ 3=11	9+ 3=12	10+ 3=13	11+ 3=14	12+ 3=15
8+ 4=12	9+ 4=13	10+ 4=14	11+ 4=15	12+ 4=16
8+ 5=13	9+ 5=14	10+ 5=15	11+ 5=16	12+ 5=17
8+ 6=14	9+ 6=15	10+ 6=16	11+ 6=17	12+ 6=18
8+ 7=15	9+ 7=16	10+ 7=17	11+ 7=18	12+ 7=19
8+ 8=16	9+ 8=17	10+ 8=18	11+ 8=19	12+ 8=20
8+ 9=17	9+ 9=18	10+ 9=19	11+ 9=20	12+ 9=21
8+10=18	9+10=19	10+10=20	11+10=21	12+10=22
8+11=19	9+11=20	10+11=21	11+11=22	12+11=23
8+12=20	9+12=21	10+12=22	11+12=23	12+12=24

LESSON II.

44. Addition is the process of finding the sum of two or more numbers.

45. The sum of two or more numbers is a number which contains as many units as there are units in the several numbers taken together.

1. What is the sum of 643 and 539 ?

ANALYSIS.

Add the column of units. If the units are more than ten, place the right-hand figure under the column of units, and carry the left-hand figure or figures to the column of tens.

OPERATION.

```
  643
  539
 ────
 1182
```

Add the column of tens, including the figure carried from the units. If there are more than ten tens, place the right-hand figure under the column of tens, and carry the left-hand figure to the next column.

Add the column of hundreds, together with the figure carried from the tens. If there are more than ten hundreds, place the right-hand figure under the column of hundreds, and carry the left-hand figure to the column of thousands.

Continue in this way until the last column is reached then write down the sum of the last column.

3

2. Analyze as follows : write the numbers—thus,	643
	539
With a line drawn between them,	——
Add the column of the units,	12
Add the column of the tens,	7
Add the column of the hundreds,	11
Sum of the three columns,	1182

3. What is the sum of 2497, 3864, 8972 ?

Begin with the column of units, and add thus: 2 and 4 are 6, and 7 are 13; place the three under the column, and carry the figure one (mentally) to the next column.

OPERATION.

2497
3864
8972
——
15333

Add the column of tens, thus : 7 and 6 are 13, and 9 are 22, and one carried from the unit column, makes 23 ; place the 3 under the column of tens, and carry the 2.

Add the column of hundreds: 9 and 8 are 17, and 4 are 21, and 2 carried from the column of tens make 23; write the 3 under the column of hundreds, and carry the 2 to the column of thousands.

Add the column of thousands: 8 and 3 are 11, and 2 are 13, and 2 carried from the colnmn of hundreds make 15. There being no more columns to add, write the number 15, with the 5 under the column of thousands, and the 1 a little to the left of it, as above.

Hence the following :

RULE FOR ADDITION.

46. I.—*Place the figures of the same value un-*
der each other.

II.—*Begin at the column of units, and add the*
figures found in that column. If their sum exceed
10, *place the right-hand figure under the column, and*
carry the left-hand one to the next column. Con-
tinue in this way until the last or left-hand *column*
is added. Place the sum of the figures in the last
column under the last column.

The figures thus found will be the answer.

PROOF.

47. There are several methods of proving ad-
dition.

1. Begin at the top and add the columns down-
wards. Then begin at the bottom and add the
columns upwards. If the two results are the same,
the addition is correct.

2. Divide the given numbers into as many parts
as may be necessary. Add the several parts. Add
the sums of the several parts. If the sum of the
several parts added together is equal to the sum
found, the work is correct.

3. Find the excess of nines in each of the num-
bers, and place it to the right of the number. Add the
numbers thus found, and note the excess of nines in

their sum. This excess of nines should equal the excess of nines in the sum of the numbers. If so, the addition is correct.

SECOND METHOD.

1st part,	246892
	134714
Sum of 1st part,	381606
2d part, . .	241381
	527123
Sum of 2d part,	768504
Sum of 1st part,	381606
Sum of 2d part,	768504
Sum of both parts,	1150110

FIRST METHOD.

246892
134714
241381
527123
————
1150110

THIRD METHOD.

246892	. . . . .	4	excess of 9's.
134714	. . . . .	2	" 9's.
241381	. . . . .	1	" 9's.
527123	. . . . .	2	" 9's.
————		—	—
1150110		0	0

LESSON III.

EXAMPLES.

(1.)	(2.)	(3.)	(4.)	(5.)
123456	123	12345	689724	18976543
24689	5734	2468	398764	24897321
3754	24698	53897	876421	98765428
123	321456	8721	578932	87639742

(6.)	(7.)	(8.)	(9.)	(10.)
5375	25846	93754	13289	52936
1203	64845	49375	91328	65293
3201	52846	54937	89132	36529
3753	85264	75493	28913	93652
5737	42583	37549	32891	29365

(11.)	(12.)	(13.)	(14.)	(15.)
482436	127618	298634	211119	169540
634284	276811	928634	112191	619450
824364	761821	863429	121911	965454
284436	618127	634298	219111	543956
824336	181271	342986	191211	487251
283346	816721	436892	912111	72893

(16.)	(17.)	(18.)	(19.)	(20.)
246810	246891	1817163	94231	678912
987641	42698	181716	9423	34567
198764	12345	31817	194	89101
387214	2489	1637	32	23456
38761	321	942	8	78910

3*

LESSON IV.

(21.)	(22.)	(23.)	(24.)
123456	2345678	3456789	4567891
789101	9101234	1012345	8765432
234567	5678910	6789101	9101234
891012	1234567	2345678	2198765
345678	8910123	9101234	3456789
910123	4567890	5678910	1012345

(25.)	(26.)	(27.)	(28.)
1234567	24689	123456789	3456789101
8910123	98642	101234567	2345678910
4567891	13579	891012345	1234567890
1234567	97513	678910123	2413456789
8910123	12345	456789101	1012345678
4567890	54321	234567891	9101234567
1234567	67890	12345678	8910123456
	9876	91012345	7891012345
			6789101234
			5678910123

(29.)	(30.)	(31.)	(32.)
1234567	2468972	12345	24764283
8910123	1234567	45678	12345678
4567891	8910111	78910	91234567
0123456	2345678	10112	89012345
7891012	9101112	13456	67890123
3456789	3456789	56789	45678901
1012345	1011123	89108	23456789
6789101	4567890		12345678
			91012345

(33.)	(34.)	(35.)	(36.)
24689754	3132333435	246892	36987
32123456	6373839404	345678	78963
78910111	1424344456	123456	12345
23456178	4748495051	789101	67891
19202122	5253545556	121356	23456
23242526	5758596061	789321	78910
27282930	6263656667	234567	24689
31323334			87654
			32112
			34567
			89101

(37.)	(38.)	(39.)	(40.)
2222222	3333333	444444	124689
3333333	4445555	444555	44444
4444444	6666777	555666	55555
5555555	7888999	666777	66777
6666666	2223333	777888	88999
7777777	4444444	888999	11222
8888888	5556666	111222	33444
9999999	7778888	222333	55666
1111111	9876543	444555	
		666777	

LESSON V.

(41.)	(42.)	(43.)	(44.)	(45.)	(46.)
867	798	321	12386	168872	168321
673	765	748	2497	241387	54592
571	743	617	3874	89764	18764
876	435	748	2976	2138	2468
901	876	286	8397	976	1234
		931	2178	22	456
					67
					8

(47.)	(48.)	(49.)	(50.)	(51.)
1	2	3	6	387462
12	22	45	78	59321
123	223	678	910	8764
1234	2233	9101	1112	213
12345	22333	23456	13141	89
123456	223344	789102	151621	
		123456	24899	

(52.)	(53.)	(54.)
87649231	123456789	87643212345
4156789	123456	6789023456
198642	43210987	678912345
23456	5678902	67891234
1987	654321	5678912
234		

(55.)	(56.)	(57.)	(58.)	(59.)	(60.)
12	123	1234	12345	45321	10
21	456	5678	67891	65432	100
22	789	9101	02123	78910	1010
23	101	2345	45678	54321	10101
24	234	6789	90123	67890	100101
25	567	1012	45678	98642	12345
26	890	3456	91012	12345	12354
27	123	7890	34567	19876	32145
72	456	1234	89101	187643	13254
82	789	8976	23450	21761	
28	101	2468			
32	234				

61. In the State of Maine there are 16 counties, in New Hampshire 10, in Vermont 14, in Massachusetts 14, in Rhode Island 5, in Connecticut 8. How many counties in the six New England States?

Ans.

62. In New York there are 60 counties, in New Jersey 21, in Pennsylvania 66, in Delaware 3. How many counties in the four Middle States?

Ans.

63. New York City has a population of 805651 persons, Philadelphia 565531, Brooklyn 266664, Baltimore 212419. How many inhabitants in the four cities? *Ans.*

64. New Orleans has 168472 inhabitants, Cincinnati 161044, St. Louis 151780, Chicago 109263, Buffalo 81131, Newark 71941, Louisville 69470, Albany

62368.   How many inhabitants in all of the above cities?   *Ans.* 875469.

65. London has 2803034 inhabitants, Paris has 1696141, Pekin 2000000, Yeddo 1500000, Canton 1000000, Constantinople 787000, Berlin 547571, York 40377, Londonderry 19888.   How many inhabitants in all of the above cities?   *Ans.* 10394011.

66. Mt. St. Elias is 17860 feet high; Mt. Fairweather, 14900 feet; Mt. Illaman, 12066 feet; Mt. Brown, 15990 feet; Mt. Hooker, 15675; Fremont's Peak, 13570.   If they were piled one on the other, how high would they reach.   *Ans.* 90061.

67. The Missouri is 3100 miles long; the Mississippi, 3160 miles; Mackenzie, 2440 miles; St. Lawrence, 2200 miles; Arkansas, 2170 miles; Rio Grande, 1800 miles; Red, 1500; Nebraska, 1500; Nelson, 1600; Columbia, 1200; Colorado, 1000; Ohio, 948; Lewis, 900; Tennessee, 800.   If they were placed one after the other, forming one long river, how many miles in length would it be?   *Ans.* 24318.

68. The Amazon is 4000 miles; La Plata, 2250 miles; Madeira, 1800 miles; Paraguay, 1600 miles; Orinoco, 1590; St. Francisco, 1260; Tocantins, 1100. How many miles in length would they be if formed into one stream?   *Ans.* 13600.

69. Asia has 696725000 inhabitants; Africa, 70670000; North America, 49280480; South America, 21356870; Oceanica, 25898788; Europe, 282959-505.   How many inhabitants on the globe?

                                                Ans. 1146890643.

70. Russia has 66891493 inhabitants; Sweden

and Norway, 5474452; Austria, 35019058; France, 37472132; Turkey, 15500000; Spain, 16560813; Great Britain and Ireland, 29335340; Italy, 22430000; Prussia, 23810743; Germany, 13774510; Portugal, 3923410; Denmark, 1592354; Greece, 1343293; Switzerland, 2510394; Holland, 3569456; Belgium, 4731957. How many inhabitants in Europe?

Ans. 282959505.

71. China has a population of 408000000 inhabitants; Siberia, 4600000; Hindoostan, 183500000; Arabia, 8000000; Farther India, 20000000; Independent Tartary, 6000000; Turkey in Asia, 15000000; Persia, 9000000; Japan Empire, 30000000; Afghanistan, 6000000; Beloochistan, 2000000; Georgia, 1625000; Corea, 2000000; Ladak, 1000000. How many inhabitants in Asia? *Ans.* 696725000.

48. To add from left to right and from right to left alternately.

1. Add 241, 362, 648, and 182.

	OPERATION.

1st. Arrange the numbers one under the other, as in the operation; begin with the first number and read as follows: 241 and 2 units make 243 units, and 60 units make 303 units, and 300 units make 603 units. 603 units and 8 units make 611 units, and

```
   241
   362
   648
   182
  1433
```

40 units make 651 units, and 600 units make 1251 units. 1251 units and 2 units make 1253 units, and 80 units make 1333 units, and 100 units make 1433 units, the answer.

Leaving the word units out, it is read more rapidly in this way.

241 and 2 are 243, 243 and 60 are 303, 303 and 300 are 603, 603 and 8 are 611, 611 and 40 are 651, 651 and 600 are 1251, 1251 and 2 are 1253, 1253 and 80 are 1333, 1333 and 100 are 1433, the answer.

49. Whole columns of a ledger can be added in this way very rapidly and correctly.

50. The examples given are simple enough to be performed by almost any beginner.

EXAMPLES.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)	(7.)	(8.)	(9.)
11	11	12	14	15	16	17	18	19
12	12	13	15	16	17	18	19	20
13	31	14	16	17	18	19	20	21
14	41	15	17	18	19	20	21	22
—	—	—	—	—	—	—	—	—

(10.)	(11.)	(12.)	(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
20	21	23	24	25	26	27	28	29
21	22	24	25	26	27	28	29	30
22	23	25	26	27	28	29	30	31
23	24	26	27	28	29	30	31	32
24	25	27	28	29	30	31	32	33
—	—	—	—	—	—	—	—	—

(19.)	(20.)	(21.)	(22.)	(23.)	(24.)	(25.)	(26.)
111	222	333	444	555	666	777	888
222	333	444	555	666	777	888	999
333	444	555	666	777	888	999	111
444	555	666	777	888	999	111	222
555	666	777	888	999	111	222	333
—	—	—	—	—	—	—	—

(27.)	(28.)	(29.)	(30.)
9999	11111	222222	123456
1111	22222	333333	78912
2222	33333	444444	12345
3333	44444	555555	6789
4444	55555	666666	123

(31.)	(32.)	(33.)	(34.)
12345	54321	987643	123456
67891	19876	543212	789012
23045	54032	678901	345678]
67890	19876	765432	901234
12345	54321	890123	908765
67890	67890	654321	123456
12345	12345	789012	789012
67890	69876	345678	123465

CHAPTER IV.

SUBTRACTION.

LESSON I.

51. SUBTRACTION is the process of finding the difference between two numbers. Thus, 5 from 7 leaves 2.

52. In the above example, the figure 5 is the

4

subtrahend, the figure 7 the minuend, and the figure 2 the remainder.

53. The sign (—) minus is the sign of subtraction, and when it is placed between two quantities signifies that they are to be subtracted.

54. The minuend is placed on the left and the subtrahend on the right of the sign.

55. When separated by the sign it is read thus:

$$7-5=2$$

7 minus 5 equals 2 ; or, better, 5 from 7 leaves 2.

56. The pupil should commit to memory the following

TABLE OF SUBTRACTION.

2—1= 1	2—2= 0	3—3= 0	4—4= 0	5—5= 0	6—6= 0
3—1= 2	3—2= 1	4—3= 1	5—4= 1	6—5= 1	7—6= 1
4—1= 3	4—2= 2	5—3= 2	6—4= 2	7—5= 2	8—6= 2
5—1= 4	5—2= 3	6—3= 3	7—4= 3	8—5= 3	9—6= 3
6—1= 5	6—2= 4	7—3= 4	8—4= 4	9—5= 4	10—6= 4
7—1= 6	7—2= 5	8—3= 5	9—4= 5	10—5= 5	11—6= 5
8—1= 7	8—2= 6	9—3= 6	10—4= 6	11—5= 6	12—6= 6
9—1= 8	9—2= 7	10—3= 7	11—4= 7	12—5= 7	13—6= 7
10—1= 9	10—2= 8	11—3= 8	12—4= 8	13—5= 8	14—6= 8
11—1=10	11—2= 9	12—3= 9	13—4= 9	14—5= 9	15—6= 9
12—1=11	12—2=10	13—3=10	14—4=10	15—5=10	16—6=10
7—7= 0	8—8= 0	9—9= 0	10—10= 0	11—11= 0	12—12= 0
8—7= 1	9—8= 1	10—9= 1	11—10= 1	12—11= 1	13—12= 1
9—7= 2	10—8= 2	11—9= 2	12—10= 2	13—11= 2	14—12= 2
10—7= 3	11—8= 3	12—9= 3	13—10= 3	14—11= 3	15—12= 3
11—7= 4	12—8= 4	13—9= 4	14—10= 4	15—11= 4	16—12= 4
12—7= 5	13—8= 5	14—9= 5	15—10= 5	16—11= 5	17—12= 5
13—7= 6	14—8= 6	15—9= 6	16—10= 6	17—11= 6	18—12= 6
14—7= 7	15—8= 7	16—9= 7	17—10= 7	18—11= 7	19—12= 7
15—7= 8	16—8= 8	17—9= 8	18—10= 8	19—11= 8	20—12= 8
16—7= 9	17—8= 9	18—9= 9	19—10= 9	20—11= 9	21—12= 9
17—7=10	18—8=10	19—9=10	20—10=10	21—11=10	22—12=10

57. To subtract one number from another, both of them must be of the same kind.

Thus, 7 inches could not be subtracted from 7 feet; but we could reduce 7 feet in the following manner, 6 feet 12 inches, which would be of the same value as 7 feet. Now subtract 7 inches from the 12 inches, and there remain 6 feet 5 inches.

The proof of subtraction depends upon this principle:

58. That the greater number is equal to the smaller number and the difference added together.

LESSON II.

59. When but one figure is used in both minuend and subtrahend:

Place the greater number above the less, with a line drawn below them. Under this line place the remainder or difference.

MENTAL EXERCISES.

7 from 8 leaves how many? 5 from 6? 3 from 7?

9 from 10? 8 from 10? 2 from 6? 5 from 8? 3 from 7?

2 from 6? 1 from 3? 5 from 7? 9 from 10? 7 from 9?

1 from 2? 2 from 4? 3 from 6? 4 from 8? 5 from 10?

2 from 3? 2 from 5? 2 from 7? 2 from 9? 2 from 8?

3 from 4? 3 from 5? 3 from 6? 3 from 7? 3 from 8?

4 from 5? 4 from 6? 4 from 7? 4 from 9? 4 from 8?

LESSON III.

60. When several figures are employed in both minuend and subtrahend.

CASE I.

When the figures of the subtrahend are of less value than the corresponding ones in the minuend.

RULE.

Subtract the figures in the subtrahend from those in the minuend, and place the difference below the line drawn under the subtrahend.

61. The pupil must be careful and see that his units tens, hundreds, etc., are in their proper places.

EXERCISES.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
456	987	79318	8987	5432	8976898
123	432	58107	2461	1321	7265787

(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
8976	64389	876492	88888	99999	77778888
7425	52273	765381	24681	36901	45632547

(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
8889	879642	976384	7643	87649	28976438
7786	768221	251271	6221	72116	12345216

(19.)	(20.)	(21.)	(22.)	(23.)	(24.)
98764	876213	76439	8749	87924	98769876
72413	245102	51216	2513	12312	12345672

(25.)	(26.)	(27.)	(28.)	(29.)	(30.)
98764	5987689	897643	89769	76834	5986873
24673	1276543	245121	21212	12121	2211331

(31.)	(32.)	(33.)	(34.)	(35.)	(36.)
8976	897643	123456	34567	88992	98765482
1312	253121	12345	3456	24681	12345121

(37.)	(38.)	(39.)	(40.)	(41.)	(42.)
9876	8765	56789	8976	98764	8976543
5432	4321	23456	3452	72315	5312312

(43.)	(44.)	(45.)	(46.)	(47.)	(48.)
8976	89764	98765	98764	87649	9876543
5213	25761	43212	21213	25517	2123431

(49.)	(50.)	(51.)	(52.)	(53.)	(54)
9876	87643	6743218	87643	99889	88888888
7253	35213	2521116	24612	24683	12345678

4*

(55.)	(56.)	(57.)	(58.)	(59.)	(60.)
2468	4689	678912	24891	89764	99999999
1234	3456	123411	12341	72213	23456789

(61.)	(62.)	(63.) ·	(64.)	(65.)
9898	87878	67676767	89764	289764
8888	77777	66666666	32512	123452 ˙

(66.)

77777777777
12345671234

LESSON IV.

CASE II.

62. When the figures in the subtrahend are in some instances greater than the corresponding ones in the minuend, proceed as follows :

Write the less number under the greater, with a line drawn beneath the lower one, as in the following

EXAMPLE.

Minuend.	3687421
Subtrahend.	2758622
	3 6973
Remainder.	928799

Begin and subtract: 2 from 1, I cannot; borrow 1 ten, which is equal to 10 units, and 1 unit in the minuend makes 11. 2 from 11 leaves 9. Place the

9 under the column of units, and add 1 to the tens of the subtrahend (or subtract 1 from the tens of the minuend), which would make 3 tens instead of 2 tens. To subtract the 3 from the 2 would be impossible; but borrow 1 from the column of hundreds, which is equal to 10 tens, and 2 tens gives 12 tens, from which subtract the 3, leaving 9. Write the figure 9 under the column of tens, and add 1 to the figure in the column of hundreds, which would give 7 hundreds. To subtract the 7 hundreds from the 4 hundreds would be impossible; but borrow 1 from the column of thousands, which would be equal to ten hundreds, and 4 hundreds gives 14 hundreds; subtract 7 from 14, and 7 remains. Write the figure 7 under the column of hundreds, and add 1 to the figure in the column of thousands, making it 9. Subtract the column of thousands. 9 from 7 impossible; borrow 1 ten thousand, which is equal to 10 one thousands, and 7 gives 17 thousands. 9 from 17 leaves 8. Write the figure 8 under the column of thousands, and add 1 to the next figure of the subtrahend, making it 6. Subtract: 6 from 8 leaves 2; write the figure 2 under the column of tens of thousands. In this case we borrowed nothing from the next column; therefore we have nothing to add to the column of hundreds of thousands, but merely to subtract the figures as we find them. Subtract: 7 from 6, impossible; borrow 1 from the next column of millions, which is equal to 10 hundreds of thousands, and 6 making 16 hundreds of thousands. 7 from 16 leaves 9. Write the figure 9 under the col-

umn of hundreds of thousands, and add 1 to the next figure in the subtrahend, making it 3. Subtract: 3 from 3 leaves 0, which figure is never written to the left of any integral number. The answer is 928799.

To prove subtraction, add the remainder to the subtrahend, and the result must be equal to the minuend.

Subtrahend.	2758622
Remainder.	928799
Minuend.	3687421

The pupil should be thoroughly drilled in reading subtraction. Thus:

From 786433 take 536248.

EXAMPLE.

```
786433
596248
------
190185
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READING.

8 from 13 leaves 5, write down the 5 and carry 1. 1 and 4 are 5. 5 from 13 leaves 8, write the 8, and carry 1. 1 and 2 are 3. 3 from 4 leaves 1. (Since we borrowed nothing, we have nothing to carry.) 6 from 6 leaves 0, write down the 0. 9 from 18 leaves 9, write the figure 9 and carry 1. 1 and 5 are 6. 6 from 7 leaves 1. Write down the 1.

PROOF.

The answer would be
190185	Remainder.
596248	Subtrahend.
786433	Minuend.

From which we have the following general

RULE.

63.—I. *Write the greater number above the less, with a line drawn beneath them.*

II. *Begin with the units, and subtract. If the figure in the subtrahend is larger than the figure in the minuend, add* 10 *to it (mentally), and subtract.*

III. *Add* 1 *to the next figure in the subtrahend, and subtract. If the figure in the subtrahend is again too large, add* 10 *to it mentally, and subtract.*

IV. *Proceed in this way until all of the figures in the subtrahend are subtracted from the corresponding figures in the minuend.*

64. NOTE.—When it becomes necessary to add 10 mentally to the figure in the minuend, 1 must be added to the next figure of the subtrahend, but in no other case.

EXAMPLES.

(1.)	(2.)	(3.)	(4.)	(5.)
678912	123456	897643	888888	2999919
589213	122567	188754	99999	898721

(6.)	(7.)	(8.)	(9.)	(10.)
89778842	246891	123456	345678	456789
8898953	157802	34567	56789	67891

(11.)	(12.)	(13.)	(14.)	(15.)
1234567	123456789	223344	334455	445566
345678	34567891	33445	44556	55667

(16.)	(17.)	(18.)	(19.)	(20.)
556677	667788	778899332	889911	991122
66778	77889	88993323	99122	99112

(21.)	(22.)	(23.)	(24.)
1122334	1223345	2233456	13579246
223345	334455	334457	5689357

(25.)	(26.)	(27.)	(28.)	(29.)
246892	123456	3456789	8764312	2689724
57903	13578	567890	876431	788935

(30.)	(31.)	(32.)	(33.)
8899776655	7642839	2286931	1869741
998877666	764289	121286	979852

(34.)	(35.)	(36.)	(37.)	(38.)
876432	228769	18976542	246891	10001
98743	39892	9897653	57902	9999

(39.)	(40.)	(41.)	(42.)
200020	202020	3030303	40404040404040
20002	20202	303030	4040404040404

(43.)	(44.)	(45.)	(46.)
50505050	6060606	70809010	6243875
6070809	708098	7080908	624897

(47.)	(48.)	(49.)	(50.)
2030020	3040506070809	1000000	2000000
203103	340510670080	246801	234567

(51.)	(52.)	(53.)	(54.)
30000000	40000000	1234567	10203040506070
11234567	12357982	891873	1234567897082

LESSON V.

55. Asia has 696725000 inhabitants. Europe has 282959505. How many more has Asia than Europe?
Ans. 413765495.

56. North America has 49280480 inhabitants.

South America has 21356870 inhabitants. How many more has North America than South America?

<div align="right">*Ans.* 27923610</div>

57. The area of the United States is 3002332 square miles. British America, 2914318 square miles. How much larger is the United States than British America?   *Ans.* 88014 square miles.

58. The area of Central America is 219000 square miles. The West Indies, 92820 square miles. How much larger is Central America than the West Indies?   *Ans.* 126180 square miles.

59. Massachusetts has 1231065 inhabitants. Louisiana, 708002. How many more has Massachusetts than Louisiana?   *Ans.* 523063.

60. The area of Louisiana is 46431 square miles. Massachusetts, 7800 square miles. How much larger is Louisiana than Massachusetts?

<div align="right">*Ans.* 38631 square miles.</div>

61. London has two millions eight hundred and three thousand and thirty-four inhabitants, and Paris has one million six hundred and ninety-six thousand one hundred and forty-one inhabitants. How many more has London than Paris?   *Ans.* 1106893.

62. Constantinople has seven hundred and eighty-seven thousand inhabitants. New York has eight hundred and five thousand six hundred and fifty-one inhabitants. Has New York more than Constantinople?   *Ans.* 18651 more.

63. From one billion take two hundred and ninety-nine millions nine hundred and ninety-nine.

<div align="right">*Ans.* 700999001.</div>

CHAPTER IV.

MULTIPLICATION.

LESSON I.

65. MULTIPLICATION is the process of taking one number as many times as there are units in another.

66. When a number is to be added to itself several times, the operation is shortened by the process of multiplication.

67. In multiplication, three terms are used: the multiplicand, the multiplier, and the product.

68. The multiplicand is the number to be multiplied.

69. The multiplier is the number by which we multiply the multiplicand.

70. The product is the result of the multiplication.

71. The multiplicand may be either an abstract or a concrete number. The multiplier must always be considered as an abstract number.

72. The product is always of the same kind as the multiplicand.

EXAMPLE.

$$\$40 \times 3 = \$120$$

That is, 3 times 40 dollars are 120 dollars.

In this example, 40 dollars is the multiplicand, 3 the multiplier, and 120 dollars the product.

5

73. The sign of multiplication is formed by two small lines crossing each other so as to form an $\times$.

74. It shows that the two numbers between which it is placed are to be multiplied together.

Thus, $6 \times 7 = 42$ is read, 6 times 7 are 42.

EXAMPLE.

If 1 barrel of flour cost 8 dollars, how much will 5 barrels cost?

It is plain that if one barrel would cost eight dollars, 5 barrels would cost 5 times as much as 1 barrel, or $5 \times 8 = 40$ dollars. Or,

If 1 barrel cost 8 dollars, 5 barrels would cost 5 times 8, or $8 + 8 + 8 + 8 + 8$, which added together gives 40.

TABLE OF MULTIPLICATION.

$2 \times 1 = 2$	$3 \times 1 = 3$	$4 \times 1 = 4$	$5 \times 1 = 5$	$6 \times 1 = 6$	$7 \times 1 = 7$
$2 \times 2 = 4$	$3 \times 2 = 6$	$4 \times 2 = 8$	$5 \times 2 = 10$	$6 \times 2 = 12$	$7 \times 2 = 14$
$2 \times 3 = 6$	$3 \times 3 = 9$	$4 \times 3 = 12$	$5 \times 3 = 15$	$6 \times 3 = 18$	$7 \times 3 = 21$
$2 \times 4 = 8$	$3 \times 4 = 12$	$4 \times 4 = 16$	$5 \times 4 = 20$	$6 \times 4 = 24$	$7 \times 4 = 28$
$2 \times 5 = 10$	$3 \times 5 = 15$	$4 \times 5 = 20$	$5 \times 5 = 25$	$6 \times 5 = 30$	$7 \times 5 = 35$
$2 \times 6 = 12$	$3 \times 6 = 18$	$4 \times 6 = 24$	$5 \times 6 = 30$	$6 \times 6 = 36$	$7 \times 6 = 42$
$2 \times 7 = 14$	$3 \times 7 = 21$	$4 \times 7 = 28$	$5 \times 7 = 35$	$6 \times 7 = 42$	$7 \times 7 = 49$
$2 \times 8 = 16$	$3 \times 8 = 24$	$4 \times 8 = 32$	$5 \times 8 = 40$	$6 \times 8 = 48$	$7 \times 8 = 56$
$2 \times 9 = 18$	$3 \times 9 = 27$	$4 \times 9 = 36$	$5 \times 9 = 45$	$6 \times 9 = 54$	$7 \times 9 = 63$
$2 \times 10 = 20$	$3 \times 10 = 30$	$4 \times 10 = 40$	$5 \times 10 = 50$	$6 \times 10 = 60$	$7 \times 10 = 70$
$2 \times 11 = 22$	$3 \times 11 = 33$	$4 \times 11 = 44$	$5 \times 11 = 55$	$6 \times 11 = 66$	$7 \times 11 = 77$
$2 \times 12 = 24$	$3 \times 12 = 36$	$4 \times 12 = 48$	$5 \times 12 = 60$	$6 \times 12 = 72$	$7 \times 12 = 84$

$8 \times 1 = 8$	$9 \times 1 = 9$	$10 \times 1 = 10$	$11 \times 1 = 11$	$12 \times 1 = 12$
$8 \times 2 = 16$	$9 \times 2 = 18$	$10 \times 2 = 20$	$11 \times 2 = 22$	$12 \times 2 = 24$
$8 \times 3 = 24$	$9 \times 3 = 27$	$10 \times 3 = 30$	$11 \times 3 = 33$	$12 \times 3 = 36$
$8 \times 4 = 32$	$9 \times 4 = 36$	$10 \times 4 = 40$	$11 \times 4 = 44$	$12 \times 4 = 48$
$8 \times 5 = 40$	$9 \times 5 = 45$	$10 \times 5 = 50$	$11 \times 5 = 55$	$12 \times 5 = 60$
$8 \times 6 = 48$	$9 \times 6 = 54$	$10 \times 6 = 60$	$11 \times 6 = 66$	$12 \times 6 = 72$
$8 \times 7 = 56$	$9 \times 7 = 63$	$10 \times 7 = 70$	$11 \times 7 = 77$	$12 \times 7 = 84$
$8 \times 8 = 64$	$9 \times 8 = 72$	$10 \times 8 = 80$	$11 \times 8 = 88$	$12 \times 8 = 96$
$8 \times 9 = 72$	$9 \times 9 = 81$	$10 \times 9 = 90$	$11 \times 9 = 99$	$12 \times 9 = 108$
$8 \times 10 = 80$	$9 \times 10 = 90$	$10 \times 10 = 100$	$11 \times 10 = 110$	$12 \times 10 = 120$
$8 \times 11 = 88$	$9 \times 11 = 99$	$10 \times 11 = 110$	$11 \times 11 = 121$	$12 \times 11 = 132$
$8 \times 12 = 96$	$9 \times 12 = 108$	$10 \times 12 = 120$	$11 \times 12 = 132$	$12 \times 12 = 144$

LESSON II.

CASE I.

75. When the multiplicand and multiplier are between 1 and 9 inclusive.

RULE.

Multiply as in the table.

Thus:

1. What cost 5 hats @ $3 ?

<div align="right">

Ans. 5 × 3 = 15 dollars.
</div>

2. What cost 6 hats @ $2 ? @ $3 ? @ $4 ? @ $5 ? @ $6 ? @ $7 ? @ $8 ?

3. What cost 2 slates @ 4 cents ? @ 5 cents ? @ 6 cents ? @ 7 cents ? @ 8 cents ? @ 9 cents ?

4. If 1 pound of sugar cost 8 cents, what will 2 pounds cost ? 3 pounds ? 4 pounds ? 5 pounds ? 6 pounds ? 7 pounds ? 8 pounds ?

5. A good pair of shoes is worth 8 dollars, what must I give for 2 pairs ? 3 pairs ? 4 pairs ? 5 pairs ? 6 pairs ? 7 pairs ? 8 pairs ? 9 pairs ?

6. If a man earns 6 dollars in 1 week, how much will he earn in 2 weeks ? 3 weeks ? 4 weeks ? 5 weeks ? 6 weeks ? 7 weeks ? 8 weeks ? 9 weeks ?

7. If 1 thousand feet of lumber cost 12 dollars, what cost 2 thousand ? 3 thousand ? 4 thousand ?

5 thousand ? 6 thousand ? 7 thousand ? 8 thousand ?
9 thousand ?

8. If 1 bottle of ink cost 5 cents, what cost 2 bottles ?   3 bottles ?   4 bottles ?   5 bottles ?   6 bottles ?
7 bottles ?   8 bottles ?   9 bottles ?

9. If 1 bushel of apples cost 2 dollars, what cost
2 bushels ?   3 bushels ?   4 bushels ?   5 bushels ?
6 bushels ?   7 bushels ?   8 bushels ?   9 bushels ?

10. If 1 hat is worth 8 dollars, what are 9 hats
worth ?

11. If 1 box of cheese cost 9 dollars, what cost 9
boxes ?

12. If a man earns 7 dollars in 1 week, how much
will he earn in 7 weeks ?

13. If 1 barrel of potatoes cost 5 dollars, what
will 8 barrels cost ?

14. If a car travels 7 miles an hour, how far will
it travel in 4 hours ?

15. If 1 pair of pantaloons cost 6 dollars, what
will 6 pair cost ?

16. If 1 yard of cotton cost 9 cents, what will 8
yards cost ?

LESSON III.

CASE II.

76. When there are several figures in the multiplicand and only one in the multiplier.

Multiply 2348 by 5.

77. Multiply each and every number in the multiplicand by the multiplier. Begin thus: 5 times 8 are 40, or 4 tens and 0 units. Write the figure 0 under the figure of units and carry the 4 tens. 5 times 4 are 20 and 4 are 24, or 2 hundreds and 4 tens. Write the figure 4 under the tens of the multiplicand and carry the 2. 5 times 3 are 15 and 2 are 17, or 1 thousand and 7 hundreds. Write the figure 7 under the column of hundreds and carry the figure 1. 5 times 2 are 10 and 1 are 11. There being no more figures in the multiplicand, write down the number found by the last multiplication.

OPERATION.

$$\begin{array}{r} 2348 \\ 5 \\ \hline 11740 \end{array}$$

EXAMPLES.

1. Multiply 12345 by 3.   *Ans.* 37035.
2. Multiply 23456 by 4.   *Ans.* 93824.
3. Multiply 34567 by 5.   *Ans.* 172835.
4. Multiply 45678 by 6.   *Ans.* 274068.
5. Multiply 56789 by 7.   *Ans.* 397523.
6. Multiply 67891 by 8.   *Ans.* 543128.
7. Multiply 78912 by 9.   *Ans.* 710208.
8. Multiply 89123 by 3.   *Ans.* 267869.
9. Multiply 91234 by 4.   *Ans.* 364936.
10. Multiply 1234567 by 2.   *Ans.* 2469134.

5*

11. Multiply 123456789 by 2.   *Ans.* 246913578.

12. Multiply 132546789 by 3.   *Ans.* 370370367.

13. Multiply 312546798 by 4.   *Ans.* 493827156.

14. Multiply 123546798 by 5.   *Ans.* 617733990.

15. Multiply 987654321 by 6.   *Ans.* 5925925926.

16. Multiply 897645123 by 7.   *Ans.* 6283515861.

17. Multiply 987612345 by 8.   *Ans.* 7900898760.

18. Multiply 123456987 by 9.   *Ans.* 1111112883.

19. Multiply 123456789 by 9.   *Ans.* 1111111101.

20. Multiply 123456789 by 8.   *Ans.* 987654312.

21. Multiply 2463516789 by 7. *Ans.* 17244617523.

22. Multiply 123456798 by 6.   *Ans.* 740740788.

23. Multiply 213546789 by 7.   *Ans.* 1494827523.

24. Multiply 312645879 by 5.   *Ans.* 1563229395·

25. Multiply 897543261 by 4.   *Ans.* 3590173044.

26. Multiply 246891357 by 3.   *Ans.* 740674071.

27. Multiply 462893157 by 2.   *Ans.* 925786314.

28. Multiply 1234579123 by 3. *Ans.* 3703737369.

29. Multiply 123456789123 by 4.
                              Ans. 493827156492.

30. Multiply 312654789312 by 5.
                             Ans. 1563273946560.

31. Multiply 3124658971321 by 6.
                            Ans. 18747953827926.

32. Multiply 18765432897654 by 7.
                           Ans. 131358030283578.

33. Multiply 876154389762541 by 8.

     Ans. 7009235118100328.

34. Multiply 7685143987614521 by 9.

     Ans. 69166295888530689.

35. Multiply 263748591221314151 by 2.

     Ans. 527497182442628302.

36. Multiply 152535455565758595 by 3.

     Ans.

37. Multiply 223344556677889911 by 4.

     Ans.

38. Multiply 123321213312132231 by 5.

     Ans.

39. Multiply 3456435645634435654 by 6.

     Ans.

40. Multiply 8767789888946398498 by 7.

     Ans.

41. Multiply 982938498999868977998 by 8.

     Ans.

42. Multiply 2864988767478394987964 by 9.

     Ans.

43. Multiply 98989898989897969587898 by 8.

     Ans.

LESSON IV.

CASE III.

78. When there are several figures in both multiplier and multiplicand.

Multiply 123456 by 234.

1. Multiply all the figures of the multiplicand by the figure representing the units in the multiplier. Thus, 1st, 4 times 6 are 24; write the figure 4 under the column of units, and carry the 2. 2d, 4 times 5 are 20 and 2 are 22; write the figure 2 and carry 2. 3d, 4 times 4 are 16 and 2 are 18; write the figure 8 and carry 1. 4th, 4 times 3 are 12 and 1 are 13; write the figure 3 and carry 1. 5th, 4 times 2 are 8 and 1 are 9; write the figure 9. In this last multiplication we have no figure representing the next higher order, therefore we have nothing to carry. 6th, 4 times 1 are 4; write the 4 a little to the left of the 9.

OPERATION.

```
  123456
     234
─────────
  493824
  370368
  246912
─────────
28888704
```

2. Begin with the figure representing the tens, which in this case is 3. Tens by units give tens. 1st, 3 times 6 are 18; write the 8 under the tens, and carry 1. Tens by tens give hundreds. 2d, 3 times 5 are 15 and 1 are 16; write the figure 6 under the column of hundreds and carry 1. Tens by hundreds give thousands. 3d, 3 times 4 are 12 and 1 are 13; write the figure 3 under the column of thousands and carry 1. Tens by thousands give tens of thousands. 4th, 3 times 3 are 9 and 1 are 10; write the figure 0 under the column of tens of thousands and carry 1. Tens by tens of thousands give hundreds of thou-

sands.   5th, 3 times 2 are 6 and 1 are 7.   In this case we have nothing to carry.   Tens by hundreds of thousands give ten hundreds of thousands, or millions.   6th, 3 times 1 are 3; write the figure 3 a little to the left of the figure 7.

   3. Begin with the figure 2 of the multiplier which represents hundreds.   Hundreds by units give hundreds.   1st, 2 times 6 are 12; write the figure 2 under the column of hundreds and carry 1.   Hundreds by tens give tens of hundreds, or thousands. 2d, 2 times 5 are 10 and 1 are 11; write the figure 1 under the column of thousands and carry 1.   Hundreds by hundreds give tens of thousands.   3d, 2 times 4 are 8 and 1 are 9; write the figure 9 under the column of tens of thousands.   In this case we have nothing to carry.   Hundreds by thousands give hundreds of thousands.   4th, 2 times 3 are 6; write the figure 6 under the column of hundreds of thousands.   In this case we have nothing to carry. Hundreds by tens of thousands give ten hundreds of thousands, or millions.   5th, 2 times 2 are 4; write the figure 4 under the column of millions.   In this case also we have nothing to carry.   Hundreds by hundreds of thousands give tens of millions.   6th, 2 times 1 are 2; write the figure 2 a little to the left of the figure 4.

   4. Draw a line under the several products, as in the example, and add the several results as they are found.

   The addition of the several results will be the answer required.   *Ans.* 28888704.

79. From which we derive the following general

RULE.

I. *Write the multiplier under the multiplicand, so that the units fall under units, tens under tens, hundreds under hundreds, etc., etc.*

II. *Multiply all the figures of the multiplicand by the units in the multiplier, write down the right-hand figure and carry the left-hand one to be added to the product of the next figure with the units.*

III. *Multiply all the figures of the multiplicand by the figure in the multiplier representing tens. Write down the right-hand figure in every case in the column of tens and carry the left one.*

IV. *Proceed in this way until you have multiplied the multiplicand by each and every figure of the multiplier.*

V. *The sum of the several products will be the answer required.*

80. NOTE.—When there are 0's between the figures of the multiplier, pass over them in the operation, but be careful to give the figure to the left of the 0 its proper place in the product.

PROOF.

81. To prove multiplication, multiply the multiplier by the multiplicand, for the reason that in whatever order the figures may be multiplied the result will be the same.

EXAMPLE.

$$3 \times 2 \times 4 = 24 \qquad 4 \times 2 \times 3 = 24$$
$$2 \times 3 \times 4 = 24 \qquad 3 \times 4 \times 2 = 24$$

etc., etc., etc.

1. Multiply 12345 by 12. *Ans.* 148140.
2. Multiply 12345 by 123. *Ans.* 1518435.
3. Multiply 23456 by 23. *Ans.* 539488.
4. Multiply 23456 by 123. *Ans.* 2885088.
5. Multiply 345678 by 1234. *Ans.* 426566652.
6. Multiply 345678 by 12. *Ans.* 4148136.
7. Multiply 345678 by 345. *Ans.* 119258910.
8. Multiply 456789 by 435. *Ans.* 198703215.
9. Multiply 456789 by 543. *Ans.* 248636427.
10. Multiply 4576982 by 5432. *Ans.* 24862166224.
11. Multiply 4567289 by 12345.

Ans. 45383182705.

12. Multiply 123456789 by 12. *Ans.* 1481481468.
13. Multiply 123456789 by 123.

Ans. 15185185047.

14. Multiply 123456789 by 21. *Ans.* 2592592569.
15. Multiply 123456789 by 321.

Ans. 39629629269.

16. Multiply 987654321 by 567.

Ans. 559999990007.

17. Multiply 987654321 by 657.

Ans. 648848888897.

18. Multiply 987654321 by 765.

Ans. 755555554565.

19. Multiply 987654321 by 7065.

Ans. 687777777786.

20. Multiply 8765432 by 70605.

Ans. 618883326360.

21. Multiply 23546798 by 789. *Ans.* 18578423622.

22. Multiply 23546798 by 7089.

Ans. 166923251022.

23. Multiply 23546798 by 70809.

Ans. 183885219582.

24. Multiply 123456789 by 12304.

Ans. 1519012331856.

25. Multiply 123456789 by 1034.

Ans. 127654319826.

26. Multiply 12345678 by 102304.

Ans. 1263012334856.

27. Multiply 23456789 by 1020304.

Ans. 23933055643856.

28. Multiply 187654321 by 90809.

Ans. 17040701235689.

29. Multiply 123456789 by 123456.

Ans. 12686058042784.

30. Multiply 24356879 by 78901.

Ans. 1921782189979.

31. Multiply 12387642 by 38976.

Ans. 482820734592.

32. Multiply 18764321 by 12345.

Ans. 231645542745.

33. Multiply 1897643 by 2468. *Ans.* 4683382924.

LESSON V.

34. Multiply one thousand four hundred and fifty-nine by twenty-three. *Ans.* 33557.

35. Multiply three hundred and seventy-nine thousand eight hundred and forty-seven by two hundred and thirty-eight. *Ans.* 90403586.

36. Multiply three hundred and forty-one thousand nine hundred and thirty-two by six hundred and eight. *Ans.* 207894656.

37. Multiply seven hundred and sixty-five thousand three hundred and twenty-five by two hundred and eighty-five. *Ans.* 218117625.

38. Multiply seven million nine hundred and forty-five thousand three hundred and one by two thousand and forty-nine. *Ans.* 16279921749.

39. Multiply eighty-seven thousand six hundred and twenty-four by two hundred and two.

Ans. 17700048.

40. A steam-car travels 360 miles a day, how far will it travel in 365 days? *Ans.* 131400.

41. One hogshead of sugar weighs 462 lbs., how many pounds in 263 hogsheads? *Ans.* 121506.

42. An army of 29286 receives 203 dollars as their annual pay; what is the amount paid the whole army. *Ans.* $5945058.

LESSON VI.

CASE IV.

72. To multiply by a composite number.

73. A composite number is the product of two or more prime numbers. 6 is a composite number, because it is composed of the factors 2 and 3.

$$2 \times 3 = 6$$

74. A prime number is one which can be divided by no number but itself and unity. They are 1, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc., etc.

6

EXAMPLE.

OPERATION.

Multiply 236 by 12.

236
2
———
472
2
———
944
3
———
2832

The prime factors of 12 are 2, 2, and 3.   First multiply 236 by 2, which gives 472 for the product. Multiply this product by the next factor, 2, which gives 944 for the product.   Multiply this product by the other factor 3, which gives 2832 for the answer.

75. This method is rather too long.   Better adopt the rule under Case III.

EXAMPLES.

1. Multiply 2674 by 6.   *Ans.* 16044.

2. Multiply 1358 by 12.   *Ans.* 16296.

3. Multiply 8531 by 14.   *Ans.* 119434.

4. Multiply 6428 by 18.   *Ans.* 115704.

5. Multiply 4628 by 16.   *Ans.* 74048.

6. Multiply 2648 by 9.   *Ans.* 23832.

7. Multiply 8462 by 24.   *Ans.* 203088.

8. Multiply 2648 by 32.   *Ans.* 84736.

LESSON VII.

CASE III.

76. To multiply by 10, or 100, or 1000, etc., etc.

EXAMPLE.

1. Multiply 236 by 10.

Annex as many 0's to the product as there are 0's in the multiplier.

OPERATION.
```
   236
    10
  ————
  2360
```

2. Multiply 123 by 10.   *Ans.* 1230.
3. Multiply 1234 by 10.   *Ans.* 12340.
4. Multiply 123 by 100.   *Ans.* 12300.
5. Multiply 1234 by 100.   *Ans.* 123400.
6. Multiply 12345 by 10.   *Ans.* 123450.
7. Multiply 12345 by 100.   *Ans.* 1234500.
8. Multiply 12345 by 1000.   *Ans.* 12345000.
9. Multiply 2345 by 1000.   *Ans.* 2345000.
10. Multiply 12345 by 10000.   *Ans.* 123450000.
11. Multiply 123 by 100000.   *Ans.* 12300000.
12. Multiply 1234 by 100000.   *Ans.* 123400000.
13. Multiply 1876 by 100000.   *Ans.* 187600000.
14. Multiply 1867 by 100000.   *Ans.* 186700000.
15. Multiply 12893 by 10.   *Ans.* 128930.
16. Multiply 18293 by 100.   *Ans.* 1829300.
17. Multiply 82193 by 1000.   *Ans.* 82193000.
18. Multiply 1897 by 100000.   *Ans.* 189700000.
19. Multiply 18862 by 1000.   *Ans.* 18862000.
20. Multiply 123456 by 100.   *Ans.* 12345600.
21. Multiply 1234 by 10000.   *Ans.* 12340000.
22. Multiply 123 by 100000.   *Ans.* 12300000.
23. Multiply 12 by 1000000.   *Ans.* 12000000.
24. Multiply 123456 by 10000000.   *Ans.*
25. Multiply 654321 by 100000000. *Ans.*

LESSON VIII.

CASE VI.

77. In which the multiplier is composed of significant figures followed by 0's.

RULÈ.

Multiply the multiplicand of the significant figures and add to the result as many 0's as there are in the multiplier.

EXAMPLE.

Multiply 1234 by 1600.

1st. Multiply the multiplicand by 16 and to the product add two 0's.

OPERATION.

```
   1234
   1600
  ─────
   7344
  1234
  ─────
1968400
```

1.	Multiply 1234 by 1200.	*Ans.* 1480800.
2.	Multiply 2345 by 230.	*Ans.* 539350.
3.	Multiply 2345 by 2300.	*Ans.* 5393500.
4.	Multiply 34567 by 3400.	*Ans.* 117527800.
5.	Multiply 4567 by 34000.	*Ans.* 155278000.
6.	Multiply 5678 by 4500.	*Ans.* 25551000.
7.	Multiply 6758 by 45000.	*Ans.* 304110000.
8.	Multiply 8999 by 120.	*Ans.* 1079880.
9.	Multiply 9998 by 129300.	*Ans.* 1292741400.
10.	Multiply 8764 by 3400.	*Ans.* 29797600.
11.	Multiply 7648 by 3450.	*Ans.* 26385600.

12. Multiply 6478 by 12000.   *Ans.* 77736000.

13. Multiply 7896 by 3400.   *Ans.* 26846400.

14. Multiply 78912 by 35000.   *Ans.* 2761920000.

15. Multiply 8912 by 5600.   *Ans.* 49907200.

16. Multiply 876432 by 12000. *Ans.* 10517184000.

17. Multiply 12345 by 1234000.

Ans. 15233730000.

18. Multiply 123456 by 123450000.

Ans. 15240643200000.

19. Multiply 1357698 by 120000.

Ans. 162923760000.

20. Multiply 12345678 by 13130000.

Ans. 162098752140000.

21. Multiply 98989898 by 9900000.

Ans. 979999990200000.

LESSON IX.

CASE VII.

78. In which 0's are found between the significant figures of the multiplier and multiplicand.

RULE.

Multiply by the significant figures only, care being taken to give to each and every 0 its proper position among the significant figures in the product.

ι*

Multiply 1023 by 203.

OPERATION.

1st. Multiply all the figures in the multiplicand by 3. Thus 3 times 3 are 9.   3 times 2 are 6.   3 times 0 are 0.   3 times 1 are 3.

2nd. Write the 0 under the column of tens and multiply by the next significant figure in the multiplier. Thus, 2 times 3 are 6.   2 times 2 are 4.   2 times 0 are 0.   2 times 1 are 2.

```
  1023
   203
 ─────
  3069
 20460
 ─────
207669
```

Add the several products for the answer.

1. Multiply 20304 by 201.   *Ans.* 4081104.

2. Multiply 302403 by 3020.   *Ans.* 913257060.

3. Multiply 50607 by 3040.   *Ans.* 153845280.

4. Multiply 60708 by 405.   *Ans.* 24586740.

5. Multiply 60708 by 4050.   *Ans.* 245867400.

6. Multiply 70809 by 5060.   *Ans.* 358293540.

7. Multiply 70809 by 605.   *Ans.* 42839445.

8. Multiply 8090 by 708.   *Ans.* 5727720.

9. Multiply 9080 by 8070.   *Ans.* 73275600.

10. Multiply 9809 by 78080.   *Ans.* 765886720.

11. Multiply 120980 by 70380.   *Ans.* 8514572400.

12. Multiply 20304 by 20304.   *Ans.* 412252416.

LESSON X.

CASE VIII.

79. In which 0's are found after the significant figures of both multiplier and multiplicand.

RULE.

Multiply the significant figures, and add to the product as many 0's as there are in the multiplier and multiplicand together.

EXAMPLE.

Multiply 1200 by 1200.

	OPERATION
12 times 12 are 144. Write 144 for your product. Annex or add to it as many 0's as you find in the multiplicand and multiplier. In this case four 0's. The answer is 1440000.	1200 1200 ——— 1440000

EXAMPLES.

1. Multiply 200 by 30.   *Ans.* 6000.
2. Multiply 320 by 300.   *Ans.* 96000.
3. Multiply 300 by 160.   *Ans.* 48000.
4. Multiply 340 by 150.   *Ans.* 51000.
5. Multiply 3450 by 1200.   *Ans.* 4140000.
6. Multiply 4500 by 340.   *Ans.* 1530000.
7. Multiply 6700 by 3400.   *Ans.* 22780000.
8. Multiply 8900 by 450.   *Ans.* 4005000.
9. Multiply 9800 by 4500.   *Ans.* 441000000.
10. Multiply 7600 by 670.   *Ans.* 5092000.
11. Multiply 5400 by 6700.   *Ans.* 36180000.

12. Multiply 3200 by 780.   *Ans.* 2496000.

13. Multiply 2100 by 7800.   *Ans.* 16380000.

14. Multiply 12000 by 8900.   *Ans.* 106800000.

15. Multiply 123000 by 9000.   *Ans.* 1107000000.

16. Multiply 12340000 by 10000.

                  Ans. 123400000000.

17. Multiply 1234500000 by 20000.

                Ans. 24690000000000.

CHAPTER V.

DIVISION.

LESSON I.

80. DIVISION is the process by which we find how many times one number is contained in another.

81. The number by which we divide is called the *divisor*.

82. The number divided is called the *dividend*.

83. The number of times the divisor is contained in the dividend is called the *quotient*.

84. If the dividend does not contain the divisor an exact number of times, the excess is called the *remainder*.

EXAMPLE.

A boy having 40 marbles, wishes to divide them equally between his 4 brothers. How many marbles does each one receive?

ILLUSTRATION.—The answer is a number which, being multiplied by 4, would give 40 for the product, or each boy will receive as many marbles as the number of boys (4), is contained in 40, which is 10 times, because 10 times 4 are 40.

85. The sign of division is a horizontal bar, with a dot below and a dot above it; thus, $\div$. It shows that the number before which it is placed is to be divided by the one after it.

86. A horizontal bar between two numbers, as $\frac{12}{2}$, indicates division, and shows that 12, the number above the line, is to be divided by the one below the line. Such numbers are called *fractions*.

87. The remainder, if any, must always be less than the divisor.

TABLE OF DIVISION.

$2\div2=\ 1$	$3\div3=\ 1$	$4\div4=\ 1$	$5\div5=\ 1$	$6\div6=\ 1$	$7\div7=\ 1$
$4\div2=\ 2$	$6\div3=\ 2$	$8\div4=\ 2$	$10\div5=\ 2$	$12\div6=\ 2$	$14\div7=\ 2$
$6\div2=\ 3$	$9\div3=\ 3$	$12\div4=\ 3$	$15\div5=\ 3$	$18\div6=\ 3$	$21\div7=\ 3$
$8\div2=\ 4$	$12\div3=\ 4$	$16\div4=\ 4$	$20\div5=\ 4$	$24\div6=\ 4$	$28\div7=\ 4$
$10\div2=\ 5$	$15\div3=\ 5$	$20\div4=\ 5$	$25\div5=\ 5$	$30\div6=\ 5$	$35\div7=\ 5$
$12\div2=\ 6$	$18\div3=\ 6$	$24\div4=\ 6$	$30\div5=\ 6$	$36\div6=\ 6$	$42\div7=\ 6$
$14\div2=\ 7$	$21\div3=\ 7$	$28\div4=\ 7$	$35\div5=\ 7$	$42\div6=\ 7$	$49\div7=\ 7$
$16\div2=\ 8$	$24\div3=\ 8$	$32\div4=\ 8$	$40\div5=\ 8$	$48\div6=\ 8$	$56\div7=\ 8$
$18\div2=\ 9$	$27\div3=\ 9$	$36\div4=\ 9$	$45\div5=\ 9$	$54\div6=\ 9$	$63\div7=\ 9$
$20\div2=10$	$30\div3=10$	$40\div4=10$	$50\div5=10$	$60\div6=10$	$70\div7=10$
$22\div2=11$	$33\div3=11$	$44\div4=11$	$55\div5=11$	$66\div6=11$	$77\div7=11$
$24\div2=12$	$36\div3=12$	$48\div4=12$	$60\div5=12$	$72\div6=12$	$84\div7=12$

$8\div8=\ 1$	$9\div9=\ 1$	$10\div10=\ 1$	$11\div11=\ 1$	$12\div12=\ 1$
$16\div8=\ 2$	$18\div9=\ 2$	$20\div10=\ 2$	$22\div11=\ 2$	$24\div12=\ 2$
$24\div8=\ 3$	$27\div9=\ 3$	$30\div10=\ 3$	$33\div11=\ 3$	$36\div12=\ 3$
$32\div8=\ 4$	$36\div9=\ 4$	$40\div10=\ 4$	$44\div11=\ 4$	$48\div12=\ 4$
$40\div8=\ 5$	$45\div9=\ 5$	$50\div10=\ 5$	$55\div11=\ 5$	$60\div12=\ 5$
$48\div8=\ 6$	$54\div9=\ 6$	$60\div10=\ 6$	$66\div11=\ 6$	$72\div12=\ 6$
$56\div8=\ 7$	$63\div9=\ 7$	$70\div10=\ 7$	$77\div11=\ 7$	$84\div12=\ 7$
$64\div8=\ 8$	$72\div9=\ 8$	$80\div10=\ 8$	$88\div11=\ 8$	$96\div12=\ 8$
$72\div8=\ 9$	$81\div9=\ 9$	$90\div10=\ 9$	$99\div11=\ 9$	$108\div12=\ 9$
$80\div8=10$	$90\div9=10$	$100\div10=10$	$110\div11=10$	$120\div12=10$
$88\div8=11$	$99\div9=11$	$110\div10=11$	$121\div11=11$	$132\div12=11$
$96\div8=12$	$108\div9=12$	$120\div10=12$	$132\div11=12$	$144\div12=12$

LESSON II.

EXAMPLES.

1. In 120 how many times 10.   *Ans.* 12 times.

2. 2 in 8 how many times? in 4? in 6? in 12? in 14? in 16? in 20? in 24?

3. A man gave 24 dollars to 8 boys. How many dollars did each boy receive?   *Ans.* 3.

4. If 10 hats cost 40 dollars what costs 1 hat?

5. Divide 25 oranges among 5 young ladies. How many does each one receive?   *Ans.* 5.

6. I gave 132 dollars for 12 chairs. How much pr. chair did they cost?   *Ans.* 11 dollars.

7. Paid 144 dollars to 12 men. How much did each man receive?   *Ans.* 12 dollars.

8. If a man earn 40 dollars in 5 weeks, how much is that pr. week.   *Ans.* 8 dollars.

9. If 21 pounds of butter cost 7 dollars, how many pounds could you buy for 1 dollar.

Ans. 3 pounds.

LESSON III.

EXAMPLES.

10. 3 in 21 how many times? in 24? in 36?

11. 7 in 21 how many times? in 42? in 56?

12. 8 in 64 how many times? in 16? in 24?

13. 9 in 27 how many times? in 36? in 45?

14. 10 in 20 how many times? in 30? in 40?

15. 11 in 22 how many times? in 44? in 88?

16. 12 in 24 how many times? in 36? in 48? in 60?

17. 3 in 36 how many times? in 36? in 48? in 60?

18. 4 in 48 how many times? in 36? in 48? in 24?

19. 5 in 60 how many times? in 50? in 25? in 45?

20. 6 in 60 how many times? in 48? in 54? in 72?

---

88. SHORT DIVISION,
or when the divisor is not greater than 12.

EXAMPLE.
Divide 8975 dollars equally between 5 men.

OPERATION.
Divisor. 5) 8975 Dividend.

1795 Quotient.

We first find how many times 5, the divisor, is contained in 8, the first figure of the dividend, which is thousands, and find it to be 1 time and 3 thousands over. Write the figure 1 under the figure 8, and carry the 3 thousands remaining to the next figure of the dividend. 3 thousands and 9 hundreds make 39 hundreds. Divide 39 hundreds by 5: 5 into 39, 7 times and 4 hundreds over. Write the figure 7 under the figure 9, and carry the 4 hundreds to the next figure. 4 hundreds and 7 tens make 47 tens. Divide 47 tens by 5: 5 into 47, 9 times and two tens

over. Write the figure 9 under the figure 7, and carry the two tens to the next column. 2 tens and 5 units make 25 units. Divide the 25 units by 5; 5 into 25, 5 times and no remainder. Write the figure 5 under the 5 of the dividend. The figures thus written make the answer. *Ans.* $1795 each.

RULE.

89. I. *Write the Divisor to the left of the Dividend with a curved line separating them.*

II. *Draw a horizontal line under the Dividend.*

III. *Then beginning at the left of the Dividend, find how many times the Divisor is contained in the first figure of the Dividend.*

IV. *If the first figure of the Dividend will not contain the Divisor, take the first two figures, and write the quotient under the Dividend.*

V. *If there be any remainder consider it as prefixed to the next figure of the Dividend.*

VI. *Divide the number thus formed by the Divisor and write the quotient under the dividend.*

VII. *If the number thus formed will not contain the Divisor write an 0 in the quotient and annex another figure.*

VIII. *Continue in this way until the last figure is reached.*

IX. *When there is a remainder write it a little to the right of the quotient.*

X. *It may be written in the form of a fraction, the remainder above the Divisor with a line drawn between them.*

90. FIRST METHOD OF PROOF.—Multiply the Quotient by the Divisor and to the product add the remainder if any, and the result obtained will be the Dividend, if the work is correct.

EXAMPLES.

2. Divide 8976 by 2.	*Ans.* 4488.
3. Divide 8974 by 2.	*Ans.* 4487.
4. Divide 3993 by 3.	*Ans.* 1331.
5. Divide 319133 by 3.	*Ans.* 106377$\frac{2}{3}$.
6. Divide 24684 by 4.	*Ans.* 6171.
7. Divide 24168 by 4.	*Ans.* 6042.
8. Divide 125125 by 5.	*Ans.* 25025.
9. Divide 125525 by 5.	*Ans.* 25105.
10. Divide 896896 by 7.	*Ans.* 128128.
11. Divide 986896 by 7.	*Ans.* 140985$\frac{1}{7}$.
12. Divide 70011608 by 8.	*Ans.* 8751451.
13. Divide 494580 by 12.	*Ans.* 41215.
14. Divide 514503 by 9.	*Ans.* 57167.
15. Divide 683489 by 9.	*Ans.* 75943$\frac{5}{9}$.
16. Divide 246892 by 8.	*Ans.* 30861$\frac{1}{2}$.
17. Divide 368492 by 4.	*Ans.* 92123.
18. Divide 634829 by 4.	*Ans.* 158707$\frac{1}{4}$.
19. Divide 436298 by 6.	*Ans.* 72716$\frac{1}{3}$.
20. Divide 580494 by 8.	*Ans.* 72561$\frac{3}{4}$.
21. Divide 684284 by 8.	*Ans.* 85535$\frac{1}{2}$.
22. Divide 864824 by 9.	*Ans.* 96091$\frac{5}{9}$.
23. Divide 123456 by 12.	*Ans.* 10288.
24. Divide 123456 by 11.	*Ans.* 11223$\frac{3}{11}$.
25. Divide 12345678 by 12.	*Ans.* 1028806$\frac{1}{2}$.

7

LESSON IV.

EXAMPLES.

26. Divide 23874 apples among 12 boys. How many will each boy receive.   *Ans.* 1989½ apples.

27. If 4 farthings make one penny, how many pennies in 242424 farthings.   *Ans.* 60606 pennies.

28. A plantation was sold for 2469876 dollars, and the proceeds were divided equally among 9 children. How much was each child's share.

<div align="right">

Ans. $274430.66⅔.

</div>

29. Divide 24680 acres of land equally among 8 persons. How many acres does each one receive?

<div align="right">

Ans. 3085 acres.

</div>

30. A school containing 368 pupils has 8 teachers. How many scholars to a teacher?   *Ans.* 46 pupils.

31. If 3 feet make 1 yard, how many yards in 15-280 feet.   *Ans.* 5093⅓ yards.

32. If 12 pence make 1 shilling, how many shillings in 121212 pence?   *Ans.* 10101 shillings.

33. 12 months make a year. How many years in 12012012 months.   *Ans.* 1001001 years.

34. 7 days make a week. How many weeks in 77077 days.   *Ans.* 11011.

35. 12 articles make a dozen. How many dozen in 123456780 articles?   *Ans.* 10288065.

LESSON V.

91.—LONG DIVISION,

or Division in which the Divisor is greater than 12.

EXAMPLE.

1. A gentleman having 9683425 acres of ground, planted a colony of 122 persons on it, and divided the land equally among them. How many acres did each one receive?

Write the divisor on the left of the dividend with a curved line between them.	OPERATION

$$122)9683425(79372$$

$$854$$

$$\overline{1143}$$
$$1098$$

On the right of the dividend draw another curved line to separate it from the quotient. Count the figures in the divisor and point off as many in the dividend begin- ning at the left. Find how many times the divisor is con- tained in the figures pointed off in the dividend and place the result in the curved line for the first figure of the quo- tient. If the corresponding

$$\overline{454}$$
$$366$$

$$\overline{882}$$
$$854$$

$$\overline{285}$$
$$244$$

$$\overline{41}$$

number of figures in the dividend are not large enough to contain the divisor, point off another fig- ure. Multiply the divisor by the quotient found and place the result under the figures pointed off. Sub- tract the result from the figures pointed off and take

down the next figure of the dividend. Enquire how many times the divisor is contained in the new dividend, and write the result in the quotient. Multiply and subtract as before. If the remainder together with the figure taken from the dividend are too small to contain the divisor, write a cipher (0) in the quotient and take down the next figure.

92. The above example is read thus: 122 into 968, 7 times. Multiply the Divisor 122 by 7 and write the product under the figures 968, and subtract. The remainder is 114, to which the next figure (3) of the Dividend is annexed making 1143. 122 in 1143, 9 times. Multiply the Divisor by 9 and subtract the product from 1143. 45 remains to which annex the next figure (4) of the Dividend, making 454. 122 into 454, 3 times. Multiply the Divisor by 3 and subtract the product from 454. 88 remains to which annex the next figure (2) of the Dividend making 882. 122 into 882, 7 times. Multiply the Divisor by 7 and subtract the product from 882. 28 remains to which annex the next figure (5) of the Dividend making 285. 122 into 285, 2 times. Multiply the Divisor by 2 and subtract the result from 285; 41 remains. The Division is now complete. The answer is 79372 and 41 remaining.

The answer may be written $79372\frac{41}{122}$.

93. To prove the example multiply the quotient (79372) by the divisor (122) and to the product add the remainder 41. The sum must be equal to the Dividend 9683425.

EXAMPLES.

1. Divide 24689 by 12. *Ans.* $2057\frac{5}{12}$.
2. Divide 123456 by 123. *Ans.* $1003\frac{87}{123}$.
3. Divide 234567 by 123. *Ans.* $1907\frac{6}{123}$.
4. Divide 345678 by 123. *Ans.* $2810\frac{48}{123}$.
5. Divide 435678 by 1234. *Ans.* $353\frac{716}{1234}$.
6. Divide 249648 by 12. *Ans.* 20804.
7. Divide 289246 by 123. *Ans.* $2351\frac{73}{123}$.
8. Divide 184392 by 234. *Ans.* 788.
9. Divide 481293 by 234. *Ans.* $2056\frac{189}{234}$.
10. Divide 418239 by 345. *Ans.* $1212\frac{99}{345}$.
11. Divide 419293 by 45. *Ans.* $9317\frac{28}{45}$.
12. Divide 196247 by 45. *Ans.* $4361\frac{2}{45}$.
13. Divide 213648 by 456. *Ans.* $468\frac{240}{456}$.
14. Divide 976413 by 456. *Ans.* $2141\frac{117}{456}$.
15. Divide 817924 by 56. *Ans.* $14605\frac{44}{56}$.
16. Divide 213876 by 56. *Ans.* $3819\frac{12}{56}$.
17. Divide 142224 by 567. *Ans.* $250\frac{474}{567}$.
18. Divide 224176 by 567. *Ans.* $395\frac{211}{567}$.
19. Divide 1869723 by 67. *Ans.* $2789\frac{60}{67}$.
20. Divide 184976 by 67. *Ans.* $2760\frac{56}{67}$.
21. Divide 184976 by 678. *Ans.* $274\frac{392}{678}$.
22. Divide 247983 by 678. *Ans.* $365\frac{513}{678}$.
23. Divide 4683214 by 78. *Ans.* $60041\frac{16}{78}$.
24. Divide 98764321 by 78. *Ans.* $1266209\frac{19}{78}$.
25. Divide 24876321 by 789. *Ans.* $31528\frac{729}{789}$.
26. Divide 18764972 by 789. *Ans.* $23780\frac{152}{789}$.
27. Divide 2487623 by 89. *Ans.* $27838\frac{41}{89}$.
28. Divide 1876491 by 89. *Ans.* $21084\frac{15}{89}$.
29. Divide 1876432 by 12345. *Ans.* $151\frac{1237}{12345}$.

7*

LESSON VI.

32. Asia has a population of 696725000. **Louisiana** has a population of 708002. How many **times** more population has Asia than the State of **Louisiana**? *Ans.* $984\frac{51032}{708002}$.

33. Australia has a population of 1431000. Rhode Island, 174621. How many times greater is the population of Australia than that of Rhode Island?

Ans. $8\frac{34032}{174621}$ times.

34. Louisiana has an area of 46431 square miles. If it was divided equally among the 48 parishes in the State, what would be the area of each parish?

Ans. $967\frac{15}{48}$ square miles.

35. New York State has 3,880,735 inhabitants, and an area of 47000 square miles. How many inhabitants is that to a square mile? *Ans.* $82\frac{26735}{47000}$.

36. The Middle States have 7571099 inhabitants, and an area of 103440 square miles. How many inhabitants to a square mile? *Ans.* $73\frac{19979}{103440}$.

37. The Southern States, 11 in number, have a population of 8321301 inhabitants and an area of 644551 square miles. If the states were of the same area, how many square miles would each one contain?

If the inhabitants were equally divided between them, how many inhabitants would each State contain?

How many inhabitants to a square mile in the Southern States?

Ans. $58595\frac{6}{11}$ square miles to each State.
       $75648\frac{9}{11}$ inhabitants to each State.
       $102\frac{587099}{644551}$ inhabitants to each sq. mile.

38. Belgium has a population of 4731957 inhabitants and an area of 11313 square miles. How many inhabitants to a square mile?   *Ans.*

39. The Eastern States have a population of 3135282 inhabitants, and an area of 65038 square miles. How many inhabitants to a square mile?

Ans. 48$\frac{13458}{65038}$.

40. A colony of 248 persons purchased 1864358 acres of land. How many acres to each person?

Ans. 7517$\frac{142}{248}$.

41. 5280 feet make one mile.   3 feet make 1 yard. How many yards in a mile?   *Ans.* 1760.

42. 63360 inches make 1 mile.   12 inches make 1 foot.   How many feet in a mile?   *Ans.* 5280.

43. 12 articles make 1 dozen.   How many dozen in 1728 articles?   *Ans.* 144.

45. A gentleman divided 567053 cents among 24 beggars.   How much did each beggar receive?

Ans. 2362$\frac{15}{24}$ cents.

46. A teacher buys 123456789 marbles.   In his school there are 348 boys.   How many marbles could he give to each one of the boys?

Supppose $\frac{1}{2}$ of the boys were absent, how many would those who were present get?

1st Ans. 35476$\frac{309}{318}$.

2d Ans.

LESSON VII.

94. To Divide by a composite number.

EXAMPLES.

1. Divide 240 by 12.

12 is composed of the factors 3 × 2×2. Divide 240 by 3. The quotient is 80. Divide 80 by 2. The quotient is 40. Divide 40 by 2. The quotient is 20. The last quotient (20) is the answer.

OPERATION.

3)240
2)80
2)40
Ans. 20

2. Divide 1230 by 6.	*Ans.* 205.
3. Divide 2340 by 4.	*Ans.* 585.
4. Divide 23460 by 4.	*Ans.* 5865.
5. Divide 1800 by 9.	*Ans.* 200.
6. Divide 3600 by 18.	*Ans.* 200.
7. Divide 1212 by 4.	*Ans.* 303.
8. Divide 2424 by 8.	*Ans.* 303.
9. Divide 36036 by 4.	*Ans.* 9009.
10. Divide 36036 by 9.	*Ans.* 4004.
11. Divide 1240 by 4.	*Ans.* 310.
12. Divide 1240 by 8.	*Ans.* 155.
13. Divide 4500 by 15.	*Ans.* 300.
14. Divide 8888 by 8.	*Ans.* 1111.
15. Divide 6464 by 64.	*Ans.* 101.
16. Divide 12012 by 6.	*Ans.* 2002.
17. Divide 3648 by 6.	*Ans.* 608.
18. Divide 3600 by 12.	*Ans.* 300.
19. Divide 8400 by 14.	*Ans.* 600.
20. Divide 8000 by 16. •	*Ans.* 500.
21. Divide 1620 by 18.	*Ans.* 90.
22. Divide 24200 by 21.	*Ans.* $1152\frac{8}{21}$.

95. To find the true remainder :

RULE.

Multiply all the remainders by all the divisors except its own, beginning at the last and multiplying upwards ; add the first remainder to the sum of the products, and the amount will be the true remainder.

(Since every remainder is multiplied by all the divisors from the one which produced it, the first remainder will have no multiplier.)

EXAMPLE.

Divide 798 by 44.

The factors of 44 are 2×2×11. Dividing by 2 gives 299 and no remainder. Dividing the quotient by 2 gives 199 and 2 remaining. Dividing the quotient by 11 gives 18 and 1 remaining.

To find the true remainder, begin with the last remainder, 1, and multiply it

OPERATION.

$$
\begin{array}{r}
2)\overline{798} \\
2)\overline{399-0} \quad \text{1st rem.} \\
11)\overline{199-1} \quad \text{2d rem.} \\
\overline{18-1} \quad \text{3d rem.}
\end{array}
$$

$$
\begin{array}{l}
1 \times 2 \times 2 = 4 \\
1 \times 2 \quad\;\; = 2 \\
\quad\; 0 \quad\quad = 0 \\
\overline{} \\
\quad\quad\quad\;\; 6
\end{array}
$$

by all the divisors except its own; the result will be 4. Multiply the second remainder by all the divisors except its own; the result will be 2. Adding the results gives 6, to which add the first remainder, 0. The amount is 6, which is the true remainder.

The answer of the sum is 18 and 6 remaining.

<div align="center">

PROOF.

44) 798 (18 and 6 remaining.

44

——

358

352

——

6

</div>

LESSON VIII.

96. When the divisor ends in 0's.

<div align="center">EXAMPLE.</div>

1. Divide 2468 by 10.

We know that to multiply any number by 10, an 0 annexed to the multiplicand will give the required product.   Division is the reverse of multiplication.

OPERATION.

10)246|8
——————
246—8 rem.

If we wish to divide by 10, we must cut off one figure from the right of the dividend.   The remaining figures on the left will be the quotient, and the figure cut off will be the remainder.   The answer in the above example is 246 and 8 remaining.

To divide by 100, 1000, etc., etc., cut off as many figures from the right of the dividend as there are 0's in the divisor.   The figures to the left of the line is the quotient, those on the right the remainder.

2. Divide 12345 by 10.

<div align="right">*Ans.* 1234 and 5 remaining.</div>

3. Divide 23456 by 100.

> *Ans.* 234 and 56 remaining.

4. Divide 34567 by 1000.

> *Ans.* 34 and 567 remaining.

5. Divide 45678 by 10000.

> *Ans.* 4 and 5678 remaining.

6. Divide 987654321 by 1000000.

> *Ans.* 987 and 654321 remaining.

7. Divide 123456789 by 1000000.

> *Ans.* 123 and 456789 remaining.

LESSON IX.

97. To buy or sell articles by the hundreds, thousands, etc.

EXAMPLE.

1. What will 1234567 bricks cost @ $12 per thousand.

OPERATION.

By the thousand means for every 1000. To find the number of thousands in any number, cut off as many figures from the dividend as there are 0's in 1000. Multiply the number by the price, care being taken to cut off as many figures from the result as there are figures cut off in the multiplicand. The figures to the left of the line in the result are dollars, cents, etc., as the case may be. Those to the right of the line are fractional parts of dollars, etc.

$$1234|577$$
$$12$$
$$\overline{}$$
$$2468734$$
$$1234567$$
$$\overline{}$$
$$\$14814|404$$

In the following examples we will treat of dollars only in the multiplier.

2. What will 12345 bricks cost @ $16 per thousand?   *Ans.* $197|520.

3. What will 234567 slates cost @ $25 per thousand?   *Ans.* $5864|175.

4. What will 267433 feet of lumber cost @ $22 per thousand?   *Ans.* $5883|526.

5. What will 34567 bricks cost @ $14 per thousand?   *Ans.* $483|938.

6. What will 123456 shingles cost @ $2 per hundred?   *Ans.* $246|90.

7. What will 654321 shingles cost @ $3 per hundred?   *Ans.* $19629|63

8. What will 8765432 bricks cost @ $2 per hundred?   *Ans.* $175308|64.

9. What will 2345678 bricks cost @ $20 per thousand?   *Ans.* $46913|560.

10. What will 9876543 slates cost @ $20 per thousand?   *Ans.*

11. What will 3456789 slates cost @ $2 per hundred?   *Ans.*

12. What will 5464748 bricks cost @ $3 per hundred?   *Ans.*

13. What will 8474645 bricks cost @ $30 per thousand?   *Ans.*

14. What will 123456789 feet of lumber cost @ $14 per thousand?   *Ans.*

LESSON X.

·98. To divide without writing the result under the dividend; that is, to subtract mentally the result from the dividend.

EXAMPLE.

Divide 934 by 33.

33 into 93, 2 times. Write the figure 2 in the quotient, and multiply. 2 times 3 are 6. Instead of writing the figure 6 under the figure 3 of the dividend, subtract it from the figure

OPERATION.

33)934(28
　　274
　　　10

3, and write the difference under the dividend; thus: 2 times 3 are 6; 6 from 3 I cannot, borrow 10; 6 from 13 leaves 7. 2 times 3 are 6. To the figure 6 I must now add the 1 ten I borrowed. 6 and 1 are 7. 7 from 9 leaves 2. Write the figure 2 under the figure 9. The remainder is 27. Take down the next figure (4) of the dividend. The new dividend is 274.

33 into 274, 8 times. 8 times 3 are 24. 4 from 4 leaves 0. Write the figure 0 under the figure 4. 8 times 3 are 24 and 2 are 26. 26 from 27 leaves 1. Write the figure 1 under the figure 7. The answer is 28 and 10 remaining.

· PROOF.

33	Divisor.
28	Quotient.
264	
66	
924	Product.
10	Remainder.
934	Dividend.

8

EXAMPLES.

1. Divide 6789 by 15. *Ans.* 452 and 9 remaining.
2. Divide 7892 by 14.
           Ans. 563 and 10 remaining.
3. Divide 7829 by 13. *Ans.* 602 and 3 remaining.
4. Divide 2234 by 12. *Ans.* 186 and 2 remaining.
5. Divide 45678 by 16.
           Ans. 2854 and 14 remaining.
6. Divide 50478 by 17.
           Ans. 3322 and 4 remaining.
7. Divide 78645 by 18.
           Ans. 4369 and 3 remaining.
8. Divide 165487 by 19.
           Ans. 8709 and 16 remaining.
9. Divide 76432 by 21.
           Ans. 3639 and 13 remaining.
10. Divide 89765 by 24.
           Ans. 3740 and 5 remaining.
11. Divide 24897 by 28.
           Ans. 889 and 5 remaining.
12. Divide 389764 by 31.
           Ans. 12573 and 1 remaining.
13. Divide 983764 by 211.
           Ans. 4662 and 82 remaining.
14. Divide 876432 by 321.
           Ans. 2730 and 102 remaining.
15. Divide 8976421 by 461.
           Ans. 19471 and 290 remaining.
16. Divide 978642 by 112.
           Ans. 8737 and 98 remaining.

17. Divide 8796431 by 213.

Ans. 41297 and 170 remaining.

18. Divide 8764921 by 318.

Ans. 27562 and 205 remaining.

19. Divide 31314151 by 541.

Ans. 39397 and 374 remaining.

20. Divide 12345678 by 1234.

Ans. 10004 and 742 remaining.

21. Divide 23456789 by 2345.

Ans. 10002 and 2099 remaining.

22. Divide 98765423 by 1345.

Ans. 73431 and 728 remaining.

23. Divide 34567898 by 1456.

Ans. 23741 and 1002 remaining.

24. Divide 123456789 by 1567.

Ans. 78785 and 649 remaining.

25. Divide 98234765 by 1678.

Ans. 58548 and 721 remaining.

26. Divide 82934569 by 2135.

Ans. 38845 and 494 remaining.

27. Divide 98276431 by 2146.

Ans. 45795 and 361 remaining.

28. Divide 897643218 by 2133.

Ans. 420836 and 30 remaining.

29. Divide 876432182 by 8697.

Ans. 100774 and 704 remaining.

30. Divide 764328973 by 213456.

Ans. 3580 and 156293 remaining.

LESSON XI.

99. EXAMPLES IN NUMERATION, NOTATION, AND THE FOUR RULES.

Write in figures the following:

1. Six hundred and ninety-six millions seven hundred and twenty-five thousand nine hundred and eighty-one.

2. Four millions three hundred and eighty-six thousand four hundred and ninety-seven.

3. Four hundred and eleven billions one hundred and fourteen millions one hundred and forty-one thousand one hundred and fourteen.

4. Two hundred and eighty millions nine hundred and fifty-nine thousand five hundred and five.

5. Three millions five hundred and forty-four thousand eight hundred and seventy-seven.

6. Two hundred and forty-three trillions three hundred and twenty-four billions one hundred and sixty-two millions two thousand one hundred and one.

7. Three trillions three billions three millions thirty thousand and three hundred.

8. One quadrillion one thousand.

Read the following numbers:

9.   123456789.

10.   1234567890.

11. 12345678901.

12. 123456789012. .

13.   12345987643.
14.   2345987643.
15.   439587643.
16.   87642182. .
17.   1876431.
18.   8796438.
19.   187643.
20.   278692.
21.   83217.
22.   9462.
23.   873.
24. 123456789101234552.

LESSON XII.

25. The following is the population of the 6 New England States: Maine, 628279; Vermont, 315098; New Hampshire, 326072; Massachusetts, 1231065; Connecticut, 460147; Rhode Island, 174621. How many inhabitants in the New England States? How many more has Massachusetts than Rhode Island. Which is the greater, Maine or Vermont, and by how many? How many more has Massachusetts than Rhode Island and Connecticut together? Than Maine and Vermont together? If Massachusetts was taken from the Eastern States, how many inhabitants would there be in the remaining States?

First Ans. 3135282.

26. The population of the 4 Middle States is as follows: New York, 3880735; Pennsylvania, 2906-115; New Jersey, 672031; Delaware, 112218. How

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many inhabitants in the Middle States? How many more has New York than Delaware? How many more has New York than New Jersey and Delaware together? Has New York more than Delaware and Pennsylvania both? If so, how many? How many times greater is the population of New York than New Jersey?   *First Ans.* 7571099.

27. The population of the Southern States is distributed as follows: Texas, 604215; Virginia, 1219-630; West Virginia, 376688; Florida, 140425; Georgia, 1057286; Alabama, 964201; Mississippi, 791395; Louisiana, 708002; North Carolina, 992622; South Carolina, 703708; Maryland, 687049; District of Columbia, 75080. How many inhabitants in the Southern States? Which has the most population, West Virginia or the District of Columbia? Virginia or West Virginia? Louisiana or Maryland? Maryland or Georgia? and by how many? How many times more inhabitants has Georgia than the District of Columbia? How many more inhabitants have the Southern States than the Middle States?

                    Last Ans. 749202.

28. The population of the Western States is distributed as follows: California, 380015; Oregon, 52465; Minnesota, 173885; Kansas, 107208; Missouri, 1182014; Michigan, 749112; Illinois, 1711951; Wisconsin, 775881; Arkansas, 435450; Iowa, 674-948; Tennessee, 1109801; Ohio, 2339511; Kentucky, 1155684; Indiana, 1350428; Nevada, 6857. How many inhabitants in the Western States? How many more in the Western than in the Southern

States? How many more has Ohio than Louisiana? How many times more has Ohio than Nevada?

First Ans. 12186210.

29. The population of the territories is divided as follows: Dakota, 4839; New Mexico, 93541; Washington, 11578; Utah, 40295; Nebraska, 28842; Colorado, 34197; Indian, 9761. How many inhabitants in the above-named territories? How many inhabitants has Louisiana more than all the above territories? How many times more inhabitants have the Western States than all of the territories?

Last Ans. 54+.

30. Russia has 2100000 square miles; Louisiana, 46431. How many times greater is Russia than Louisiana?

31. The area of the United States is 1147485 square miles; Great Britain and Ireland, 122550. How many times larger is the United States than Great Britain and Ireland?   *Ans.* 9+.

32. I buy from John Smith 73 hogsheads @ 22 dollars per hogshead, and sold them for 25 dollars per hogshead. Do I gain or lose, and how much?

Ans. Gains 219 dollars.

33. J. Brown owes to one man 286 dollars, to another he owes 798 dollars, to a third he owes 685 dollars. John Smith owes him 3452 dollars. If John Smith pays him, and he pays his debts, how much will he have left?   *Ans.* $1683.

34. A butcher bought 2 cows, eight hundred and forty-nine pounds each, at 11 cents a pound, and sold them @ 12 cents a pound. How much did he gain?

Ans. $16.98.

35. The butcher had four partners. What was the share of each partner?   *Ans.* $4.24.

36. The wheel of my car is 4 feet in circumference. If I travel 5280 feet, how many times will the wheel have turned?   *Ans.* 1320 times.

37. The wheel of my sulky is 4 feet in circumference. In one of my recent drives, the wheel turned 22640 times. How many miles did I drive, allowing 5280 feet to the mile?   *Ans.* 17 miles.

38. I sold 180 spools thread @ 20 cents, 10 yards calico @ 15 cents, 46 gallons oil @ 28 cents, 17 pounds tea @ 46 cents, 102 pounds coffee @ 32 cents. How much was my bill? (100 cents make a dollar.)   *Ans.* $90.84.

LESSON XIII.

39. The distance from New York to Canton is 19-450 miles. How many days would a steamer require to reach Canton traveling at the rate of 200 miles a day?   *Ans.* 97+.

40. Bought of Sam. Johnson, 200 acres of land @ $3 per acre, and paid him as follows; 2 ponies valued @ $75 each, 2 small cottages valued @ $100 each, 12 thousand feet of lumber @ $12. How much do I owe him yet?

41. Four boys find a mulberry tree and agree to pick in partnership. In one hour they picked up 18-964. How many should each boy receive as his share?   *Ans.* 4741.

42. A hotel cost 186897 dollars to build it. The expense was equally borne by 18 men. How much did each man have to pay? *Ans.* 10383+.

43. The building of the Chattanooga Railroad will cost about ten million of dollars. There are employed on the road about 2864 men. How much is that to each man? *Ans.* 3491+.

44. What is the amount of the following bill? 200 boxes soap @ $3, 50 kegs lard @ $12, 25 boxes candles @ $8, 4 bbls. sugar @ $12. If I gave the grocer three five-hundred dollar notes, how much change should I get back? *Ans.* $52.

45. Mt. St. Elias is 17860 feet high. How long would it take to climb to the top, at the rate of 3800 feet per hour? *Ans.* 4 hrs. 42 min.

46. Jamestown, Va., was settled in 1607. How old is Jamestown? *Ans.* 262 years old.

47. In the city of London there are 2803034 inhabitants. How many regiments of 980 men each, could London furnish, providing the inhabitants were all men? *Ans.* 2860+ reg.

48. Mt. Everest is twenty-nine thousand and two feet in height. Mt. St. Elias is seventeen thousand eight hundred and sixty. How much higher is Mt. Everest than Mt. St. Elias? *Ans.* 11142.

49. Lake Sirikol is 15600 above the level of the sea. The Dead Sea is 1312 feet below the level of the sea. How high is Lake Sirikol above the Dead Sea? *Ans.* 16912.

50. Lake Itasca is one thousand six hundred and seventy-five feet above the level of the sea. The

Caspian Sea is eighty-three feet below the level of the sea? How much higher is Lake Itasca?

Ans. 1658.

51. Washington died in 1799. La Fayette visited America in 1824. How long after Washington's death? *Ans.* 25 years.

52. Make the following bill:

2 doz. slates	@	20 cts. apiece.
2 boxes chalk	@	75 cts. "
4 inkstands	@	12 cts. "
8 doz. penholders	@	1 ct. "
4 qrs. paper	@	25 cts. "

$8.74

53. Make the following bill:

2 pr. shoes	@	$4 . . . .
6 " boots	@	$8 . . . .
5 " brogans	@	$3 . . . .

$71.00

54. If I give the merchant $150, how much must I get back? *Ans.* $79.00

55. What is the amount of the following bill:

doz. grammars	@	$2 each	. . .
6 " readers	@	$1 each	. . .
9 " spellers	@	$1 each	. . .
7 " geographies	@	$3 each	. . .

Amount $480

56. If I give the merchant $600, how much must he give me in return?   *Ans.* $120.

57. The building of a railroad cost 987654321 dollars. There were 864321 shares sold to realize that sum. How much is each share worth? If the road cost $55000 per mile, how long would it be?

First Ans. 1142.69+.

CHAPTER VI.

QUESTIONS INVOLVING FRACTIONS.

LESSON I.

100. If a unit or a single thing is divided into parts, each part is a fraction of that unit; or, better,

101. A *fraction* is a part of a unit.

102. If an apple is cut into 2 parts, each part is equal to one-half of the apple. One-half is written ($\frac{1}{2}$) with the 1 above the figure 2, and a line drawn between them.

103. The figure below the line is called the *denominator*, because it shows into how many parts the unit is divided.

104. The figure above the line is called the *numerator*, and shows how many of those parts into which the unit is divided, are used. Thus, $\frac{5}{8}$ shows that the unit is divided into 8 equal parts, and that 5 of these parts are taken.

105. If a unit is divided into thirds, fourths, fifths, sixths, etc., and one of the equal parts is taken, the fraction is written thus : $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, etc.

106. When the numerator is equal to the denominator, the fraction represents a whole unit or thing; because, if an object is divided into 10 equal parts, and 10 of those parts are taken, the whole unit is taken.

107. If a unit is divided into 4 equal parts, and 2 of those parts are taken, there remains 2 parts, or $\frac{2}{4}$; because, if a unit is divided into 4, $\frac{4}{4}$ is equal to the unit, and if 2 fourths are taken away, there remains $\frac{2}{4}$.

EXAMPLES.

1. If a unit is divided into 14 equal parts, what is 3 of those parts called?   6 parts?   8 parts?

2. Write in figures three-fourteenths, six-fourteenths, eight-fourteenths.

108. To find the half, third, fourth, etc.. of any number, divide the number by 2, 3, 4, etc.

EXAMPLES.

3. What is one-half of 14?   16?   18?   20?   22? 24?   26?   28?   30?

4. What is one-third of 3?   6?   9?   12?   15? 18?   21?   24?   27?   30?   33?

5. What is one-fourth of 4?   8?   12?   16?   20? 24?   28?   32?   36?   40?

6. What is one-fifth of 5?   15?   25?   35?

LESSON II.

109. When several of the equal parts are taken.

EXAMPLE.

7. What is $\frac{2}{3}$ of 6 ?

SOLUTION.

$2 \times 6 = 12$ $\frac{12}{3} = 4$, the answer.

RULE.

Multiply the whole number by the numerator and divide the result by the denominator.

EXAMPLES.

8. What is $\frac{3}{4}$ of 4 ?   8 ?   12 ?   16 ?   20 ?   24 ?
9. What is $\frac{4}{5}$ of 5 ?   10 ?   15 ?   20 ?   25 ?   30 ?
10. What is $\frac{5}{6}$ of 6 ?   12 ?   18 ?   24 ?   30 ?   36 ?
11. What is $\frac{6}{7}$ of 7 ?   14 ?   21 ?   28 ?   35 ?   42 ?
12. What is $\frac{5}{7}$ of 42 ?   84 ?   168 ?   336 ?   672 ?
13. What is $\frac{5}{8}$ of 8 ?   16 ? 24 ? 32 ? 40 ? 48 ? 56 ?
14. What is $\frac{5}{8}$ of 112 ?   224 ?   448 ?   896 ?
15. What is $\frac{3}{8}$ of 9 ?   81 ?   27 ?   36 ?   45 ?   90 ?
16. James Brown gave $\frac{1}{8}$ of his cake to his sister and $\frac{4}{8}$ to his brother.   What part of the cake did he keep for himself?

SOLUTION.

$\frac{1}{8}$ and $\frac{4}{8}$ are $\frac{5}{8}$, the part given away.

$\frac{8}{8}$ make one whole cake.   From $\frac{8}{8}$ take $\frac{5}{8}$, and there remains (5 from 8) $\frac{3}{8}$.

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17. From one whole one take $\frac{4}{5}$, $\frac{3}{4}$, $\frac{5}{6}$.

18. From a load of pumpkins I gave Jones $\frac{4}{9}$. How many ninths had I remaining. ?

19. In a school, $\frac{2}{12}$ study grammar; $\frac{3}{12}$, geography; $\frac{4}{12}$, history; and the balance study rhetoric. What part of the school study rhetoric ?

20. From a hogshead of sugar I sold $\frac{9}{11}$. How many elevenths remained ?

21. If 1 yard of cloth costs 10 dollars, what will $\frac{1}{2}$ a yard cost ?

<div align="center">SOLUTION.</div>

If 1 yard costs 10 dollars, $\frac{1}{2}$ yard will cost $\frac{1}{2}$ of 10 dollars, or 5 dollars.

22. If 1 acre of land costs $40, what will $\frac{1}{4}$ of an acre cost? $\frac{3}{4}$ of an acre cost ?

23. If 1 acre of land costs $240, what will $\frac{1}{6}$ of an acre cost ? $\frac{2}{6}$ cost ? $\frac{3}{6}$ cost ? $\frac{4}{6}$ cost ?

<div align="center">LESSON III.</div>

24. If $\frac{5}{6}$ of a hogshead of sugar is worth 120 dollars, what is $\frac{1}{6}$ worth? $\frac{4}{6}$ worth ?

<div align="center">SOLUTION.</div>

Since $\frac{5}{6}$ is equal to 120 dollars, $\frac{1}{6}$ is equal to $\frac{1}{5}$ of 120 which is 24.   If $\frac{1}{6}$ is equal to 24, $\frac{4}{6}$ is equal to 96.

25. If $\frac{3}{7}$ of an acre of land is worth $210, how much is $\frac{1}{7}$ worth? $\frac{5}{7}$? $\frac{4}{7}$? $\frac{2}{7}$?

26. If $\frac{4}{5}$ of a farm is worth $1000, how much is $\frac{1}{5}$ worth? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{5}{5}$?

27. If $\frac{9}{10}$ of a farm is worth 9000 dollars, how much is $\frac{1}{10}$ worth? $\frac{3}{10}$? $\frac{4}{10}$? $\frac{5}{10}$? $\frac{6}{10}$? $\frac{7}{10}$? $\frac{8}{10}$?

LESSON IV.

28. How many half pounds in 4 pounds and $\frac{1}{2}$ of a pound?

SOLUTION.

In 1 pound there are 2 halves.   In 4 pounds there are 4×2 halves or 8 halves.   In 4 and 1 half pounds there would be 8 halves and 1 half or 9 halves $\frac{9}{2}$.

. Or, better:

$4\frac{1}{2}$ is called a mixed number.   A mixed number consists of a whole number and a fraction. $\frac{9}{2}$ is an improper fraction.   To reduce a mixed number to an improper fraction:

Multiply the whole number by the denominator of the fraction, and to the sum add the numerator.   Under the result thus found write the denominator.

29. How many thirds in $6\frac{2}{3}$.   *Ans.* $\frac{20}{3}$.

SOLUTION.

$[(6 \times 3) + 2] \div 3$ $6 \times 3 = 18$ $18 + 2 = 20$ $20 \div 3 = \frac{20}{3}$.

30. How many thirds of a dollar in $3\frac{1}{3}$? $4\frac{1}{3}$? $5\frac{2}{3}$? $6\frac{1}{3}$? $7\frac{2}{3}$? $8\frac{1}{3}$? $9\frac{2}{3}$? $10\frac{1}{3}$?

31. How many fourths of an ounce in $1\frac{1}{4}$? $2\frac{3}{4}$? $3\frac{3}{4}$? $4\frac{1}{4}$? $5\frac{3}{4}$? $6\frac{1}{4}$? $7\frac{1}{4}$? $8\frac{1}{4}$? $9\frac{3}{4}$? $10\frac{3}{4}$?

LESSON V.

32. How many boxes cheese in 18 half boxes? 20? 22? 24? 26? 28? 30? 32? 34? 36? 38? 40? Why?

The above is reducing an improper fraction to a mixed number. To reduce an improper fraction to a mixed number:

110. *Divide the numerator by the denominator. If there is a remainder write it above the denominator for the fractional part.* Thus:

33. How many thirds in 25 thirds?

<div align="center">SOLUTION.</div>

$25 \div 3 = \frac{25}{3}$ or $8\frac{1}{3}$, the answer.

34. How many fifths in 18 fifths? 22 fifths? 25 fifths? 30 fifths? 32 fifths? 40 fifths? 38 fifths?

35. How many ninths in 24 ninths? 36? 84? 97? 43? 96? 43? 82? 46? 22? 28? 64? 73?

36. How many boxes candles in $\frac{24}{3}$? $\frac{26}{4}$? $\frac{28}{5}$? $\frac{29}{7}$? $\frac{34}{6}$? $\frac{45}{8}$? $\frac{23}{9}$? $\frac{32}{10}$? $\frac{48}{11}$? $\frac{21}{2}$?

LESSON VI.

37. If one pound of butter costs $3\frac{1}{2}$ cents, what is 3 pounds worth?

> *Ans.* 3 times as much as 1 pound, or 3 times $3\frac{1}{2}$ cents.

SOLUTION.

3 pounds at 3 cents are 9 cents. 3 pounds at $\frac{1}{2}$ cent are $\frac{3}{2}$ cents or $1\frac{1}{2}$ cents. 9 cents and $1\frac{1}{2}$ cents$=$ $10\frac{1}{2}$ cents. Or, better, reduce the mixed numbers to an improper fraction. Multiply the numerator by the given number and divide the product by the denominator.

Thus: $3\frac{1}{2}$ cents$=\frac{7}{2}$ cents. $\frac{7}{2}$ cents $\times 3 = \frac{21}{2}$ cents or $10\frac{1}{2}$, the answer.

38. If one hat cost $3\frac{1}{4}$ dollars what is the value of 4 hats? 5 hats? 6 hats? 7 hats? 8 hats? 9 hats? 10 hats?

39. If 1 cap cost $2\frac{1}{4}$ dollars, what is the value of 2 caps? 3 caps? 4 caps? 5 caps? 6 caps?

40. If 1 pound of butter is worth $12\frac{1}{2}$ cents, what is the value of 20 pounds? 22 pounds? 18 pounds? 36 pounds? 3 pounds? 6 pounds? 4 pounds?

LESSON VII.

41. If $2\frac{2}{3}$ boxes of cheese are worth 14 dollars, how much is 1 box worth?

If $2\frac{2}{3}$ boxes cheese are worth 14 dollars, 1 box would be worth as many dollars as $2\frac{2}{3}$ are contained in 14 dollars.

1st reduce $2\frac{2}{3}$ to an improper fraction, $2\frac{2}{3}=\frac{8}{3}$.

To divide by a fraction, *invert* the divisor, that is, make the numerator take the place of the denominator, and the denominator the place of the numerator.

9*

Multiply the numerator thus found by the given number, and divide the result by the new denominator.

$$2\tfrac{2}{3} = \tfrac{8}{3}.$$

$$14 \div \tfrac{8}{3} = 14 \times \tfrac{3}{8} = \frac{14 \times 3}{8} = 4\tfrac{2}{8}. \quad 42 \div 8 = 5\tfrac{2}{8}, \text{ the answer.}$$

42. I gave 22 dollars for 12 yards and $\tfrac{1}{2}$ of cloth. What was the cost per yard? *Ans.* $1.76.

43. Divide 60 by $2\tfrac{1}{2}$. *Ans.* 24.

44. Divide 30 by $3\tfrac{1}{4}$. *Ans.* $9\tfrac{3}{13}$.

45. Divide 46 by $4\tfrac{1}{6}$. *Ans.* $11\tfrac{1}{25}$.

46. Divide 89 by $2\tfrac{1}{8}$. *Ans.* $41\tfrac{15}{17}$.

47. Divide 73 by $9\tfrac{3}{8}$. *Ans.* $7\tfrac{22}{48}$.

48. Divide 24 by $5\tfrac{4}{8}$. *Ans.* $4\tfrac{20}{43}$.

49. Divide 92 by $6\tfrac{1}{4}$. *Ans.* $14\tfrac{42}{48}$.

50. Divide 100 by $8\tfrac{1}{10}$. *Ans.* $12\tfrac{28}{81}$.

51. Divide 200 by $12\tfrac{3}{4}$. *Ans.* 16.

52. Divide 400 by $13\tfrac{1}{4}$. *Ans.* $30\tfrac{20}{13}$.

53. Divide 288 by $8\tfrac{9}{10}$. *Ans.* $32\tfrac{32}{89}$.

54. Divide 673 by $9\tfrac{8}{10}$. *Ans.* $68\tfrac{33}{49}$.

55. Divide 941 by $8\tfrac{1}{2}$. *Ans.* $110\tfrac{14}{17}$.

56. Divide 896 by $8\tfrac{3}{4}$. *Ans.* $102\tfrac{2}{5}$.

57. A mile is 5280 feet long. A rod is $16\tfrac{1}{2}$ feet. How many rods in a mile? *Ans.* 320.

58. Divide 2264 rods by $5\tfrac{1}{2}$ rods. *Ans.* $411\tfrac{7}{11}$.

59. A mile is 1760 yards long. $5\tfrac{1}{2}$ yards make a rod. How many rods in 1 mile? *Ans.* 320.

60. I paid 250 dollars for $20\tfrac{3}{8}$ yards silk. How much is that per yard? *Ans.* 5.22\tfrac{1}{4}$.

61. I gave 22 cents for 13½ crackers. How much is 1 cracker worth? *Ans.* 1¹⁷⁄₂₇.

62. I sold 22½ pounds cheese for 215 cents. At what price per pound was it sold? *Ans.* 9⁵⁄₉.

CONTRACTIONS.

LESSON VIII.

111. CONTRACTIONS are short methods of performing multiplication or division.

CASE I.

112. To multiply any number by 25.

EXAMPLE.

Multiply 246 by 25.

25 is one-fourth of 100. To multiply any number by 100, annex two 0's. By multiplying 246 by 100, we increase its value 4 times. By dividing by 4, we get the true product.

OPERATION.

4)24600
—————
6150

Ans. 6150.

From which we derive the following

RULE.

113. *Annex two 0's to the multiplicand, and divide by 4.*

EXAMPLES.

1.	Multiply 1234 by 25.	*Ans.* 30850.
2.	Multiply 12345 by 25.	*Ans.* 308625.
3.	Multiply 2345 by 25.	*Ans.* 58625.
4.	Multiply 23456 by 25.	*Ans.* 586400.
5.	Multiply 3456 by 25.	*Ans.* 86400.
6.	Multiply 34567 by 25.	*Ans.* 864175.
7.	Multiply 4567 by 25.	*Ans.* 114175.
8.	Multiply 45678 by 25.	*Ans.* 1141950.
9.	Multiply 5678 by 25.	*Ans.* 141950.
10.	Multiply 56789 by 25.	*Ans.* 1419725.

LESSON IX.

CASE II.

114. To multiply by 12½.

EXAMPLES.

1. Multiply 126 by 12½.

Annex two ciphers to the multi-plicand (126), and divide the number thus formed by 8, because 12½ is the eighth part of 100.   *Ans.* 1575.

OPERATION.

8)12600
‾‾‾‾‾‾
1575

2.	Multiply 1234 by 12½.	*Ans.* 15425.
3.	Multiply 12340 by 12½.	*Ans.* 154250.
4.	Multiply 2340 by 12½.	*Ans.* 29250.
5.	Multiply 3456 by 12½.	*Ans.* 42200.
6.	Multiply 34560 by 12½.	*Ans.* 432000.
7.	Multiply 345600 by 13½.	*Ans.* 4320000.

8. Multiply 386400 by 12½.   *Ans.* 4830000.
9. Multiply 246800 by 12½.   *Ans.* 3085000.
10. Multiply 3698700 by 12½   *Ans.* 46233750.
11. Multiply 692460 by 12½.   *Ans.* 8655750.

LESSON X.

CASE III.

115. To multiply by 33⅓.

33⅓ is ⅓ of 100.   To multiply by 33⅓:

Annex two 0's to the multiplicand and divide by 3.

EXAMPLES.

1. Multiply 12345 by 33⅓.

OPERATION.

3)1234500

    411500, the answer.

2. Multiply 123456 by 33⅓.   *Ans.* 4115200.
3. Multiply 23456 by 33⅓.   *Ans.* 781866⅔.
4. Multiply 70368 by 33⅓.   *Ans.* 2345600.
5. Multiply 211104 by 33⅓.   *Ans.* 7036800.
6. Multiply 633312 by 33⅓.   *Ans.* 21110400.
7. Multiply 1899936 by 33⅓.   *Ans.* 63331200.
8. Multiply 5699808 by 33⅓.   *Ans.* 189993600.
9. Multiply 17099424 by 33⅓.   *Ans.* 569980800.
10. Multiply 51298272 by 33⅓.   *Ans.* 1709942400.
11. Multiply 153894816 by 33⅓.   *Ans.* 5129827200.

LESSON XI.

116. To multiply by 125.

125 is $\frac{1}{8}$ of 1000.   To multiply by 1000, annex 3 ciphers to the multiplicand.   To multiply by 125 :

Annex three 0's to the multiplicand and divide by 8.

EXAMPLES.

1. Multiply 12344 by 125.

OPERATION.

8)12344000

1543000

2. Multiply 98752 by 125.   *Ans.* 12344000.
3. Multiply 790016 by 125.   *Ans.* 98752000.
4. Multiply 6320128 by 125.   *Ans.* 790016000.
5. Multiply 50460984 by 125. *Ans.* 6307623000.
6. Multiply 888888.by 125.   *Ans.* 111111000.
7. Multiply 7111104 by 125.   *Ans.* 888888000.
8. Multiply 56888832 by 125. *Ans.* 7111104000.
9 Multiply 404040 by 125.   *Ans.* 50505000.
10. Multiply 3232320 by 125.   *Ans.* 404040000.
11. Multiply 25858560 by 125. *Ans.* 3232320000.

LESSON XII.

117. To multiply by any number of 9's.

By adding 1 to any number consisting wholly of 9's, we obtain a number the first figure of which is

1, followed by as many 0's as there are 9's in the first number. Therefore, to multiply by any number of 9's, add as many 0's to the multiplicand as there are 9's in the multiplier. This will give a number greater than the required product by 1 times the original multiplicand. Subtract the first multiplicand from the new multiplicand, and the difference will be the true result.

<div align="center">**EXAMPLES.**</div>

1. Multiply 12345 by 99.   *Ans.* 1222155.

<div align="center">**OPERATION.**</div>

1234500	Minuend.
12345	Subtrahend.
1222155	Remainder.

Ans. 1222155.

2.	Multiply 12 by 9.	*Ans.* 108.
3.	Multiply 12 by 99.	*Ans.* 1188.
4.	Multiply 123 by 9.	*Ans.* 1107.
5.	Multiply 123 by 99.	*Ans.* 12177.
6.	Multiply 1234 by 99.	*Ans.* 122166.
7.	Multiply 1234 by 999.	*Ans.* 1232766.
8.	Multiply 12345 by 999.	*Ans.* 12332655.
9.	Multiply 12345 by 9999.	*Ans.* 123437655.
10.	Multiply 123456 by 9999.	*Ans.* 1234436544.
11.	Multiply 123456 by 99999.	*Ans.* 12345476544.
12.	Multiply 123456 by 999.	*Ans.* 123332544.
13.	Multiply 1234567 by 9999.	*Ans.* 12344435433.
14.	Multiply 23456 by 99999.	*Ans.* 2345576544.

15. Multiply 34567 by 999.   *Ans.* 34532433.
16. Multiply 87643 by 9999.   *Ans.* 876342357.
17. Multiply 76422 by 9999.   *Ans.* 764143578.
18. Multiply 28764 by 99999.   *Ans.* 2876371236.
19. Multiply 876492 by 99999. *Ans.* 87648323508.
20. Multiply 332233 by 99999. *Ans.* 33222967767.
21. Multiply 8764321 by 99999.

Ans. 876423335679.

22. Multiply 123456 by 999999.

Ans. 123455876544.

23. Multiply 2345678 by 999999.

Ans.

24. Multiply 3456789 by 99999999.

Ans.

25. Multiply 123456789 by 99999999.

Ans.

26. Multiply 213141516171 by 99999999.

Ans.

27. Multiply 31451681923 by 99999999.

Ans.

LESSON XIII.

CONTRACTIONS IN DIVISION.

118. To divide by 25.

If we multiply the dividend by 4, we get a product 4 times too great. To find the true quotient, we must divide by a number 4 times greater than the given divisor. To divide by 25, multiply the divi-

dend by 4 and divide the product by 100, which is the same as cutting off two figures from the right of the result.

EXAMPLES.

OPERATION.

1. Divide 1234 by 25.

Multiply by 4, and from the product cut off 2 figures. The answer is 49 times and $\frac{36}{100}$ remaining.

$$
\begin{array}{r}
1234 \\
4 \\
\hline
49|36
\end{array}
$$

2. Divide 12345 by 25.   *Ans.* $493\frac{80}{100}$.
3. Divide 23456 by 25.   *Ans.* $938\frac{24}{100}$.
4. Divide 34567 by 25.   • *Ans.* $1382\frac{68}{100}$.
5. Divide 45678 by 25.   *Ans.* $1827\frac{12}{100}$.
6. Divide 56789 by 25.   *Ans.* $2271\frac{56}{100}$.
7. Divide 678910 by 25.   *Ans.* $27156\frac{40}{100}$.
8. Divide 78910 by 25.   *Ans.* $3156\frac{40}{100}$.
9. Divide 891011 by 25.   *Ans.* $35640\frac{44}{100}$.
10. Divide 910100 by 25.   *Ans.* 36404.
11. Divide 12000 by 25.   *Ans.* 480.
12. Divide 23000 by 25.   *Ans.* 920.
13. Divide 23400 by 25.   *Ans.* 936.
14. Divide 345000 by 25.   *Ans.* 1380.
15. Divide 456000 by 25.   *Ans.* 18240.

LESSON XIV.

119. To divide by $33\frac{1}{3}$.

If we multiply the dividend by 3, we get a number three times too great. To find the true quotient, we must divide by a number three times greater than the given divisor.

10

120. 33⅓ is ⅓ of 100. Multiplying by 3 and dividing by 100 is the same as dividing by 33⅓. To divide by 33⅓, multiply the dividend by 3 and divide the product by 100, by taking off two figures from the right.

EXAMPLES.

1. Divide 2468 by 33⅓.

	OPERATION.
1st. Multiply the dividend by 3. 2d. From the product strike off two figures. The true answer is 74 and $\frac{4}{100}$ remaining.	2468 3 ——— 74|04

2. Divide 12345 by 33⅓. *Ans.* $370\frac{35}{100}$.

3. Divide 23456 by 33⅓. *Ans.* $703\frac{68}{100}$.

4. Divide 34567 by 33⅓. *Ans.* $1037\frac{1}{100}$.

5. Divide 45678 by 33⅓. *Ans.* $1370\frac{34}{100}$.

6. Divide 56789 by 33⅓. *Ans.* $1703\frac{67}{100}$.

7. Divide 678910 by 33⅓. *Ans.* $20367\frac{30}{100}$.

8. Divide 78912 by 33⅓. *Ans.* $2367\frac{36}{100}$.

9. Divide 89123 by 33⅓. *Ans.* $2673\frac{69}{100}$.

10. Divide 91234 by 33⅓. *Ans.* $2737\frac{2}{100}$.

11. Divide 43219 by 33⅓. *Ans.* $1296\frac{57}{100}$.

12. Divide 32189 by 33⅓. *Ans.* $965\frac{67}{100}$.

13. Divide 198756 by 33⅓. *Ans.* $5962\frac{68}{100}$.

14. Divide 246890 by 33⅓. *Ans.* $7406\frac{70}{100}$.

LESSON XV.

121. To divide by $12\frac{1}{2}$:

Multiply the dividend by 8 *and divide the product by* 100 (because $12\frac{1}{2}$ is $\frac{1}{8}$ of 100).

EXAMPLES.

1. Divide 86432 by $12\frac{1}{2}$.

1st. Multiply by 8.   2d. Cut off two figures from the right of the result.   The answer is 6914 and $\frac{56}{100}$ remaining.

OPERATION.

$$
\begin{array}{r}
86432 \\
8 \\
\hline
6914|56
\end{array}
$$

2. Divide 987 by $12\frac{1}{2}$.   *Ans.* $78\frac{96}{100}$.

3. Divide 8976 by $12\frac{1}{2}$.   *Ans.* $718\frac{8}{100}$.

4. Divide 6798 by $12\frac{1}{2}$.   *Ans.* $543\frac{84}{100}$.

5. Divide 7968 by $12\frac{1}{2}$.   *Ans.* $637\frac{44}{100}$.

6. Divide 4312 by $12\frac{1}{2}$.   *Ans.* $344\frac{96}{100}$.

7. Divide 4132 by $12\frac{1}{2}$.   *Ans.* $320\frac{56}{100}$.

8. Divide 4321 by $12\frac{1}{2}$.   *Ans.* $345\frac{68}{100}$.

9. Divide 76425 by $12\frac{1}{2}$.   *Ans.* 6114.

10. Divide 46725 by $12\frac{1}{2}$.   *Ans.* 5738.

11. Divide 67452 by $12\frac{1}{2}$.   *Ans.* $5396\frac{16}{100}$.

12. Divide 47625 by $12\frac{1}{2}$.   *Ans.* 3810.

13. Divide 18793 by $12\frac{1}{2}$.   *Ans.* $1503\frac{44}{100}$.

14. Divide 78139 by $12\frac{1}{2}$.   *Ans.* $6251\frac{12}{100}$.

15. Divide 17893 by $12\frac{1}{2}$.   *Ans.* $1431\frac{44}{100}$.

16. Divide 38719 by $12\frac{1}{2}$.   *Ans.* $3097\frac{52}{100}$.

17. Divide 248763 by $12\frac{1}{2}$.   *Ans.* $19901\frac{4}{100}$.

18. Divide 427836 by $12\frac{1}{2}$.   *Ans.* $34226\frac{88}{100}$.

19. Divide 742886 by $12\frac{1}{2}$.   *Ans.* $59430\frac{88}{100}$.

20. Divide 3217546 by $12\frac{1}{2}$.   *Ans.* $257403\frac{68}{100}$.

LESSON XVI.

122. To divide by 125.

125 is ⅛ of 1000.

Multiply the dividend by 8 *and divide the product by* 1000, *by taking three figures from the right of the result.*

EXAMPLES.

1. Divide 186254 by 125.

1st. Multiply 186254 by 8. 2d. From the result, 1490032 cut off three figures. The answer is $1490\frac{32}{1000}$.

OPERATION.

$$\begin{array}{r} 186254 \\ 8 \\ \hline 1490|032 \end{array}$$

2. Divide 12345 by 125.   *Ans.* $98\frac{760}{1000}$.

3. Divide 23456 by 125.   *Ans.* $187\frac{648}{1000}$.

4. Divide 34567 by 125.   *Ans.* $276\frac{536}{1000}$.

5. Divide 45678 by 125.   *Ans.* $365\frac{424}{1000}$.

6. Divide 56789 by 125.   *Ans.* $454\frac{312}{1000}$.

7. Divide 678910 by 125.   *Ans.* $5431\frac{280}{1000}$.

8. Divide 7891011 by 125.   *Ans.* $63128\frac{88}{1000}$.

9. Divide 891234 by 125.   *Ans.* $7129\frac{872}{1000}$.

10. Divide 9123456 by 125.   *Ans.* $72987\frac{648}{1000}$.

11. Divide 1111111 by 125.   *Ans.* $8888\frac{888}{1000}$.

12. Divide 222222 by 125.   *Ans.* $1777\frac{776}{1000}$.

13. Divide 3333333 by 125.   *Ans.* $26666\frac{664}{1000}$.

14. Divide 6666666 by 125.   *Ans.* $52222\frac{228}{1000}$.

15. Divide 46644664 by 125.   *Ans.* $373157\frac{312}{1000}$.

16. Divide 8282828282 by 125.   *Ans.* $66262626\frac{256}{1000}$.

17. Divide 544554455445 by 125.

Ans. 4356435643$\frac{560}{1000}$.

18. Divide 38765433456783 by 125.

Ans. 310123467654$\frac{264}{1000}$.

19. Divide 12345677654321 by 125.

Ans. 98765221236$\frac{668}{1000}$.

•

EXAMPLES INVOLVING THE FOREGOING RULES.

LESSON XVI

123. 1. I bought 20 hogsheads sugar @ $30 per hhd., and sold them for $45 per hhd. What did I gain? *Ans.* $300.

2. I bought 25 hhds. sugar, 250 pounds each, @ 12½ cents per pound, and sold 20 hhds. of it for 15 cents. How much did I pay for the sugar? How much did I receive for what I sold? How many pounds of sugar had I left, and what was it worth, @ 20 cents per pound. *First Ans.* $781.25.

3. A grocer bought 14 hogsheads rice, 200 pounds each, @ 33⅓ cents per pound, and sold it for 34 cents. How much did he gain? *Ans.* $18.66⅔.

4. Bought 27 oxen @ 40 dollars, 32 cows @ 25 dollars, and gave in payment 25 horses, valued @ 75 dollars each. How much do I owe? *Ans.* $5.

5. The distance between two cities is 840 miles.

10*

If I can travel 42 miles a day on foot, in how many days will I accomplish the journey? *Ans.* 20 days.

6. I sold S. Brown 200 kegs nails, @ 3 dollars; 100 thousand feet lumber, @ 12½ dollars per thousand. How much does he owe me? *Ans.* $725.

7. In one year there are 365 days. How many days in 12 years? *Ans.* 4380 days.

8. Bought of Jas. Philips 200 acres of land, @ $25 per acre, and gave him in payment 25 horses, valued at $200 each. How much does he owe me?

Ans. Nothing.

9. Make out the following bill: 50 barrels flour, @ $5; 40 hundredweight cheese, @ 8 dollars per hundredweight; 15 boxes salmon, @ 7 dollars per box. How much did the bill amount to? *Ans.* $675.

10. I bought 2 casks oil, 184 gallons each, @ $1 per gallon. 22 gallons having leaked out, I sell the remainder for 105 cents a gallon. Do I gain or lose, and how much? *Ans.* Lost $27.80.

11. I bought 240 acres of land, @ $10 an acre, and sold 120 acres of it to Wm. Brown, @ $20 an acre. How much is the balance worth, at $13 an acre, and how much do I gain? *Ans.* $1560.

13. I gave 1728 dollars for a certain number of barrels of flour, valued at 12 dollars a barrel. I sold the flour for 14 dollars a barrel. How many barrels did I buy? How much did I receive? How much profit did I make?

Ans. Bought 144 bbls.; profit, $288.

LESSON XVIII.

14. The sum of two numbers is 123456. The smaller number is 1234. What is the larger number?   *Ans.* 122222.

15. The dividend is 1864. The divisor is 2. What is the quotient?   *Ans.* 932.

16. The quotient is 932. The divisor is 2. What is the dividend?   *Ans.* 1864.

17. The quotient is 932. The dividend is 1864. What is the divisor.   *Ans.* 2.

18. I have a farm of 21000 acres, valued at 2 dollars per acre. I bequeath $\frac{1}{4}$ of it to my son, Robert; $\frac{1}{4}$ of the remainder to my daughter, Jane; and the remainder I reserve for myself. How much is the whole farm worth? How many acres did Jane receive for her share, and what is her share worth, at 5 dollars per acre? How many acres did my son Robert receive, and what is the value of his share, at 6 dollars an acre? How many acres did I reserve, and what is the value of my portion, at $12\frac{1}{2}$ dollars an acre? What is the value of the whole farm as divided above?   *Ans.*

19. I buy molasses to the amount of 1800 dollars, at the rate of 4 dollars per barrel. How many barrels did I buy?   *Ans.* 450.

20. A train of twelve cars brought down cotton on the Jackson Railroad. Each car contained 20 bales, and every bale weighed 420 pounds. What was the value of the cotton, at $12\frac{1}{2}$ cents per pound?   *Ans.* $12600.

21. The distance from the earth to the moon is two hundred and forty thousand miles. How long would it take a steam-car to get there, at the rate of 100 miles per hour, allowing 24 hours to the day?

Ans. 100 days.

22. The population of Pekin is estimated at two millions. If each person in Pekin should eat one and a half pound of rice per day, how many casks of rice would be consumed, allowing 380 lbs. to a cask? What would be the value of the rice, @ 12½ cents a pound? *First Ans.* 3000000.

23. London has two millions eight hundred thousand inhabitants. How many pounds of mutton would be consumed, if every inhabitant consumed 1¾ pound per day? How much per week? How many sheep would be required to supply London per day, if a sheep weighed 65 pounds? per week?

First Ans. 4900000.

24. I purchased 23640 gallons of molasses. How many barrels did I buy, if every barrel contained 40 gallons? If I sold the molasses for 20 dollars a barrel, how much would I have to receive? If I sold the molasses at 55 cents a gallon, how much would I receive, and what would my profit be, if I bought the molasses at 45 cents a gallon?

First Ans. 591 barrels.

25. The building of a bridge cost as follows: 25 carpenters, 18 days, @ 3 dollars per day; 30 bricklayers, 18 days, @ 3 dollars per day; 20 laborers, 18 days, @ 2 dollars per day; 10000 bricks, @ $13 per thousand; 64000 feet lumber, @ 18 dollars per thou-

sand; 2 plans of the bridge, @ 25 dollars each. I received 30000 dollars for making the bridge. Do I gain or lose, and how much? What would my salary as contractor be, if I was 20 days superintending the work?   *Ans.*

26. Horrel, Gayle & Co., bought of Jas. Brown: 200 sacks of corn, @ 3 dollars per sack; 100 sacks of bran, @ 2 dollars per sack; 25 barrels of pork, @ 28 dollars per barrel; 30 barrels of beef, @ 21 dollars per barrel. What is the amount of the bill?

Ans. $2130.

LESSON XIX.

27. If there are 365 days, of 24 hours each, in one year, how many hours in ten years?   *Ans.* 87600.

28. If there are 5280 feet, of 12 inches each, in 1 mile, how many inches in 10 miles?   *Ans.* 633600.

29. There 320 boys in the Bienville school. If each boy would contribute 10 cents for charitable purposes, how much would each beggar receive, if 25 made application for his share?   *Ans.* $1.28.

30. In one mile there are 63360 inches. A brick is 8 inches long. How many bricks would make 1 mile? 2 miles?   *Last Ans.* 15840.

31. My school books cost as follows: 1 reader, 135 cents; 1 speller, 45 cents; 1 slate, 20 cents; 1 arithmetic, 100 cents; 1 Scholar's Companion, 95 cents; 1 geography, 150 cents; 1 history, 200 cents. What did all of them cost?   *Ans.* $7.95.

32. If 12 inches make 1 foot, how many feet in 63360 inches?   *Ans.* 5280.

33. If 3 feet make 1 yard, how many yards in 5280 feet ?   *Ans.* 1760.

34. Make the following bill:

    2 Grammars, @ 55 cents   .   .   :   .
    4 Histories U. S., @ 150 cents .   .   .
    5 Scholar's Companions, at 80 cents .
    8 Arithmetics, @ 70 cents .   .   .   .
                       —————
      Total **$16.70**

35. If 1 grammar costs 55 cents, how many grammars can I buy for 5555 cents ?   *Ans.* 101.

36. What cost 250 boxes chalk, at 12½ cents per box?   *Ans.* $31.25.

37. What cost 250 barrels beef, at 25 dollars per barrel ?   *Ans.* $6250.

38. Multiply 2468 by 999, by contraction.

                                Ans. 2465532.

40. If a ship costs 40000 dollars, what will ⅛ of a ship cost ?   What is ⅝ worth ?

                     Last Ans. 25000 dollars.

41. 20 men work 70 days, at 125 cents per day. How much will their wages amount to ?   *Ans.* $175.

42. Bought 4267 slates, at 25 cents apiece.   How much did they cost?   *Ans.* $1066.75.

43. If 1 hhd. sugar costs 200 dollars, what is ¾ of it worth ?   *Ans.* 150 dollars.

44. If a steamboat costs 42000 dollars, how much is ¾ of it worth ?   *Ans.* 18000.

45. If a school-house costs 12000 dollars, how much is ⅘ of it worth ?   *Ans.* $9600.

LESSON XX.

46. If in 1 mile there are 5280 feet, how many miles in 158400 feet?   *Ans.* 30 miles.

124. BY CONTRACTION.

47. I bought 2460 lbs. butter, at 12½ cents.   Required the amount of my bill.   *Ans.* $307.50.

48. I sold 98 dozen of hats, at 12½ dollars a dozen. How much was my bill?   *Ans.* $1225.

49. To 25 men I gave 43 dollars apiece.   How much did I give to all of them?   *Ans.* $1075.

50. John Smythe bought 46 green mules, for which he paid 125 dollars apiece.   How much did he have to spend?   *Ans.* $5750.

51. James Brown bought 64 quires of paper, at 33⅓ cents a quire.   How much was the amount of the bill?   *Ans.* $21.33⅓.

52. If 1 hat costs 8 dollars, how many hats can I buy for 640 dollars?   *Ans.* 80.

53. How much will 200000 feet of lumber cost, at 12½ dollars per thousand?   *Ans.* $5000.

54. How much will 264000 bricks cost, at 12½ dollars per thousand?   *Ans.* $3300.

55. I received 24 loads of brick, each load containing 500 bricks.   How much are they worth, at 33⅓ dollars per thousand?   *Ans.* $400.

56. How much will the following bill amount to:
24 thousand bricks, @ 12½ dollars per thousand; 18
thousand feet lumber, @ 25 dollars per thousand?

Ans. $750.

57. In 12 days of 24 hours each, every hour 60
minutes, every minute 60 seconds, how many sec-
onds?   *Ans.* 1036800.

58. In 25 carts there are 50 planks, and each
plank measures 14 feet. How much is the lumber
worth, at 12½ cents per foot?   *Ans.* $2187.50.

59. If I buy lumber at 12½ cents per foot, and sell
at 125 dollars per thousand, do I gain or lose, and
how much? Why?   *Ans.*

60. If I buy a calf weighing 100 pounds, at 12½
cents a pound, and sell it for 12½ dollars, how much
do I gain or lose? Why?   *Ans.*

61. I require 240000 feet of lumber and 300000
bricks. John Brown says that he will sell me lum-
ber at 25 dollars per thousand, and bricks at 12½
dollars per thousand. Robert Smith says that he
can sell cheaper than Brown. He says that I can
have his lumber for 2½ cents per foot, and his bricks
for 1¼ cents apiece. How much do I save by buying
from Smith, and why?   *Ans.*

CHAPTER VII.

PRACTICE IN UNITED STATES MONEY.

LESSON I.

125. UNITED STATES MONEY was established by the Congress of the United States in 1796 as the legal currency of the United States.

In 1862, Congress issued bills of credit called United States Treasury notes, or, as they are commonly called, " *Greenbacks*."

126. The units increase ten times for every figure from right to left, and decrease in the same proportion from left to right.

127. The dollars are separated from the cents by a period (.) called the decimal point. All the figures to the left of the period are dollars. The first figure after the period represents dimes, the second, cents, and the third, mills. Thus, $18.625 is read, 18 dollars 6 dimes (or 60 cents) 2 cents and 5 mills; or, 18 dollars 62 cents and 5 mills.

128. Since it takes 10 cents to make 1 dime, when the cents are less than 10, a cipher must be placed after the period and before the figure representing cents.

EXAMPLE.

Write 1 dollar and 5 cents.   *Ans.* $1.05.

11

129. The gold coins of the United States are the double eagle, or $20 gold piece; the eagle, or $10 gold piece; the half eagle, or $5 gold piece; the quarter eagle, or $2.50 gold piece; the $3 gold piece; and the $1 gold piece.

130. The silver pieces are the dollar, half dollar, quarter dollar, dime, half dime or picayune, and 3-cent piece. There is also the 1-cent piece, made of copper. Nickels, or 5-cent pieces, are now made of nickel and copper.

131. This sign, $, is the sign of dollars.

TABLE OF UNITED STATES MONEY.

10	mills	make	1	cent,	marked	*ct.*
10	cents	"	1	dime,	"	*d.*
10	dimes	"	1	dollar,	"	$.
10	dollars	"	1	eagle,	"	*E.*

REDUCTION I.

132. To reduce dollars to dimes.

RULE.

Annex 1 cipher.

EXAMPLES.

1. Reduce $25 to dimes.

OPERATION.

$$25 \times 10 = 250 \text{ dimes.}$$

Since it takes 10 dimes to make 1 dollar, there will be 10 times as many dimes as dollars. To multiply by 10, annex 1 cipher to the multiplicand.

2. How many dimes in $2?   *Ans.* 20 dimes.
3. How many dimes in $3?   *Ans.* 30 dimes.
4. How many dimes in $4?   *Ans.* 40 dimes.
5. How many dimes in $15?   *Ans.* 150 dimes.
6. How many dimes in $20?   *Ans.* 200 dimes.
7. How many dimes in $200? *Ans.* 2000 dimes.
8. How many dimes in $250? *Ans.* 2500 dimes.
9. How many dimes in $300? *Ans.* 3000 dimes.
10. How many dimes in $208? *Ans.* 2080 dimes.
11. How many dimes in $309? *Ans.* 3090 dimes.
12. How many dimes in $390? *Ans.* 3900 dimes.
13. How many dimes in $486? *Ans.* 4860 dimes.
14. How many dimes in $846? *Ans.* 8460 dimes.
15. How many dimes in $246? *Ans.* 2460 dimes.
16. How many dimes in $642? *Ans.* 6420 dimes.
17. How many dimes in $462? *Ans.* 4620 dimes.
18. How many dimes in $8972?

Ans. 89720 dimes.

LESSON II.

REDUCTION II.

133. To reduce dollars to cents.

RULE.

Multiply the dollars by 100.

Because it takes 100 cents to make a dollar, therefore there will be 100 times as many cents as there are dollars. To multiply by 100, annex two ciphers to the multiplicand.

1. How many cents in 25 dollars?

$$\textit{Ans. } 2500 \text{ cents.}$$

OPERATION.

$$\$25 \times 100 = 2500 \text{ cents.}$$

2. How many cents in 24 dollars?

$$\textit{Ans. } 2400 \text{ cents.}$$

3. How many cents in 42 dollars?

$$\textit{Ans. } 4200 \text{ cents.}$$

4. How many cents in 58 dollars?

$$\textit{Ans. } 5800 \text{ cents.}$$

5. How many cents in 85 dollars?

$$\textit{Ans. } 8500 \text{ cents.}$$

6. How many cents in 976 dollars?

$$\textit{Ans. } 97600 \text{ cents.}$$

7. How many cents in 679 dollars?

$$\textit{Ans. } 67900 \text{ cents.}$$

8. How many cents in 796 dollars?

$$\textit{Ans. } 79600 \text{ cents.}$$

9. How many cents in 769 dollars?

$$\textit{Ans. } 76900 \text{ cents.}$$

10. How many cents in 287 dollars?

$$\textit{Ans. } 28700 \text{ cents.}$$

11. How many cents in 827 dollars?

$$\textit{Ans. } 82700 \text{ cents.}$$

12. How many cents in 872 dollars?

$$\textit{Ans. } 87200 \text{ cents.}$$

13. How many cents in 728 dollars?

$$\textit{Ans. } 72800 \text{ cents.}$$

14. How many cents in 348 dollars ?
                              Ans. 34800 cents.

15. How many cents in 976 dollars ?
                              Ans. 97600 cents.

16. How many cents in 834 dollars ?
                              Ans. 83400 cents.

17. How many cents in 679 dollars ?
                              Ans. 67900 cents.

18. How·many cents in 796 dollars ?
                              Ans. 79600 cents.

19. How many cents in 643 dollars ?
                              Ans. 64300 cents.

20. How many cents in 897 dollars ?
                              Ans. 89700 cents.

21. How many cents in 241 dollars ?
                              Ans. 24100 cents.

22. How many cents in 873 dollars ?
                              Ans. 87300 cents.

23. How many cents in 946 dollars ?
                              Ans. 94600 cents.

24. How many cents in 638 dollars ?
                              Ans. 63800 cents.

LESSON III.

REDUCTION III.

134. To reduce dollars to mills.

RULE.

Annex 3 ciphers·to the dollars.
11*

Since it takes 10 mills to make 1 cent, and 100 cents to make 1 dollar, it requires 10 × 100, or 1000 mills to make 1 dollar. To multiply by 1000, annex 3 ciphers to the multiplicand.

EXAMPLES.

1. How many mills in 4 dollars ?

OPERATION.

$$4 \times 1000 = 4000. \quad \textit{Ans.}$$

2. How many mills in 185 dollars ? *Ans.* 185000.
3. How many mills in 851 dollars ? *Ans.* 851000.
4. How many mills in 581 dollars ? *Ans.* 581000.
5. How many mills in 518 dollars ? *Ans.* 518000.
6. How many mills in 815 dollars ? *Ans.* 815000.
7. How many mills in 723 dollars ? *Ans.* 723000.
8. How many mills in 327 dollars ? *Ans.* 327000.
9. How many mills in 237 dollars ? *Ans.* 237000.
10. How many mills in 732 dollars? *Ans.* 732000.
11. How many mills in 914 dollars ? *Ans.* 914000.
12. How many mills in 419 dollars ? *Ans.* 419000.
13. How many mills in 149 dollars ? *Ans.* 149000.
14. How many mills in 941 dollars ? *Ans.* 941000.
15. How many mills in 186 dollars? *Ans.* 186000.
16. How many mills in 681 dollars ? *Ans.* 681000.
17. How many mills in 618 dollars ? *Ans.* 618000.
18. How many mills in 816 dollars ? *Ans.* 816000.
19. How many mills in 861 dollars ? *Ans.* 861000.
20. How many mills in 732 dollars ? *Ans.* 732000.
21. How many mills in 723 dollars ? *Ans.* 723000.
22. How many mills in 327 dollars ? *Ans.* 327000.
23. How many mills in 372 dollars ? *Ans.* 372000.

LESSON IV.

REDUCTION IV.

135. To reduce dimes to cents.

RULE.

Annex 1 cipher to the multiplicand.

Since it takes 10 cents to make 1 dime, there will be 10 times as many cents as dimes. To multiply by 10, annex 1 cipher.

EXAMPLES.

1. Reduce 10 dimes to cents.

OPERATION.

$$10 \times 10 = 100 \text{ cents.}$$

2. Reduce 4 dimes to cents.   *Ans.* 40 cents.

3. Reduce 5 dimes to cents.   *Ans.* 50 cents.

4. Reduce 6 dimes to cents.   *Ans.* 60 cents.

5. Reduce 7 dimes to cents.   *Ans.* 70 cents.

6. Reduce 8 dimes to cents.   *Ans.* 80 cents.

7. Reduce 9 dimes to cents.   *Ans.* 90 cents.

8. Reduce 10 dimes to cents.   *Ans.* 100 cents.

9. Reduce 11 dimes to cents.   *Ans.* 110 cents.

10. Reduce 12 dimes to cents.   *Ans.* 120 cents.

LESSON V.

REDUCTION V.

136. To reduce cents to mills.

RULE.

Annex 1 cipher.

Since it takes 10 mills to make 1 cent, there will be 10 times as many mills as cents.   To multiply by 10, annex 1 cipher.

EXAMPLES.

1. Reduce 5 cents to mills.

OPERATION.

$$5 \times 10 = 50 \text{ mills.}$$

2. Reduce 6 cents to mills.   *Ans.* 60 mills.
3. Reduce 7 cents to mills.   *Ans.* 70 mills.
4. Reduce 8 cents to mills.   *Ans.* 80 mills.
5. Reduce 9 cents to mills.   *Ans.* 90 mills.
6. Reduce 10 cents to mills.   *Ans.* 100 mills.

LESSON VI.

REDUCTION VI.

137. To reduce mills to cents.

RULE.

Divide by 10.

To divide by 10, cut off 1 figure from the right.

EXAMPLES.

1. Reduce 200 mills to cents.

OPERATION.

200 ÷ 10 = 20$\emptyset$ or 20 cents.

2. Reduce 340 mills to cents.   *Ans.* 34 cents.
3. Reduce 430 mills to cents.   *Ans.* 43 cents.
4. Reduce 640 mills to cents.   *Ans.* 64 cents.
5. Reduce 460 mills to cents.   *Ans.* 46 cents.
6. Reduce 280 mills to cents.   *Ans.* 28 cents.
7. Reduce 820 mills to cents.   *Ans.* 82 cents.
8. Reduce 490 mills to cents.   *Ans.* 49 cents.
9. Reduce 940 mills to cents.   *Ans.* 94 cents.
10. Reduce 62800 mills to cents. *Ans.* 6280 cents.

LESSON VII.

REDUCTION VII.

138. To reduce mills to dimes.

RULE.

Divide by 100.

To divide by 100, cut off 2 figures from the right.

EXAMPLES.

1. Reduce 2400 mills to dimes.

OPERATION.

2400 ÷ 100 = 24 dimes.

2. Reduce 1800 mills to dimes.   *Ans.* 18 dimes.

3. Reduce 2800 mills to dimes.   *Ans.* 28 dimes.
4. Reduce 1600 mills to dimes.   *Ans.* 16 dimes.
5. Reduce 18600 mills to dimes. *Ans.* 186 dimes.
6. Reduce 98700 mills to dimes. *Ans.* 987 dimes.
7. Reduce 27400 mills to dimes. *Ans.* 274 dimes.
8. Reduce 87400 mills to dimes. *Ans.* 874 dimes.

LESSON VIII.

REDUCTION VIII.

139. To reduce mills to dollars.

RULE.

Divide by 1000.

To divide by 1000, cut off 3 figures from the right of the number.

1000 mills make 1 dollar.

EXAMPLES.

1. Reduce 24000 mills to dollars.

OPERATION.

$$24000 \div 1000 = 24 \text{ dollars.}$$

2. Reduce 64000 mills to dollars.

Ans. 64 dollars.

3. Reduce 78000 mills to dollars.

Ans. 78 dollars.

4. Reduce 96000 mills to dollars.

Ans. 96 dollars.

5. Reduce 89000 mills to dollars.

Ans. 89 dollars.

6. Reduce 746000 mills to dollars.

Ans. 746 dollars.

7. Reduce 740000 mills to dollars.

Ans. 740 dollars.

8. Reduce 860000 mills to dollars.

Ans. 860 dollars.

9. Reduce 980000 mills to dollars.

Ans. 980 dollars.

10. Reduce 8989000 mills to dollars.

Ans. 8989 dollars.

LESSON IX.

REDUCTION IX.

140. To reduce cents to dimes.

RULE.

Divide by 10.

To divide by 10, cut off 1 figure from the right.

10 cents make 1 dime.

EXAMPLES.

1. Reduce 280 cents to dimes.

OPERATION.

$$280 \div 10 = 28 \text{ dimes.}$$

2. Reduce 760 cents to dimes.   *Ans.* 76 dimes.
3. Reduce 890 cents to dimes.   *Ans.* 89 dimes.
4. Reduce 980 cents to dimes.   *Ans.* 98 dimes.
5. Reduce 1760 cents to dimes. *Ans.* 176 dimes.
6. Reduce 4300 cents to dimes. *Ans.* 430 dimes.
7. Reduce 870 cents to dimes.   *Ans.* 87 dimes.
8. Reduce 960 cents to dimes.   *Ans.* 96 dimes.

LESSON X.

REDUCTION X.

141. To reduce cents to dollars.

RULE.

Divide by 100.

To divide by 100, cut off 2 figures from the right.

100 cents make 1 dollar.

EXAMPLES.

1. Reduce 248000 cents to dollars.

OPERATION.

248000 ÷ 100 = 2480 dollars.

2. Reduce 18600 cents to dollars.

Ans. 186 dollars.

3. Reduce 98700 cents to dollars.

Ans. 987 dollars.

4. Reduce 87900 cents to dollars.

Ans. 879 dollars.

5. Reduce 34200 cents to dollars.

Ans. 342 dollars.

6. Reduce 672800 cents to dollars.

Ans. 6728 dollars.

7. Reduce 246200 cents to dollars.

Ans. 2462 dollars.

8. Reduce 297600 cents to dollars.

Ans. 2976 dollars.

9. Reduce 249600 cents to dollars.

Ans. 2496 dollars.

10. Reduce 22643 cents to dollars.

Ans. 226.43 dollars.

The answer to the last example is read: Two hundred and twenty-six dollars 4 dimes and 3 cents, or 43 cents.

LESSON XI.

REDUCTION XI.

142. To reduce dimes to dollars.

RULE.

Divide by 10.

To divide by 10, cut off 1 figure from the right.

10 dimes make 1 dollar.

EXAMPLES.

1. How many dollars in 200 dimes ?

OPERATION.

$200 \div 10 = .20$ dollars.

2. In 6000 dimes how many dollars ?

Ans. 600 dollars.

3. In 8000 dimes how many dollars?

Ans. 800 dollars.

4. In 98643 dimes how many dollars ?

Ans. 9864.3 dollars.

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LESSON XII.

ADDITION OF UNITED STATES MONEY.

143. To add United States money, proceed as in simple addition.

RULE.

Write the dollars, cents, and mills, so that the units of the same denomination will stand under each other. Add as in simple numbers, and place the period under the periods above.

144. The proof of addition of United States money is the same as that of simple addition.

EXAMPLES.

1. Add $1.253, $3.28, $4.962, $21.38.

OPERATION.

$$1.253$$
$$3.280$$
$$4.962$$
$$21.380$$
$$\overline{}$$

Ans. $30.875

145. Note.—In the second and fourth lines a 0 was written to supply the place of the mills. This 0 is of no value, and may be omitted. It was written to fill up the space.

The above answer is read : Thirty dollars eighty-seven cents and five mills.   5 mills is $\frac{5}{10}$, or $\frac{1}{2}$, of a cent.

2. Add $12.3, $12.34, $12.345.   *Ans.* $36.985.

3. Add $2.345, $23.45, $2.345.   *Ans.* $28.14.

4. Add $3.456, $34.56, $345.6.   *Ans.* $383.616.

5. Add $456.7, $45.67, $4.567.   *Ans.* $506.937.

6. Add $567.8, $5678, $5.678.   *Ans.* $6251.478.

7. Add $67.89, $6.789, $6789.   *Ans.* $6863.679.

8. Add $7.891, $78.91, $789.1.   *Ans.* $875.901.

9. Add $8912, $7.891, $891.2.   *Ans.* $9811.091.

10. Add $12345.6, $123.456.   *Ans.* $12469.056.

11. Add $23.4567, $234.567, $2345.67.

    Ans. $2603.6937.

12. Add $345.678, $3.45678, $345678.

    Ans. $346027.13478.

13. Add $4.56789, $4567.89, $45678.9.

    Ans. $50251.35789.

14. Add $576.43, $5764.3, $5.7543.

    Ans. $6346.4943.

15. Add $12.3, $1.23, $.123, $123, $12.3.

    Ans. $148.953.

16. Add $234, $23.4, $2.34, $.234, $234.

    Ans. $493.974.

17. Add $34.5, $3.45, $.345, $345, $3.45.

    Ans. $386,745.

18. Add $456, $4.56, $456, $45.6, $45.6.

    Ans. $1007.76.

19. Add $56.78, $56.78, $56.78, $56.78, $56.78.

    Ans. $283.90.

20. Add $678.9, $678.9, $678.9, $678.9, $678.9.

    Ans. $3394.5.

21. Add $78.91, $78.91, $78.91, $78.91, $78.91.

    Ans. $394.55.

22. Add $8.912, $8.912, $8.912, $8.912, $8.912.

Ans. $44.560.

23. Add $91.23, $91.23, $91.23, $91.23, $91.23.

Ans. $456.15.

24. Add $246.89, $246.89, $246.89, $246.89.

Ans. $987.56.

25. Add $46.892, $46.892, $46.892, $46.892.

Ans. $187.568.

26. Add $9.6842, $9.6842, $9.6842, $9.6842.

Ans. $38.7368.

27. Add $2.4869, $2.4869, $2.4869, $2.4869.

Ans. $9.9476.

28. Add $48.692, $48.692, $48.692, $48.692.

Ans. $194.768.

29. Add $29.684, $29.684, $29.684, $29.684.

Ans. $118.736.

30. Add $123.4567, $123.4567, $123.4567, $123.4567.

Ans. $493.8268.

31. Add $21.43567, $21.43567, $21.43567, $21.43567.

Ans. $857.4268.

32. Add $41.25637, $41.25637, $41.25673, $41.25673.

Ans. $165.02620.

33. Add $189, $981, $918, $819, $198.

Ans. $3105.

34. Add $24.68, $24.68, $24.68, $24.68, $24.68.

Ans. $123.40.

35. Add $4.268, $4.268, $4.268, $4.268, $4.268.

Ans. $21.340.

36. Add $862.4, $862.4, $862.4, $862.4. $862.4.

Ans. $4312.

37. Add $.2684, $.2684, $.2684, $.2684, $.2682.

Ans. $1.2418.

38. Add $68.42, $68.42, $68.42, $68.42, $68.42.

Ans. $342.10.

39. Add $28.64, $28.64, $28.64, $28.64, $28.64.

Ans. $143.20.

40. Add two dollars eighty-six cents and four mills, twenty-eight dollars and sixty-four cents, two hundred and eighty-six dollars and four dimes, two thousand eight hundred and sixty-four dollars.

Ans. $3181.904.

41. I bought a slate for seventy-five cents, one grammar for one dollar and five cents, one history for two dollars and ten cents, one speller for thirty-six cents and five mills. How much did I have to pay.

Ans. $4.265.

LESSON XIII.

SUBTRACTION OF UNITED STATES MONEY.

RULE.

146. *Write the numbers so that the figures of the same denomination will be found under each other, and subtract as in simple numbers.*

147. The proof of subtraction of United States money is the same as that of simple numbers.

EXAMPLES.

1. From $29.62 take $13.025.

OPERATION.

29.620

13.025

———

Ans. $16.595

12*

148. It will be seen from the preceding exam-
ple that the minuend has no mills. Supply the
deficiency by annexing a 0 for the mills. In prac-
tice, the 0 is supposed to occupy the place. The
same remarks apply to the subtrahend.

2. From $615.81 take $615.81.   *Ans.* 0.
3. From $1919.7 take $191.97.   *Ans.* $1727.73.
4. From $718.62 take $718.62.   *Ans.* 0.
5. From $1991.7 take $199.17.   *Ans.* $1792.53.
6. From $9109.17 take $910.917.*Ans.* $8198.253.
7. From $950.99 take $950.99.   *Ans.* 0.
8. From $1876.42 take $187.642.*Ans.* $1688.778.
9. From $987.342 take $987.342.   *Ans.* 0.
10. From $8937.24 take $8937.24.   *Ans.* 0.
11. From $9837.24 take $9837.24.   *Ans.* 0.
12. From $4983.27 take $498.327.

Ans. $4484.943.

13. From $287.643 take $287.643.   *Ans.* 0.
14. From $89726.4 take $897.264.

Ans. $88829.136.

15. From $82764.3 take $8276.43. *Ans.* $74487.87.
16. From $98728.43 take $9872.843.

Ans. $88855.587.

17. From $8764.321 take $8764.321.   *Ans.* 0.
18. From $98743.6 take $9874.36. *Ans.* $88869.24.
19. From $89724.3 take $897.243.

Ans. $88627.057.

20. From $87643.28 take $8764.328.

Ans. $78878.952.

LESSON XIV.

21. I received $284.63.   I owe I. Browne $81.02, and Samuel Smith $98.34.   How much have I left ?
Ans. $105.27.

22. The pay-roll of my laborers amounts to $867.45.   If I draw one thousand dollars from bank, how much will I have remaining after paying the roll?   *Ans.* $132.55.

23. From 200 dollars I gave James Smith $25, Wm. Burns, $28.05, and James Murphy, $26.45. How much have I left?   *Ans.* $20.50.

24. A lady went to town to make some purchases. She bought one shawl for twenty-five dollars, two dresses for twelve dollars each, twenty pairs of gloves, @ two dollars each.   Her father gave her two hundred and fifty dollars for her birthday present.   How much had she left after buying the above bill of goods?   *Ans.* $161.

25. A little boy who was the best scholar in his class, was presented with the following books for his devotion to study: One grammar, worth fifty-five cents; one history, worth one dollar and fifty cents; one Scholar's Companion, worth eighty cents; one arithmetic, worth eighty cents; one geography, worth one dollar and fifty cents; one reader, worth ninety cents; one speller, worth twenty cents; one slate, worth twenty cents; one French book, worth fifty-five cents.   How much did his set of books cost?   How much change did his father receive from a ten-dollar bill which he gave in payment for the books.   *Ans.*

LESSON XV.

MULTIPLICATION OF UNITED STATES MONEY

RULE.

149. *Multiply as in simple numbers. Cut off as many figures from the right of the product as there are figures to the right of the decimal point in the multiplicand.*

EXAMPLES.

1. Multiply $2.485 by 3.

150. 1st. Multiply 2.485 by 3 as in simple numbers. 2d. Count the number of figures in the multiplicand on the right of the decimal point. In the above example there are 3 figures. Cut off 3 figures from the result, counting from right to left. The answer is 7 dollars 45 cents and 5 mills.

OPERATION.

$2.485
3
———
$7.455

2. Multiply $1234.5 by 12.	*Ans.* $14814.
3. Multiply $234.56 by 21.	*Ans.* $4925.76.
4. Multiply $67.891 by 13.	*Ans.* $882.583.
5. Multiply $7.8912 by 32.	*Ans.* $252.5184.
6. Multiply $8912.3 by 14.	*Ans.* $124772.2.
7. Multiply $912.34 by 46.	*Ans.* $41967.64.
8. Multiply $43.219 by 21.	*Ans.* $907.599.
9. Multiply $3.4219 by 18.	*Ans.* $61.5942.
10. Multiply $4319.2 by 97.	*Ans.* $418962.4.
11. Multiply $291.34 by 10.	*Ans.* $2913.40.
12. Multiply $67.289 by 28.	*Ans.* $1884.092.

13. Multiply $7.6829 by 19.   *Ans.* $145.9751.

14. Multiply $9762.8 by 98.   *Ans.* $956754.4.

15. Multiply $679.28 by 18.   *Ans.* $12227.04.

16. Multiply $76.928 by 27.   *Ans.* $2077.056.

17. Multiply $8.9267 by 86.   *Ans.* $767.6962.

18. Multiply $2438.7 by 14.   *Ans.* $34141.8.

19. Multiply $23.486 by 36.   *Ans.* $845.496.

LESSON XVI.

20. I sold 26 bushels of potatoes, at $2.45 a bushel. How much was the bill?   *Ans.* $63.70.

21. I bought from Jas. Brown 12 sacks of bran, at $3.40 per sack. What was the amount of the bill?   *Ans.* $40.80.

22. James Smith bought 2 hogs, each weighing 96 pounds, at twelve cents a pound. How much were they worth?   *Ans.* $23.04.

23. Jos. Brown sold 20 grammars, worth $1.50 each, and 30 spellers, worth 25 cents each. What was the amount of the sale?   *Ans.* $37.50.

24. What is the value of 2 hogsheads of sugar, each weighing 900 pounds, at 14 cents per pound?
    Ans. $252.

25. What will 45 acres of land cost, at $3.375 per acre?   *Ans.* $151.875.

26. What will 240 yards of cloth cost, at two dollars forty-five cents and five mills per yard?
    Ans. $589.20.

27. I employ 40 men for 20 days, at $2.25 **per** day. How much must I pay?   *Ans.* **$2000.**

28. I received 2 loads of lumber, each load 2500 feet, at 2 cents and 5 mills per foot. How **much** was it worth?   *Ans.* **$125.**

29. A merchant sold 24 barrels of beef, each **barrel** weighing 196 pounds, at 11 cents and 5 mills **per** pound. What did he receive?   *Ans.* **$540.96.**

LESSON XVII.

DIVISION OF UNITED STATES MONEY.

RULE.

151. *Divide as in simple numbers. From the right of the quotient cut off (counting from right to left), as many figures as there are figures in the dividend to the right of the decimal point. The figures in the quotient to the left of the decimal point represent dollars ; those on the right of the decimal point represent dimes, cents, mills, etc., according to the position they occupy.*

152. The proof of division is multiplication.

153. Note.—If the dividend is dollars, and is smaller than the divisor, reduce it to cents, and divide. In this case, the answer would be cents. If not large enough when reduced to cents, reduce the cents to mills and divide. The answer in this case would be mills.

EXAMPLES.

1. Divide $25.50 among 5 men. How much will each man receive?

154. Divide as in division of simple numbers. Count the number of figures to the right of the decimal point in the dividend, and cut off as many in the quotient, counting from right to left. The quotient in the above is 510. The number of figures in the dividend to the right of the decimal point is 2. By cutting off 2 figures from the quotient, the answer will be, 5 dollars and 10 cents.

OPERATION.

5) 25.50
     ――――
     5.10

PROOF.

Quotient.   Divisor.   Dividend.
5.10   ×   5   =   25.50

2. Divide $1.50 between 5 boys.

$1.50 is equal to one hundred and fifty (150) cents. The question is now, Divide 150 cents between 5 boys. 5 into 150, 30 times, or 30 cents each boy.

OPERATION.

5)1.50
  ――――
  30 cents.

155. When the highest denomination of the dividend is not divisible by the divisor, consider the dividend reduced to the lowest denomination, and divide.

3. Divide $123.45 by 5.　　　　*Ans.* $24.69.

4. Divide $234.56 by 2.　　　　*Ans.* $117.28.

5. Divide $345.670 by 10.　　　*Ans.* $34.567.

6. Divide $4.5678 by 2.　　　　*Ans.* $2.2879.

7. Divide $5678.90 by 5.　　　　*Ans.* $1135.78.

8. Divide $6789.10 by 10.　　　*Ans.* $678.91.

9. Divide $78.912 by 2.　　　　*Ans.* $39.456.

10. Divide $891.230 by 5.　　　*Ans.* $178.246.

11. Divide $9123.4 by 2.　　　　*Ans.* $4561.7.

12. Divide $123.456 by 2.　　　*Ans.* $61.728.

13. Divide $5643.210 by 5.　　*Ans.* $1128.642.

14. Divide $678.90 by 5.　　　*Ans.* $135.78.

15. Divide $5432.80 by 5.　　　*Ans.* $1086.56.

16. Divide $7643.200 by 100.　*Ans.* $76.432.

17. Divide $97654.30 by 10.　*Ans.* $9765.43.

18. Divide $46.800 by 100.　　*Ans.* $.468.

19. Divide $96.8700 by 100.　*Ans.* $.9687.

20. Divide $247.250 by 10.　　*Ans.* $24.725.

21. Divide $3376.420 by 10.　*Ans.* $337.642.

22. Divide $286.7310 by 10.　*Ans.* $28.6731.

23. Divide $98764.20 by 5.　*Ans.* $19752.84.

24. Divide $1876.5400 by 100.　*Ans.* $18.7654.

LESSON XVIII.

25. If 25 acres of land cost $125.25, what is the cost of 1 acre ?   *Ans.*

26. If 18 barrels of apples cost 99 dollars, what is 1 barrel worth ?   *Ans.*

27. If 6 cords of wood cost $28.50, what is 1 cord worth ?   *Ans.* $6.75.

28. If 20 barrels of beef cost $375, what is 1 barrel worth ?   *Ans.* $18.75.

29. A merchant paid $5006.25 for 25 bales of cotton.   What was the price of 1 bale ?   *Ans.* $200.25.

30. A grocer pays $31.25 for 250 pounds of cheese.   What is 1 pound worth ?   *Ans.* 125 mills.

31. I gave 250 dollars for 25 gallons of whiskey. How much is the whiskey per gallon ?
                    Ans. 10 dollars.

32. I paid $110.25 for 63 pounds of tea.   How much is 1 pound worth?   *Ans.* $1.75.

33. Sam. Browne bought 240 gallons of wine, for which he paid $444.   How much was the wine worth per gallon?   *Ans.* $1.85.

34. If 33 sheep cost $92.50, what is the price of 1 sheep ?   *Ans.* $2.50.

35. If 60 barrels of cider cost $144, how much is 1 barrel worth ?   *Ans.* $2.40.

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CHAPTER VIII.

QUESTIONS BY ANALYSIS.

LESSON I.

156. ANALYSIS is the solution of a problem, step by step, resolving it into parts, thereby rendering it more plain and intelligible.

157. The price of one article being given, to find the value of several articles.

RULE.

Multiply the number of articles or things by the price of one of those articles or things.

EXAMPLES.

1. If 1 pound of beef costs 4 cents, how much will 20 pounds cost?

ANALYSIS.

158. Since 1 pound is worth 4 cents, 20 pounds are worth 20 times 4 cents, or 80 cents.

2. If 1 apple costs 2 cents, what will 20 apples cost? *Ans.* 40 cents.

3. If 1 pound of meat costs 12 cents, what will 12 pounds cost? *Ans.* 144 cents.

4. If 1 ton of hay costs 22 dollars, what will 8 tons cost? *Ans.* $176.

5. If 1 hat costs $8.75, how much will 20 hats cost? 25 hats? 4 hats? 40 hats? 3 hats?

Last Ans. $26.25.

6. If 1 barrel of flour is worth $6.50, how much are 2 barrels worth? 3 barrels? 4 barrels? 5 barrels? 6 barrels? 7 barrels? 8 barrels? *Last Ans.* $52.00.

LESSON II.

159. The number of articles and the price of all of them being given, to find the price of one article.

RULE.

Divide the cost of all by the number of things.

EXAMPLES.

1. If 150 bushels of corn cost $103.50, what is 1 bushel worth?

Since 150 bushels are worth $103.50, 1 bushel is worth 1 one hundred and fiftieth part of $103.50, which is 69 cents.

$103.50÷150=$0.69.

OPERATION.

150) 103.50 (69 cents.
      90 0
      —————
      13 50
      13 50
      —————

2. Bought 650 barrels of flour for 4225 dollars. What is the price of 1 barrel? of 150 barrels?

Last Ans. 975 dollars.

3. If I paid $20250 for 450 acres of land, what is 1 acre worth? *Ans.* $4.50.

4. Mrs. Brown paid 125 dollars for 25 dresses. What is cost of 1 dress?   *Ans.* 5 dollars.

5. If 25 bushels of potatoes can be bought for $12.50, how much is 1 bushel worth? *Ans.* 50 cents.

6. If 2 dozen of hats cost $4.80, how much is 1 hat worth?   *Ans.* 20 cents.

7. Samuel Jones bought 10 spellers for $2.50. What was the price of 1 speller?   *Ans.* 25 cents.

8. Seventeen boys contribute $85.85 towards defraying the expenses of a picnic. How much did each boy contribute?   *Ans.* $5.05.

9. Robert bought 250 readers and paid $62.50 for them. How much did each reader cost?

Ans. 25 cents.

10. A school requires 100 grammars, the cost of which is $55. What is 1 grammar worth?

Ans. 55 cents.

11. The second class of the same school require 50 grammars, the cost of which is $7.50. What is the cost of 1 grammar?   *Ans.* 15 cents.

12. If 10 histories cost $15, what is the price of 1 history?   *Ans.* $1.50.

13. If 20 arithmetics cost $14, what is the value of 1 arithmetic?   *Ans.* 70 cents.

14. If 250 large geographies cost $375, how much is 1 geography worth?   *Ans.* $1.50.

15. A grocer sold 20 pounds of peas for $3.00. What is 1 pound worth?   *Ans.* 15 cents.

16. I received for rent of my house, for 1 year, $330. How much per month did I rent my house for?   How much per day?   *First Ans.* $27.50.

17. If a man spends $1500 a year for the support of his house, how much does he spend per month?

Ans. $125.

How much per day? *Ans.*

18. A gentleman receives $3000 a year for his salary, ⅓ of which he puts aside and saves; the balance he spends. How much does he spend every month? *Ans.*

LESSON III.

160. The price of several articles and the price of one being given, to find the number of articles.

RULE.

Divide the price of all of the articles by the price of one article. The quotient will be the answer.

EXAMPLES.

1. How much sugar can I buy for $125, if sugar is worth 125 mills a pound?

ANALYSIS.

First reduce the dollars to mills. 1000 mills make 1 dollar. In 125 dollars there will be 125 times 1000. To multiply any number by 1000, annex three 0's. The sum can now be read: How many pounds of sugar can I buy for 125000 mills, if sugar is worth 125 mills per pound. Divide 125000 by 125, the answer will be pounds.

$$125000 \div 125 = 1000 \text{ pounds.}$$
13*

$$1000 \text{ lbs.} \times .125 = \$125.000$$

Take off 3 figures from the right for the mills, or divide the product by 1000, which is the same as taking off 3 figures from the right.

2. If coal was worth 10 dollars a load, how many loads could I buy for 100 dollars? Why?

Ans. 10 loads.

3. If I pay $103.50 for corn, at 96 cents a bushel, how many bushels do I buy?   *Ans.* 150 bushels.

4. If flour is worth $6.50 a barrel, how many barrels can I buy for 4225 dollars?   *Ans.* 650 barrels.

5. If land is worth $4.50 an acre, how many acres can I buy for 20250 dollars?   *Ans.* 450 acres.

6. If 1 dress costs 5 dollars, how many dresses can I buy for 125 dollars?   *Ans.* 25 dresses.

7. If 1 hat costs 20 cents, how many hats can I buy for $4.80?   *Ans.* 2 doz. or 24 hats.

8. If 1 speller is worth 25 cents, how many spellers can I buy for $2.50?   *Ans.* 10 spellers.

9. A number of boys contribute $85.85 towards defraying the expenses of a picnic. Each boy contributes $5.05. How many boys contributed?

Ans. 17 boys.

10. If 1 reader costs 25 cents, how many readers can be bought for $62.50?   *Ans.* 250.

11. If 1 grammar is worth 55 cents, how many grammars can be purchased for 55 dollars? *Ans.* 100.

12. If a person spends 125 dollars a month, how many months will 1500 dollars last? *Ans.* 12 months.

CHAPTER IX.

PROPERTIES OF NUMBERS.

LESSON I.

161. An INTEGRAL NUMBER is a whole number.

162. An *even number* is one which is exactly divisible by 2; as, 4, 6, 8, 10, etc.

163. An *odd number* is one which is not exactly divisible by 2; as, 3, 5, 9, 15, etc.

164. A *prime number* is one which is divisible by no number but itself and 1; as, 5, 7, 11, 13.

165. No even number can be a prime number, except the figure 2.

166. The prime numbers between 1 and 100 are: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

167. A *composite number* is the product of two or more prime numbers; thus, 12 is a composite number, because it is the product of $2 \times 2 \times 3$, which are prime numbers.

168. Any composite number is exactly divisible by any of its prime numbers, or by the product of several of its prime numbers; thus, 12 is divisible by 3, one of its prime factors, or by $3 \times 2 = 6$, the product of two of its prime factors.

169. When two numbers are so constituted

that they cannot be exactly divided by the same number, the two numbers are said to be prime to each other; thus, 36 and 17 are prime to each other, because no number will exactly divide both of them without a remainder.

170. Every number is equal to the product of its prime numbers. For example, 32, the prime numbers of which are $2 \times 2 \times 2 \times 2 \times 2$.

171. If we multiply the prime numbers together, their product must be equal to the composite number.

172. A *common divisor* between two or more numbers is a number which will exactly divide them; thus, 3 is a common divisor of 6, 12, 18, 24.

173. Any set of numbers which are prime to each other, have no common divisor except unity.

174. The *greatest common divisor* is the *greatest* number which will divide a series of numbers without a remainder; thus, 6 is the greatest common divisor for 6, 12, 18, 24, 36, etc., because it is the largest number which will divide them without a remainder.

175. A *multiple* is the product of two or more prime numbers; thus, 6 is a multiple, whose prime factors are 2 and 3.

176. A *common multiple* is a number which will exactly contain a series of numbers without a remainder; thus, 24 is a common multiple for 6 and 12, because it will contain both of them without a remainder.

177. The *least common multiple* for two or

more numbers is the smallest number which will contain them without a remainder; thus, 24 is the least common multiple for 6, 12, and 24, because no smaller number will contain every one of them without a remainder.

178. Every number must be the least common multiple of each and every one of its prime numbers.

DIVISIBILITY OF NUMBERS.

179. Any number ending in 0, 2, 4, 6, or 8, is divisible by 2.

180. Any number the sum of whose figures is divisible by 3, can be divided by 3.

181. Four will divide any number whose two last figures can be divided by 4.

182. Four will divide any number ending in two or more 0's.

183. Any number ending in 0 or 5 is divisible by 5.

184. Five will divide any number ending in one or more 0's.

185. Ten divides any number ending in 0.

186. Any number, the sum of whose figures is divisible by 3 or 9, can be divided by 3 or 9.

FACTORING.

LESSON II.

187. To find the prime factors of any number.

RULE.

Divide the number by the smallest prime number which will exactly divide it. Divide the quotient by the smallest prime number which will exactly divide it without a remainder. Divide as before until the quotient becomes 1. The divisors are the prime numbers.

EXAMPLES.

1. Find the prime factors of 1800.

OPERATION.

$$
\begin{array}{r}
2\,)\,\underline{1800} \\
2\,)\,\underline{900} \\
2\,)\,\underline{450} \\
5\,)\,\underline{225} \\
5\,)\,\underline{45} \\
3\,)\,\underline{9} \\
3\,)\,\underline{3} \\
1
\end{array}
$$

The prime factors are 2, 2, 2, 5, 5, 3, 3, 1.

2. Find the prime factors of 200.

Ans. 2, 2, 2, 5, 5.

3. Find the prime factors of 250.

Ans. 2, 2, 2, 2, 3, 5.

4. Find the prime factors of 300.

Ans. 2, 2, 3, 5, 5.

5. Find the prime factors of 350. *Ans.* 2, 5, 5, 7.

6. Find the prime factors of 400.

Ans. 2, 2, 2, 2, 5, 5.

7. Find the prime factors of 450.

Ans. 2, 5, 5 3 3.

8. Find the prime factors of 500.

Ans. 2, 2, 5, 5, 5.

9. Find the prime factors of 550.

Ans. 2, 5, 5, 11.

10. Find the prime factors of 600.

Ans. 2, 2, 2, 3, 5, 5.

11. Find the prime factors of 650.

Ans. 2, 5, 5, 13.

12. Find the prime factors of 700.

Ans. 2, 2, 5, 5, 7.

13. Find the prime factors of 750.

Ans. 2, 3, 5, 5, 5.

14. Find the prime factors of 800.

Ans. 2, 2, 2, 2, 2, 5, 5.

15. Find the prime factors of 850.

Ans. 2, 5, 5, 17.

16. Find the prime factors of 900.

Ans. 2, 2, 3, 3, 5, 5.

17. Find the prime factors of 950.

Ans. 2, 5, 5, 19.

18. Find the prime factors of 1000.

Ans. 2, 2, 2, 5, 5, 5.

19. Find the prime factors of 1100.

Ans. 2, 2, 5, 5, 11.

20. Find the prime factors of 1200.

Ans. 2, 2, 2, 2, 3, 5, 5.

21. Find the prime factors of 1250.

Ans. 2, 5, 5, 5, 5.

22. Find the prime factors of 1300.

Ans. 2, 2, 5, 5, 13.

23. Find the prime factors of 1350.

Ans. 2, 3, 3, 3, 5, 5.

24. Find the prime factors of 1400.

Ans. 2, 2, 2, 5, 5, 7.

25. Find the prime factors of 1450. *Ans.* 2, 5, 5, 29.

26. Find the prime factors of 1500.

Ans. 2, 2, 3, 5, 5, 5.

27. Find the prime factors of 1550.

Ans. 2, 5, 5, 31.

28. Find the prime factors of 1600.

Ans. 2, 2, 2, 2, 2, 2, 5, 5.

29. Find the prime factors of 1650.

Ans. 2, 3, 5, 5, 11.

30. Find the prime factors of 1700.

Ans. 2, 2, 5, 5, 17.

31. Find the prime factors of 1750.

Ans. 2, 5, 5, 5, 7.

32. Find the prime factors of 1800.

Ans. 2, 2, 2, 3, 3, 5, 5.

33. Find the prime factors of 1850.

Ans. 2, 5, 5, 37.

34. Find the prime factors of 1900.

Ans. 2, 2, 5, 5, 19.

35. Find the prime factors of 1950.

Ans. 2, 3, 5, 5, 13.

36. Find the prime factors of 2000.

Ans. 2, 2, 2, 2, 5, 5, 5.

37. Find the prime factors of 2100.

Ans. 2, 2, 3, 5, 5, 7.

38. Find the prime factors of 2150.

Ans. 2, 5, 5, 43.

39. Find the prime factors of 2200.

Ans. 2, 2, 2, 5, 5, 11.

40. Find the prime factors of 2250.

Ans. 2, 3, 3, 5, 5, 5.

41. Find the prime factors of 3000.

Ans. 2, 2, 2, 3, 5, 5, 5.

42. Find the prime factors of 3150 .

Ans. 2, 3, 3, 5, 5, 7.

43. Find the prime factors of 3250.

Ans. 2, 2, 5, 167.

44. Find the prime factors of 3350.

Ans. 2, 5, 5, 67.

45. Find the prime factors of 4000.

Ans. 2, 2, 2, 2, 3, 5, 17.

46. Find the prime factors of 4500.

Ans. 2, 5, 5, 91.

47. Find the prime factors of 8050.

Ans. 2, 5, 5, 161.

48. Find the prime factors of 9050.

Ans. 2, 5, 5, 181.

14

GREATEST COMMON DIVISOR.

LESSON III.

188. To find the greatest common divisor between two numbers:

RULE.

Divide the greater number by the less. If there is a remainder, make the remainder the divisor, and the divisor the dividend. If there is a remainder again, make the last remainder the divisor and the first remainder the dividend. Proceed in this way until there is no remainder. The last divisor is the greatest common divisor.

EXAMPLES.

1. What is the greatest common divisor of 24 and 36 ?

OPERATION.

```
24 ) 36 ( 1
     24
     ──
     12 ) 24 ( 2
          24
          ──
```

12 is the greatest common divisor.

2. What is the greatest common divisor of 6 and 12 ?   *Ans.* 6.

3. What is the greatest common divisor of 12 and 24 ?   *Ans.* 12.

4. What is the greatest common divisor of 24 and 36?   *Ans.* 12.

5. What is the greatest common divisor of 18 and 40 ?   *Ans.* 2.

6. What is the greatest common divisor of 21 and 30 ?   *Ans.* 3.

7. What is the greatest common divisor of 30 and 45?   *Ans.* 15.

8. What is the greatest common divisor of 15 and 40?   *Ans.* 5.

9. What is the greatest common divisor of 16 and 32.   *Ans.* 16.

10. What is the greatest common divisor of 19 and 38?   *Ans.* 19.

11. What is the greatest common divisor of 25 and 40 ?   *Ans.* 5.

12. What is the greatest common divisor of 30 and 21 ?   *Ans.* 3.

13. What is the greatest common divisor of 42 and 20?   *Ans.* 2.

14. What is the greatest common divisor of 16 and 30 ?   *Ans.* 2.

15. What is the greatest common divisor of 120 and 200 ?   *Ans.* 40.

16. What is the greatest common divisor of 200 and 450 ?   *Ans.* 50.

17. What is the greatest common divisor of 650 and 700.   *Ans.* 50.

18. What is the greatest common divisor of 780 and 2900.   *Ans.* 60.

LESSON IV.

189. When there are more than two numbers.

RULE.

I. *Find the greatest common divisor of any two of them.*

II. *Find the greatest common divisor between the divisor found and one of the other numbers.*

III. *Find the greatest common divisor between the last divisor and the next number.*

IV. *Proceed in this way until all of the numbers are divided.*

V. *The last divisor is the greatest common divisor of all the numbers.*

EXAMPLES.

1. What is the greatest common divisor between 12, 24, and 38.

OPERATION.

$$12 \;) \; 24 \; (\; 2 \qquad\qquad 12 \;) \; 38 \; (\; 3$$
$$\underline{24} \qquad\qquad\qquad\quad \underline{36}$$
$$\qquad\qquad\qquad\qquad\qquad 2 \;) \; 12 \; (\; 6$$
$$\qquad\qquad\qquad\qquad\qquad\quad \underline{12}$$

First find the divisor between 12 and 24, which is 12. Find the divisor between 12 and 38. The last divisor, 2, is the greatest common divisor.

2. What is the greatest common divisor of 12, 36, and 48?   *Ans.*

3. What is the greatest common divisor of 18, 24, and 36?   *Ans.*

4. What is the greatest common divisor of 22, 30, and 40?   *Ans.*

5. What is the greatest common divisor of 23, 46, and 56?   *Ans.* 1.

6. What is the greatest common divisor of 2, 5, and 7?   *Ans.* 1.

7. What is the greatest common divisor of 16, 18, and 20?   *Ans.* 2.

8. What is the greatest common divisor of 20, 22, and 24?   *Ans.* 2.

9. What is the greatest common divisor of 22, 24, and 26?   *Ans.* 2.

10. What is the greatest common divisor of 17, 19, and 21?   *Ans.* 1.

11. What is the greatest common divisor of 13, 18, 22, and 26?   *Ans.* 1.

12. What is the greatest common divisor of 5, 7, 9, and 11?   *Ans.* 1.

13. What is the greatest common divisor of 10, 14, 18, and 22?   *Ans.* 2.

14. What is the greatest common divisor of 20, 28, 36, and 44?   *Ans.* 4.

15. What is the greatest common divisor of 40, 56, 72, and 88?   *Ans.* 8.

16. What is the greatest common divisor of 80, 112, 144, and 176 ?   *Ans.* 16.

14*

17. What is the greatest common divisor of 12, 24, and 36 ?   *Ans.* 12.

18. What is the greatest common divisor of 24, 48, and 72 ?   *Ans.* 24.

19. What is the greatest common divisor of 24, 96, and 144 ?   *Ans.* 24.

20. What is the greatest common divisor of 3, 12, 15, and 18 ?   *Ans.* 3.

21. What is the greatest common divisor of 27, 108, 135, and 162 ?   *Ans.* 27.

LESSON V.

22. What is the greatest common divisor of 2, 4, and 6 ?   *Ans.* 2.

23. What is the greatest common divisor of 4, 6, and 8 ?   *Ans.* 2.

24. What is the greatest common divisor of 8, 10, and 12 ?   *Ans.* 2.

25. What is the greatest common divisor of 12, 14, and 16 ?   *Ans.* 2.

26. What is the greatest common divisor of 14, 16, and 18 ?   *Ans.* 2.

27. What is the greatest common divisor of 142, 421, and 28 ?   *Ans.* 1.

28. What is the greatest common divisor of 364, 472, and 983 ?   *Ans.* 1.

29. What is the greatest common divisor of 867, 1734, and 3468 ?   *Ans.* 867.

30. What is the greatest common divisor of 24, 240, and 2160 ?   *Ans.* 24.

31. What is the greatest common divisor of 81, 405, and 2430 ? *Ans.* 81.

32. What is the greatest common divisor of 72, 648, and 2592 ? *Ans.* 72.

33. What is the greatest common divisor of 1800, 2000, and 2500 ? *Ans.* 100.

34. What is the greatest common divisor of 2100, 2600, and 2800 ? *Ans.* 100.

35. What is the greatest common divisor of 2400, 1728, and 1200 ? *Ans.* 48.

36. What is the greatest common divisor of 195, 200, and 205 ? *Ans.* 5.

37. What is the greatest common divisor of 218, 318, 418, 518, and 286 ? *Ans.* 2.

38. What is the greatest common divisor of 126, 248, 428, 648, and 288 ? *Ans.* 2.

LEAST COMMON MULTIPLE.

LESSON VI.

190. To find the least common multiple of two or more numbers.

RULE.

I. *Write the numbers in a row.*

II. *Divide the numbers in the row by any prime number which will exactly divide two or more of them without a remainder.*

III. *The numbers in the row not divisible by the factor used must be written below, among the quotients.*

IV. *Divide the quotients and the numbers taken down by any prime number which will exactly divide two or more of them without a remainder.*

V. *Write the quotients and the numbers not divisible by the factor used below and divide as before.*

VI. *Continue in this way until there are no two numbers exactly divisible by any prime factor.*

VII. *Multiply all the divisors and the numbers remaining undivided, if any, together, and the result will be the least common multiple.*

NOTE.—The least common multiple and the least common denominator are the same.

EXAMPLES.

1. Find the least common multiple of 2, 4, 6, 8, and 10.

OPERATION.

$$
\begin{array}{r}
2\ )\ 2 - 4 - 6 - 8 - 10 \\
\hline
2\ )\ 1 - 2 - 3 - 4 - \ 5 \\
\hline
1 - 1 - 3 - 2 - \ 5
\end{array}
$$

$2 \times 2 \times 3 \times 2 \times 5 = 120$, the least common multiple.

2. Find the least common multiple of 3, 6, and 9.
Ans. 18.

3. Find the least common multiple of 12, 18, and 24. *Ans.* 72.

4. Find the least common multiple of 12, 24, and 32. *Ans.* 96.

5. Find the least common multiple of 8, 16, and 48. *Ans.* 384.

6. Find the least common multiple of 9, 18, and 36. *Ans.* 36.

7. Find the least common multiple of 2, 4, and 12. *Ans.* 12.

8. Find the least common multiple of 9, 8, and 7. *Ans.* 504.

9. Find the least common multiple of 8, 7, and 14. *Ans.* 56.

10. Find the least common multiple of 3, 7, and 21. *Ans.* 21.

11. Find the least common multiple of 4, 9, and 22. *Ans.* 396.

12. Find the least common multiple of 5, 10, and 31. *Ans.* 310.

13. Find the least common multiple of 8, 7, and 24. *Ans.* 168.

14. Find the least common multiple of 2, 4, and 6. *Ans.* 12.

15. Find the least common multiple of 4, 6, and 8. *Ans.* 24.

16. Find the least common multiple of 6, 8, and 10. *Ans.* 120.

17. Find the least common multiple of 8, 10, and 12. *Ans.* 120.

18. Find the least common multiple of 10, 12, and 14. *Ans.* 420.

LESSON VII.

19. Find the least common multiple of 12, **14,** and 16.   *Ans.* **96.**

20. Find the least common multiple of 14, **16,** and 18.   *Ans.* **1008.**

21. Find the least common multiple of 16, **18,** and 20.   *Ans.* **720.**

22. Find the least common multiple of 2, 3, 4, 5, and 6.   *Ans.* **60.**

23. Find the least common multiple of 3, 4, 5, 6, 7, and 8.   *Ans.* **840.**

24. Find the least common multiple of 4, 5, 6, 7, 8, and 9.   *Ans.* **2620.**

25. Find the least common multiple of 5, 6, 7, 8, 9, and 10.   *Ans.* **2520.**

26. Find the least common multiple of 6, 7, 8, 9, 10, and 14.   *Ans.* **2520.**

27. Find the least common multiple of 7, 8, 9, 12, 14, and 16.   *Ans.* **2016.**

28. Find the least common multiple of 8, 9, 12, 14, 16, and 18.   *Ans.* **6048.**

29. Find the least common multiple of 12, 14, and 16.   *Ans.* **336.**

30. Find the least common multiple of 14, 16, 18, and 20.   *Ans.* **5040.**

31. Find the least common multiple of 16, 18, and 20.   *Ans.* **720.**

32. Find the least common multiple of 1, 2, 4, 6, 10, and 12.   *Ans.* **360.**

33. Find the least common multiple of 5, 7, 9, **and** 11.   *Ans.* 3465.

34. Find the least common multiple of 7, 11, 9, **and** 13.   *Ans.* 9009.

35. Find the least common multiple of 11, 9, 13, **and** 17.   *Ans.* 21879.

EXAMPLES ON THE PRECEDING RULES.

LESSON VIII.

191. 1. The sum of two numbers is 12, and their difference is 2.   What are the numbers.

1st. Add the difference to the sum and divide by 2.   This gives the greater number.

2d. Subtract the difference from the sum, and divide by 2.   This gives the smaller number.

Ans. 5 and 7.

OPERATION.

$$
\begin{array}{cc}
12 & \quad\quad 12 \\
2 & \quad\quad 2 \\
\hline
2\,)\,14 & \bullet \quad 2\,)\,10 \\
\hline
7 \text{ Greater number.} & 5 \text{ Less number.}
\end{array}
$$

192. Note.—Such sums are founded on algebraic problems.

2. The sum of two numbers is 125, and their difference is 25. What are the numbers?

Ans. 75 and 50.

3. The sum of two numbers is 15, and their difference is 1. What are the numbers?   *Ans.* 8 and 7.

4. The sum of two numbers is 20, and their difference is 12. What are the numbers?

Ans. 16 and 4.

5. The sum of two numbers is 25, and their difference is 11. What are the numbers?

Ans. 18 and 7.

6. The sum of two numbers is 30, and their difference is 20. What are the numbers?

Ans. 25 and 5.

7. If a certain number is multiplied by 5, and the product divided by 4, the quotient will be 25. What is the number?   *Ans.* 20.

8. If a certain number is multiplied by 4, and the product divided by 5, the quotient will be $3\frac{1}{5}$. What is the number?   *Ans.* 4.

9. If a certain number is multiplied by 7, and the product divided by 2, the quotient will be 70. What is the number?   *Ans.* 20.

10. If a certain number is divided by 4, and the the quotient is then multiplied by 2, the product increased by 20, and the sum diminished by 10, the result will be 20. What is the number?   *Ans.* 20.

11. A can dig 2 rods per day, B can dig 4 rods per day, and C can dig 8 rods per day. What is the least number of rods that will make a number of full days' work for each of the three men?   *Ans.* 8.

12. A gentleman has 250 gallons of sherry, 200 gallons of Madeira, and 210 gallons of claret wine, and he desires to fill a number of casks of equal size, without mixing the wine. How many gallons must each cask hold?   *Ans.* 10.

13. Sam. Brown has 24 half-dollars, 48 dollars, and 200 quarter-dollars. He wishes to place an equal number of each in several drawers, not mixing them or leaving any out. How many will each drawer contain, and what is the least number of drawers that will answer the purpose?

First Ans. 8 pieces in each drawer.

14. What is the least number which will divide 1, 2, 3, 4, 5, 6, 7, 8, and 9 without a remainder?

Ans.

15. A, B, and C start together, and travel in the same direction around an island 1000 miles in circumference. A travels 8 miles a day, B travels 16 miles, and C travels 32 miles. What is the least number of hours that will bring them together? How many times around the island will each one have traveled?

Last Ans. A, 1; B, 2; C, 4.

15

CANCELLATION.

LESSON IX.

EXAMPLES.

193. 1. Divide 100 by 25.

OPERATION.

$$2 \,)\, \overline{100} \qquad 5 \,)\, \overline{25} \qquad \frac{2 \times 2 \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5}}$$
$$2 \,)\, \overline{50} \qquad 5 \,)\, \overline{5}$$
$$5 \,)\, \overline{25} \qquad \overline{1}$$
$$5 \,)\, \overline{5}$$
$$\overline{1}$$

Find the prime factors of the dividend, and place them above the line. Find the prime factors of the divisor, and place them below the line. Cancel those which are common to both. The product of the remaining numbers is the quotient.   *Ans.* 4.

2. Divide 200 by 100.

OPERATION.

$$2 \,)\, \overline{200} \qquad \frac{\cancel{2} \times \cancel{2} \times 2 \times \cancel{5} \times \cancel{5}}{\cancel{2} \times \cancel{2} \times \cancel{5} \times \cancel{5}} \qquad 2 \,)\, \overline{100}$$
$$2 \,)\, \overline{100} \qquad 2 \,)\, \overline{50}$$
$$2 \,)\, \overline{50} \qquad 5 \,)\, \overline{25}$$
$$5 \,)\, \overline{25} \qquad 5 \,)\, \overline{5}$$
$$5 \,)\, \overline{5} \qquad \overline{1}$$
$$\overline{1}$$

Ans. 2.

3. Divide 24 by 8.   *Ans.* 3.
4. Divide 48 by 16.   *Ans.* 3.
5. Divide 96 by 32.   *Ans.* 3.
6. Divide 192 by 64.   *Ans.* 5.
7. Divide 384 by 128.   *Ans.* 3.
8. Divide 768 by 256.   *Ans.* 3.
9. Divide 1536 by 128.   *Ans.* 12.
10. Divide 2072 by 24.   *Ans.*
11. Divide 4144 by 48.   *Ans.*
12. Divide 8288 by 96.   *Ans.*
13. Divide 16576 by 192.   *Ans.*
14. Divide 33152 by 384.   *Ans.*
15. Divide 66304 by 768.   *Ans.*
16. Divide 132608 by 1536.   *Ans.*
17. Divide 265216 by 3072.   *Ans.*
18. Divide 530432 by 1536.   *Ans.*
19. Divide 1060864 by 3072.   *Ans.*
20. Divide 2121728 by 1536.   *Ans.*
21. Divide 200000 by 2000.   *Ans.* 100.
22. Divide 40400 by 400.   *Ans.* 11.
23. Divide 50000 by 5000.   *Ans.* 10.
24. Divide 50500 by 500.   *Ans.* 11.
25. Divide 60000 by 6000.   *Ans.* 10.
26. Divide 70000 by 7000.   *Ans.* 10.
27. Divide 4243556 by 1536.   *Ans.*
28. Divide 90000 by 9000.   *Ans.* 10.
29. Divide 8487112 by 3072.   *Ans.*

CHAPTER X.

FRACTIONS.

LESSON I.

194. The unit 1 means one entire object, or a whole object.

195. If that object is cut into two equal parts, each part is called one half of the object, and is thus written, $\frac{1}{2}$, the figure 1 above the 2, with a line drawn between them.

196. If the object is divided into 3 equal parts, each part is called ($\frac{1}{3}$) one third.

197. If the object is divided into four equal parts, each part is called ($\frac{1}{4}$) one fourth, etc., etc.

$\frac{1}{2}$	is read	one half.	$\frac{1}{11}$	is read	one eleventh.
$\frac{1}{3}$	"	one third.	$\frac{1}{12}$	"	one twelfth.
$\frac{1}{4}$	"	one fourth.	$\frac{1}{13}$	"	one thirteenth.
$\frac{1}{5}$	"	one fifth.	$\frac{1}{14}$	"	one fourteenth.
$\frac{1}{6}$	"	one sixth.	$\frac{1}{15}$	"	one fifteenth.
$\frac{1}{7}$	"	one seventh.	$\frac{1}{16}$	"	one sixteenth.
$\frac{1}{8}$	"	one eighth.	$\frac{1}{17}$	"	one seventeenth.
$\frac{1}{9}$	"	one ninth.	$\frac{1}{18}$	"	one eighteenth.
$\frac{1}{10}$	"	one tenth.	$\frac{1}{19}$	"	one nineteenth.

198. A *fraction* is an equal part of a unit. The number below the line is called the *denominator*, and shows into how many parts the object is divided.

199. The number above the line is called the *numerator*, and shows how many of the equal parts of the fraction are used.

200. In $\frac{3}{4}$, the figure 4 is the denominator, and the 3 is the numerator. It shows that the object was divided into 4 equal parts, of which 3 parts were used.

201. A fraction is less than a whole one.

There are several kinds of fractions :

202. A *proper fraction* is one whose numerator is less than the denominator; thus, $\frac{1}{4}$.

203. An *improper fraction* is one whose numerator is equal to or greater than the denominator; thus, $\frac{2}{3}$ or $\frac{3}{3}$.

204. A *compound fraction* is a fraction of a fraction, or a fraction connected to another by the sign *of ;* thus, $\frac{2}{3}$ of $\frac{1}{4}$.

205. A *mixed fraction* is a fraction and a whole number; thus, $2\frac{1}{2}$, and is read, two and one half.

206. A *complex fraction* is one which has a fraction for the numerator and a fraction for the denominator, or a fraction in either numerator or denominator; thus $\dfrac{\frac{1}{2}}{\frac{3}{4}}$ is read, one half divided by three fourths; or $\dfrac{1}{\frac{1}{2}}$ is read, one divided by one half.

207. The value of a fraction is the quotient found by dividing the numerator by the denominator. The value of a proper fraction must be less than 1. The value of an improper fraction must be equal to or greater than 1.

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LESSON II.

PROPOSITIONS.

208. If the numerator and denominator of a fraction are divided by the same number, the value of the fraction is not changed; thus $\frac{8}{10}$, divide both numerator and denominator by 2, and the fraction becomes $\frac{4}{5}$.

209. If the numerator and denominator of a fraction are multiplied by the same number, the value of the fraction is not changed; thus $\frac{4}{5}$, multiply both numerator and denominator by 2, and the fraction becomes $\frac{8}{10}$.

210. By multiplying the denominator of a fraction, the value of the fraction is decreased.

211. By dividing the numerator of a fraction, the value of the fraction is decreased.

LESSON III.

212. To reduce a fraction to its lowest terms.

A fraction is reduced to its lowest terms when the numerator and denominator are prime to each other.

213. The *terms* of a fraction are the numerator and denominator.

214. To reduce fractions:

RULE.

Divide the numerator and denominator by their greatest common divisor.

EXAMPLES.

1. Reduce $\frac{9}{12}$ to its lowest terms.

OPERATION.

3 is the common divisor.   Divide the numerator and denominator by 3.

$$3\,)\,\tfrac{9}{12} = \tfrac{3}{4},\; Ans.$$

2. Reduce $\frac{8}{12}$, $\frac{9}{27}$, $\frac{24}{36}$ to their lowest terms.
   Ans. $\frac{2}{3}$, etc.

3. Reduce $\frac{16}{24}$, $\frac{18}{54}$, $\frac{48}{108}$ to their lowest terms.
   Ans. $\frac{2}{3}$, etc.

4. Reduce $\frac{2}{4}$, $\frac{4}{8}$, $\frac{8}{16}$, $\frac{16}{32}$ to their lowest terms.
   Ans. $\frac{1}{2}$, etc.

5. Reduce $\frac{5}{10}$, $\frac{25}{30}$, $\frac{19}{38}$, $\frac{40}{76}$ to their lowest terms.
   Ans. $\frac{1}{2}$, etc.

6. Reduce $\frac{15}{40}$, $\frac{60}{90}$, $\frac{33}{84}$, $\frac{80}{150}$ to their lowest terms.
   Ans. $\frac{3}{8}$, etc.

7. Reduce $\frac{16}{42}$, $\frac{28}{32}$, $\frac{92}{100}$, $\frac{24}{36}$ to their lowest terms.
   Ans. $\frac{8}{21}$, etc.

8. Reduce $\frac{90}{210}$, $\frac{80}{46}$, $\frac{33}{78}$, $\frac{200}{180}$ to their lowest terms.
   Ans. $\frac{3}{7}$, etc.

LESSON IV.

9. Reduce $\frac{2000}{3000}$ to its lowest terms.

OPERATION.

Reduce it to its prime factors :

Numerator.

$2) \overline{2000}$
$2) \overline{1000}$
$2) \overline{500}$
$2) \overline{250}$
$5) \overline{125}$
$5) \overline{25}$
$5) \overline{5}$
1

$$\frac{2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5}{2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 3}$$

$$= \frac{2}{3}$$

Denominator.

$2) \overline{3000}$
$2) \overline{1500}$
$2) \overline{750}$
$5) \overline{375}$
$5) \overline{75}$
$5) \overline{15}$
$3) \overline{3}$
1

10. Reduce $\frac{750}{1000}$ to its lowest terms.   *Ans.* $\frac{3}{4}$.

11. Reduce $\frac{1750}{10000}$ to its lowest terms.   *Ans.* $\frac{7}{40}$.

12. Reduce $\frac{175}{200}$ to its lowest terms.   *Ans.* $\frac{7}{8}$.

13. Reduce $\frac{50}{100}$ to its lowest terms.   *Ans.* $\frac{1}{2}$.

14. Reduce $\frac{100}{500}$ to its lowest terms.   *Ans.* $\frac{1}{5}$.

15. Reduce $\frac{250}{300}$ to its lowest terms.   *Ans.* $\frac{5}{6}$.

16. Reduce $\frac{560}{700}$ to its lowest terms.   *Ans.* $\frac{4}{5}$.

17. Reduce $\frac{840}{900}$ to its lowest terms.   *Ans.* $\frac{14}{15}$.

18. Reduce $\frac{1500}{3000}$ to its lowest terms.   *Ans.* $\frac{1}{2}$.

19. Reduce $\frac{1875}{2500}$ to its lowest terms.   *Ans.* $\frac{3}{4}$.

20. Reduce $\frac{2640}{2740}$ to its lowest terms.   *Ans.* $\frac{132}{137}$.

21. Reduce $\frac{1880}{2000}$ to its lowest terms.   *Ans.* $\frac{188}{200}$.

22. Reduce $\frac{1850}{3000}$ to its lowest terms.   *Ans.* $\frac{37}{60}$.

23. Reduce $\frac{1200}{3000}$ to its lowest terms.   *Ans.* $\frac{12}{30}$.

24. Reduce $\frac{7640}{21000}$ to its lowest terms.   *Ans.* $\frac{191}{525}$.

25. Reduce $\frac{7400}{30000}$ to its lowest terms.   *Ans.* $\frac{4}{225}$.

26. Reduce $\frac{12500}{13500}$ to its lowest terms.   *Ans.* $\frac{25}{57}$.

27. Reduce $\frac{13500}{14000}$ to its lowest terms.   *Ans.* $\frac{27}{28}$.

28. Reduce $\frac{14500}{15500}$ to its lowest terms.   *Ans.* $\frac{29}{31}$.

29. Reduce $\frac{16000}{16500}$ to its lowest terms.   *Ans.* $\frac{1}{11}$.

30. Reduce $\frac{18400}{18700}$ to its lowest terms.   *Ans.* $\frac{186}{187}$.

31. Reduce $\frac{12800}{13800}$ to its lowest terms.   *Ans.* $\frac{192}{207}$.

32. Reduce $\frac{24680}{37020}$ to its lowest terms.   *Ans.* $\frac{1234}{1851}$.

33. Reduce $\frac{7404}{3138}$ to its lowest terms.   *Ans.* $\frac{1234}{1523}$.

34. Reduce $\frac{1234}{1404}$ to its lowest terms.   *Ans.* $\frac{617}{3702}$.

35. Reduce $\frac{1234}{3138}$ to its lowest terms.   *Ans.* $\frac{617}{3069}$.

LESSON V.

215. To reduce fractions to fractions having a proposed denominator.

RULE.

I. *Reduce the fraction to its lowest terms.*

II. *Divide the given denominator by the denominator of the fraction reduced.*

III. *Multiply the numerator and denominator of the reduced fraction by the quotient arising from dividing the reduced denominator into the proposed denominator.*

IV. *The result will be the required fraction.*

EXAMPLES.

1. Reduce $\frac{4}{8}$ to a fraction whose denominator will be 12.

OPERATION.

Reducing $\frac{4}{8}$ to its lowest terms gives us $\frac{2}{3}$. Dividing the proposed denominator, 12, by the denominator of the reduced fraction, 3, the quotient is 4. Multiplying both numerator and denominator of the reduced fraction $\frac{2}{3}$ by 4, we get $\frac{8}{12}$ for the required fraction.

216. Note.—The above is the first step towards addition and subtraction of fractions, and should be carefully considered.

2. Reduce $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{5}$, $\frac{6}{12}$, $\frac{12}{15}$ to fractions whose denominators will be 60.   *Ans.* $\frac{30}{60}$, $\frac{30}{60}$, $\frac{48}{60}$, $\frac{30}{60}$, $\frac{48}{60}$.

3. Reduce $\frac{1}{3}$, $\frac{3}{4}$, $\frac{6}{9}$, $\frac{9}{12}$ to fractions whose denominators will be 36.   *Ans.* $\frac{12}{36}$, $\frac{18}{36}$, $\frac{24}{36}$, $\frac{27}{36}$.

4. Reduce $\frac{3}{6}$, $\frac{6}{9}$, $\frac{9}{12}$, $\frac{12}{15}$ to fractions whose denominators will be 180.   *Ans.* $\frac{90}{180}$, $\frac{120}{180}$, $\frac{135}{180}$, $\frac{144}{180}$.

5. Reduce $\frac{21}{24}$, $\frac{7}{35}$ to fractions whose denominators are 30.   *Ans.* $\frac{15}{30}$, $\frac{6}{30}$.

6. Reduce $\frac{3}{7}$, $\frac{4}{6}$, $\frac{17}{21}$ to fractions whose denominators shall be 21.   *Ans.* $\frac{9}{21}$, $\frac{14}{21}$, $\frac{17}{21}$.

Reduce $\frac{1}{4}$, $\frac{1}{5}$, $\frac{3}{6}$, $\frac{1}{8}$ to fractions whose denominators will be 120.   *Ans.* $\frac{30}{120}$, $\frac{24}{120}$, $\frac{60}{120}$, $\frac{15}{120}$.

8. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ to fractions whose denominators will be 60.   *Ans.* $\frac{30}{60}$, $\frac{20}{60}$, $\frac{15}{60}$, $\frac{12}{60}$, $\frac{10}{60}$.

9. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ to fractions whose denominators will be 60.   *Ans.* $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$.

10. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{6}$, $\frac{7}{12}$, $\frac{12}{15}$ to fractions whose denominators will be 60.   *Ans.* $\frac{40}{60}$, $\frac{45}{60}$, $\frac{50}{60}$, $\frac{35}{60}$, $\frac{48}{60}$.

11. Reduce $4 = \frac{4}{1}$ to a fraction whose denominator will be 12.   *Ans.* $\frac{48}{12}$.

12. Reduce 2, 3, 4, 5, 6, 7, 8, 9 to a fraction whose denominator will be 5.

Ans. $\frac{10}{5}$, $\frac{15}{5}$, $\frac{20}{5}$, $\frac{25}{5}$, $\frac{30}{5}$, $\frac{35}{5}$, $\frac{40}{5}$, $\frac{45}{5}$

13. Reduce 3, 4, 5, 6, 7, 8, 9, 10 to fractions whose denominators will be 8.   *Ans.* $\frac{24}{8}$, $\frac{32}{8}$, $\frac{40}{8}$, etc.

14. Reduce 12, 13, 14, 15, 16 to fractions whose denominators will be 10.   *Ans.* $\frac{120}{10}$, $\frac{130}{10}$, $\frac{140}{10}$, etc.

LESSON VI.

217. To reduce a mixed number to an improper fraction.

RULE.

I. *Reduce the fractional part to its lowest terms.*

II. *Multiply the whole number by the reduced denominator, and to the result add the reduced numerator.*

III. *Write the sum over the reduced denominator.*

EXAMPLES.

1. Reduce $12\frac{5}{10}$ to an improper fraction.

OPERATION.

First reduce $\frac{5}{10}$ to its lowest terms, which is $\frac{1}{2}$. Multiply the whole number, 12, by the reduced denominator, 2, making 24, and add the reduced numerator, 1, making 25. Write 25 for the new numerator, and the reduced denominator, 2, for the new denominator. The fraction required will be $\frac{25}{2}$.

2. Reduce $2\frac{5}{10}$, $6\frac{4}{8}$, $9\frac{2}{4}$, to improper fractions.

Ans. $\frac{5}{2}$, $\frac{13}{2}$, $\frac{19}{2}$.

3. Reduce $3\frac{7}{21}$, $4\frac{8}{32}$, $19\frac{12}{24}$, $24\frac{5}{15}$, to improper fractions. *Ans.* $\frac{10}{3}$, $\frac{17}{4}$, $\frac{39}{2}$, $\frac{73}{3}$.

4. Reduce $11\frac{15}{45}$, $21\frac{20}{60}$, $84\frac{5}{10}$, to improper fractions. *Ans.* $\frac{34}{3}$, $\frac{64}{3}$, $\frac{169}{2}$.

5. Reduce $7\frac{17}{34}$, $6\frac{38}{57}$, $2\frac{66}{99}$, $9\frac{118}{...}$, to improper fractions. *Ans.* $\frac{15}{2}$, $\frac{20}{3}$, $\frac{11}{4}$, $\frac{42}{5}$.

6. Reduce $4\frac{32}{56}$, $25\frac{51}{85}$, $61\frac{69}{115}$, to improper fractions. *Ans.* $\frac{23}{5}$, $\frac{128}{5}$, $\frac{308}{5}$.

7. Reduce $5\frac{1}{2}$, $6\frac{1}{3}$, $7\frac{1}{4}$, $8\frac{1}{5}$, $9\frac{1}{6}$, to improper fractions. *Ans.* $\frac{11}{2}$, $\frac{19}{3}$, $\frac{29}{4}$, $\frac{41}{5}$, $\frac{55}{6}$.

8. Reduce $10\frac{1}{7}$, $11\frac{1}{8}$, $12\frac{1}{9}$, $13\frac{1}{10}$, to improper fractions. *Ans.* $\frac{71}{7}$, $\frac{89}{8}$, $\frac{109}{9}$, $\frac{131}{10}$.

9. Reduce $14\frac{1}{12}$, $15\frac{1}{13}$, $16\frac{1}{14}$, to improper fractions. *Ans.* $\frac{169}{12}$, $\frac{196}{13}$, $\frac{225}{14}$.

10. Reduce $21\frac{1}{2}$, $22\frac{1}{3}$, $23\frac{1}{4}$, $24\frac{1}{5}$, to improper fractions. *Ans.* $\frac{43}{2}$, $\frac{67}{3}$, $\frac{93}{4}$, $\frac{121}{5}$.

11. Reduce $31\frac{1}{2}$, $32\frac{1}{3}$, $33\frac{1}{4}$, $34\frac{1}{5}$, to improper fractions. *Ans.* $\frac{63}{2}$, $\frac{97}{3}$, $\frac{133}{4}$, $\frac{171}{5}$.

12. Reduce $41\frac{1}{3}$, $42\frac{1}{4}$, $43\frac{1}{5}$, $44\frac{1}{6}$, to improper fractions. *Ans.* $\frac{124}{3}$, $\frac{169}{4}$, $\frac{216}{5}$, $\frac{269}{6}$.

13. Reduce $51\frac{2}{3}$, $52\frac{3}{4}$, $53\frac{4}{5}$, $55\frac{5}{6}$, to improper fractions. *Ans.* $\frac{155}{3}$, $\frac{211}{4}$, $\frac{269}{5}$, $\frac{335}{6}$.

14. Reduce $61\frac{5}{6}$, $62\frac{7}{8}$, $63\frac{8}{9}$, to improper fractions. *Ans.* $\frac{371}{6}$, $\frac{503}{8}$, $\frac{575}{9}$.

15. Reduce $71\frac{8}{9}$, $81\frac{9}{10}$, $91\frac{11}{12}$, to improper fractions. *Ans.* $\frac{647}{9}$, $\frac{819}{10}$, $\frac{1103}{12}$.

16. Reduce $81\frac{4}{7}$, $82\frac{5}{8}$, $83\frac{5}{12}$, $84\frac{5}{10}$, to improper fractions. *Ans.* $\frac{573}{7}$, $\frac{247}{3}$, $\frac{167}{2}$, $\frac{169}{2}$.

17. Reduce $91\frac{1}{2}$, $92\frac{1}{3}$, $93\frac{1}{4}$, $94\frac{1}{5}$,
fractions.   Ans. $\frac{183}{2}$,

18. Reduce $1\frac{99}{100}$, $2\frac{99}{100}$, $3\frac{99}{100}$,
proper fractions.   Ans. $\frac{199}{100}$,

19. Reduce $6\frac{99}{100}$, $7\frac{99}{100}$, $8\frac{99}{100}$,
tions.

20. Reduce $2\frac{1}{99}$, $3\frac{1}{99}$, $4\frac{1}{99}$, $5\frac{1}{99}$,
tions.   Ans.

21. Reduce $2\frac{20}{33}$, $3\frac{21}{33}$, $4\frac{22}{33}$, to im

22. Reduce $1\frac{24}{100}$, $1\frac{25}{200}$, $1\frac{26}{300}$,
tions.

23. Reduce $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$, to in

24. Reduce $12\frac{1}{2}$, $13\frac{1}{2}$, $14\frac{1}{2}$, $15\frac{1}{2}$,
tions.

25. Reduce $121\frac{1}{3}$, $131\frac{1}{4}$, $141\frac{1}{5}$,
tions.

LESSON VII.

218. To reduce improper fr
numbers.

RULE.

Divide the numerator by the
write the remainder, if any, over th

EXAMPLES.

1. Reduce $\frac{21}{4}$ to a mixed numb

Divide 21 by 4. The answer w
be 5 and 1 over, which is written,

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219. To prove the sum correct, multiply the whole number by the denominator, and add the numerator. $4 \times 5 = 20$, $20 + 1 = 21$, or $\frac{21}{4}$.

2. Reduce $\frac{5}{2}$, $\frac{13}{2}$, $\frac{19}{2}$, to mixed numbers.

3. Reduce $\frac{10}{3}$, $\frac{11}{4}$, $\frac{39}{2}$, $\frac{73}{3}$, to mixed numbers.

4. Reduce $\frac{34}{3}$, $\frac{64}{3}$, $\frac{189}{2}$, to mixed numbers.

5. Reduce $\frac{15}{2}$, $\frac{20}{3}$, $\frac{11}{4}$, $\frac{49}{5}$, to mixed numbers.

6. Reduce $\frac{23}{5}$, $\frac{128}{5}$, $\frac{308}{5}$, to mixed numbers.

7. Reduce $\frac{11}{2}$, $\frac{19}{3}$, $\frac{29}{4}$, $\frac{41}{5}$, $\frac{55}{6}$, to mixed numbers.

8. Reduce $\frac{71}{7}$, $\frac{89}{8}$, $\frac{109}{9}$, $\frac{131}{10}$, to mixed numbers.

9. Reduce $\frac{169}{12}$, $\frac{196}{13}$, $\frac{125}{14}$, to mixed numbers.

10. Reduce $\frac{43}{2}$, $\frac{67}{3}$, $\frac{93}{4}$, $\frac{121}{5}$. to mixed numbers.

11. Reduce $\frac{63}{2}$, $\frac{97}{3}$, $\frac{133}{4}$, $\frac{171}{5}$, to mixed numbers.

12. Reduce $\frac{124}{3}$, $\frac{169}{4}$, $\frac{216}{5}$, $\frac{269}{6}$, to mixed numbers.

13. Reduce $\frac{155}{3}$, $\frac{211}{4}$, $\frac{260}{5}$, $\frac{335}{6}$, to mixed numbers.

14. Reduce $\frac{371}{6}$, $\frac{503}{8}$, $\frac{575}{9}$, to mixed numbers.

15. Reduce $\frac{647}{9}$, $\frac{819}{10}$, $\frac{1103}{12}$, to mixed numbers.

16. Reduce $\frac{573}{7}$, $\frac{247}{8}$, $\frac{167}{9}$, $\frac{169}{2}$, to mixed numbers.

17. Reduce $\frac{183}{2}$, $\frac{277}{3}$, $\frac{373}{4}$, $\frac{471}{5}$, to mixed numbers.

18. Reduce $\frac{571}{6}$, $\frac{100}{100}$, $\frac{200}{100}$, $\frac{300}{100}$, $\frac{400}{100}$, to mixed numbers.

19. Reduce $\frac{600}{100}$, $\frac{700}{100}$, $\frac{800}{100}$, to mixed numbers.

20. Reduce $\frac{199}{99}$, $\frac{298}{99}$, $\frac{397}{99}$, $\frac{496}{99}$, to mixed numbers.

21. Reduce $\frac{218}{99}$, $\frac{318}{99}$, $\frac{418}{99}$, to mixed numbers.

[*For answers, see reduction to mixed numbers.*]

ADDITION OF FRACTIONS.

LESSON VIII.

220. When the fractions have the same denominator.

RULE.

Add the numerators and place the sum over the common denominator. If an improper fraction is the result, reduce it to a mixed number.

EXAMPLES.

1. Add $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$.

OPERATION.

Adding the numerators gives 5, which written over the denominator gives $\frac{5}{2}$, which reduced is equal to $2\frac{1}{2}$.

2. Add $\frac{1}{3}, \frac{1}{3}, \frac{2}{3}, \frac{1}{3}$.   *Ans.* $\frac{5}{3}$ or $1\frac{2}{3}$.

3. Add $\frac{2}{4}, \frac{3}{4}, \frac{1}{4}, \frac{3}{4}$.   *Ans.* $\frac{9}{4}$ or $2\frac{1}{4}$.

4. Add $\frac{5}{6}, \frac{4}{6}, \frac{3}{6}, \frac{2}{6}$.   *Ans.* $\frac{14}{6}$ or $2\frac{2}{6}$ or $2\frac{1}{3}$.

5. Add $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}$.   *Ans.* $\frac{10}{7}$ or $1\frac{3}{7}$.

6. Add $\frac{1}{8}, \frac{5}{8}, \frac{7}{8}, \frac{7}{8}$.   *Ans.* $\frac{20}{8}$ or $2\frac{4}{8}$ or $2\frac{1}{2}$.

7. Add $\frac{1}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}$.   *Ans.* $\frac{17}{9}$ or $1\frac{8}{9}$.

8. Add $\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}$.   *Ans.* $\frac{20}{10}$ or 2.

9. Add $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{10}{11}$.   *Ans.* $\frac{16}{11}$ or $1\frac{5}{11}$.

10. Add $\frac{5}{12}, \frac{7}{12}, \frac{3}{12}, \frac{4}{12}$.   *Ans.* $\frac{19}{12}$ or $1\frac{7}{12}$.

11. Add $\frac{1}{13}, \frac{5}{13}, \frac{4}{13}, \frac{5}{13}$.   *Ans.* $\frac{15}{13}$ or $1\frac{2}{13}$.

12. Add $\frac{1}{14}, \frac{5}{14}, \frac{9}{14}, \frac{11}{14}$.   *Ans.* $\frac{24}{14}$ or $1\frac{12}{14}$ or $1\frac{5}{7}$.

13. Add $\frac{1}{15}, \frac{4}{15}, \frac{9}{15}, \frac{7}{15}$.   *Ans.* $\frac{21}{15}$ or $1\frac{6}{15}$ or $1\frac{2}{5}$.

14. Add $\frac{5}{16}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$.　　　　*Ans.* $\frac{32}{16}$ or 2.

15. Add $\frac{8}{17}$, $\frac{9}{17}$, $\frac{10}{17}$, $\frac{11}{17}$.　　　*Ans.* $\frac{38}{17}$ or $2\frac{4}{17}$.

16. Add $\frac{5}{18}$, $\frac{7}{18}$, $\frac{11}{18}$, $\frac{13}{18}$.　　　*Ans.* $\frac{36}{18}$ or 2.

17. Add $\frac{8}{19}$, $\frac{9}{19}$, $\frac{10}{19}$, $\frac{11}{19}$.　　　*Ans.* $\frac{38}{19}$ or 2.

18. Add $\frac{1}{20}$, $\frac{3}{20}$, $\frac{7}{20}$, $\frac{19}{20}$.　　*Ans.* $\frac{30}{20}$ or $1\frac{10}{20}$ or $1\frac{1}{2}$.

19. Add $\frac{2}{21}$, $\frac{4}{21}$, $\frac{5}{21}$, $\frac{8}{21}$.　　　　*Ans.* $\frac{19}{21}$.

20. Add $\frac{3}{22}$, $\frac{7}{22}$, $\frac{9}{22}$, $\frac{13}{22}$.　*Ans.* $\frac{32}{22}$ or $1\frac{10}{22}$ or $1\frac{8}{11}$.

21. Add $\frac{18}{23}$, $\frac{19}{23}$, $\frac{20}{23}$, $\frac{21}{23}$.　　　*Ans.* $\frac{78}{23}$ or $3\frac{9}{23}$.

LESSON IX.

221. When the denominators are prime to each other.

RULE.

Multiply the numerators by all the denominators except its own for a new numerator.

Multiply all the denominators together for a new denominator.

Add the new numerators and write their sum over the common denominator.

EXAMPLES.

1. Add $\frac{1}{4}$ and $\frac{1}{5}$.

OPERATION.

$$\frac{1}{4} + \frac{1}{5} = \frac{(5\times1)}{20} + \frac{(4\times1)}{20} = \frac{5+4}{20} = \frac{9}{20}$$

2. Add $\frac{1}{2}+\frac{1}{3}+\frac{3}{4}$.　　　　*Ans.* $\frac{19}{12}$ or $1\frac{7}{12}$.

3. Add $\frac{1}{3}+\frac{1}{4}+\frac{4}{5}$.　　　　*Ans.* $\frac{83}{60}$ or $1\frac{23}{60}$.

4. Add $\frac{1}{4}+\frac{1}{5}+\frac{5}{6}$.　　*Ans.* $\frac{154}{120}$ or $1\frac{34}{120}$ or $1\frac{17}{60}$.

5. Add $\frac{1}{5}+\frac{1}{6}+\frac{4}{7}$.　　　*Ans.* $\frac{257}{210}$ or $1\frac{47}{210}$.

6. Add $\frac{1}{6}+\frac{1}{7}+\frac{7}{11}$. *Ans.* $\frac{437}{462}$.

7. Add $\frac{1}{7}+\frac{1}{8}+\frac{3}{5}$. *Ans.* $\frac{583}{504}$.

8. Add $\frac{1}{8}+\frac{1}{9}+\frac{9}{13}$. *Ans.* $\frac{865}{936}$.

9. Add $\frac{1}{9}+\frac{1}{10}+\frac{10}{17}$. *Ans.* $\frac{1223}{1530}$.

10. Add $\frac{1}{10}+\frac{1}{11}+\frac{5}{7}$. *Ans.* $\frac{697}{770}$.

11. Add $\frac{1}{11}+\frac{1}{12}+\frac{4}{5}$. *Ans.* $\frac{843}{660}$.

12. Add $\frac{1}{12}+\frac{1}{13}+\frac{6}{17}$. *Ans.* $\frac{1361}{2652}$.

13. Add $\frac{1}{13}+\frac{1}{14}+\frac{8}{23}$. *Ans.* $\frac{2017}{4186}$.

14. Add $\frac{1}{14}+\frac{1}{15}+\frac{9}{17}$. *Ans.* $\frac{2383}{3570}$.

15. Add $\frac{1}{15}+\frac{1}{16}+\frac{12}{19}$. *Ans.* $\frac{3468}{4560}$.

16. Add $\frac{1}{16}+\frac{1}{17}+\frac{4}{5}$. *Ans.* $\frac{1263}{1360}$.

17. Add $\frac{1}{17}+\frac{1}{18}+\frac{5}{7}$. *Ans.* $\frac{1115}{1142}$.

18. Add $\frac{1}{18}+\frac{1}{19}+\frac{3}{11}$. *Ans.* $\frac{1433}{3762}$.

19. Add $\frac{1}{19}+\frac{1}{20}+\frac{18}{27}$. *Ans.* $\frac{7893}{10260}$.

20. Add $\frac{1}{20}+\frac{1}{21}+\frac{19}{31}$. *Ans.* $\frac{11251}{13020}$.

21. Add $\frac{1}{21}+\frac{1}{22}+\frac{2}{17}$. *Ans.* $\frac{1655}{7854}$.

LESSON X.

222. When the denominators are not all prime to each other.

RULE.

I. *Find the least common multiple of the denominators.*

II. *Divide the least common multiple by the denominator of every one of the fractions.*

III. *Multiply the numerator of every one of the fractions by the quotient found by dividing the least common multiple by the denominator. The result will be the numerator of the fraction.*

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IV. *Add the several numerators thus found, and write the sum over the least common multiple.*

V. *If the sum makes an improper fraction, reduce it to a mixed number, and reduce the fractional part to its lowest terms if possible.*

EXAMPLES.

1. Add $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{5}{6}$.

OPERATION.

The common multiple of 2, 3, 4 and 6 is 12. Since the multiple or denominator will be the same for them all, it would do as well to write it under one line of sufficient length to hold all of the new numerators; thus:

$$\frac{6+8+9+10}{12}$$

Divide the denominator 2 into the multiple: 2 in 12, 6 times. Multiply the numerator 1 by 6: 6 times 1 are 6. Write the figure 6 on the line above the multiple, as above. Take the next denominator: 3 in 12, 4 times. 4 times 2 are 8. Write the figure 8 above the line. Take the next denominator: 4 in 12, 3 times. 3 times 3 are 9. Write the figure 9 on the line. Take the next denominator: 6 in 12, 2 times. 2 times 5 are 10. Write the figure 10 above the line. The pupil must not omit to write the sign (+) plus between the numerators. Add the several numerators:

$$6+8+9+10=33$$

The answer is 33 for the numerator and 12 for the denominator, or $\frac{33}{12}$, which is an improper fraction. Reduce the improper fraction to a mixed number by dividing the numerator by the denominator: 12 in 33, 2 times and 9 over. The answer as found is $\left(2\frac{9}{12}\right)$ 2 whole ones and $\left(\frac{9}{12}\right)$ nine twelfths. Reducing $\frac{9}{12}$ to its lowest terms by dividing both numerator and denominator, 9 and 12, by 3, we get 3 and 4. 3 for the numerator of the reduced fraction, and 4 for its denominator. The true answer is $2\frac{3}{4}$.

223. NOTE.—All answers should be reduced to their lowest terms.

224. NOTE.—By making the pupil analyse a sum as given above, all the preceding rules will be repeated, and the pupil will begin to understand the meaning of what he has learned in what is called "Properties of Numbers."

2. Add $\frac{1}{2} + \frac{1}{3}$.   *Ans.* $\frac{5}{6}$.

3. Add $\frac{1}{3} + \frac{1}{4}$.   *Ans.* $\frac{7}{12}$.

4. Add $\frac{1}{4} + \frac{1}{5}$.   *Ans.* $\frac{9}{20}$.

5. Add $\frac{1}{5} + \frac{1}{6}$.   *Ans.* $\frac{11}{30}$.

6. Add $\frac{1}{6} + \frac{1}{7}$.   *Ans.* $\frac{13}{42}$.

7. Add $\frac{1}{7} + \frac{1}{8}$.   *Ans.* $\frac{15}{56}$.

8. Add $\frac{1}{8} + \frac{1}{9}$.   *Ans.* $\frac{17}{72}$.

9. Add $\frac{1}{9} + \frac{1}{10}$.   *Ans.* $\frac{19}{90}$.

10. Add $\frac{1}{10} + \frac{1}{11}$.   *Ans.* $\frac{21}{110}$.

11. Add $\frac{1}{11} + \frac{1}{12}$.   *Ans.* $\frac{23}{132}$.

12. Add $\frac{1}{12} + \frac{1}{13}$.   *Ans.* $\frac{25}{156}$.

13. Add $\frac{1}{13} + \frac{1}{14}$.   *Ans.* $\frac{27}{182}$.

14. Add $\frac{1}{14} + \frac{1}{15}$.   *Ans.* $\frac{29}{210}$.

15. Add $\frac{7}{14} + \frac{7}{15}$.   *Ans.* $\frac{2}{3}\frac{2}{3}$.

16. Add $\frac{4}{13}+\frac{4}{14}$.   *Ans.* $\frac{54}{91}$.

17. Add $\frac{2}{12}+\frac{6}{13}$.   *Ans.* $\frac{49}{78}$.

18. Add $\frac{8}{11}+\frac{6}{12}$.   *Ans.* $1\frac{5}{22}$.

19. Add $\frac{7}{10}+\frac{3}{11}$.   *Ans.* $\frac{107}{110}$.

20. Add $\frac{2}{8}+\frac{3}{6}+\frac{4}{8}$.   *Ans.* $1\frac{1}{4}$.

21. Add $\frac{1}{9}+\frac{2}{18}+\frac{4}{36}$.   *Ans.* $\frac{1}{3}$.

22. Add $\frac{1}{3}+\frac{3}{9}+\frac{4}{12}+\frac{5}{15}$.   *Ans.* $1\frac{1}{3}$.

23. Add $\frac{1}{4}+\frac{4}{16}+\frac{5}{20}+\frac{6}{24}+\frac{7}{28}$.   *Ans.* $1\frac{1}{4}$.

24. Add $\frac{1}{5}+\frac{2}{10}+\frac{3}{15}+\frac{4}{20}+\frac{5}{25}+\frac{6}{30}$.   *Ans.* $1\frac{1}{5}$.

25. Add $\frac{2}{6}+\frac{4}{12}+\frac{9}{18}+\frac{27}{54}$.   *Ans.* $1\frac{2}{3}$.

26. Add $\frac{3}{8}+\frac{4}{8}+\frac{6}{16}+\frac{8}{16}$.   *Ans.* $1\frac{3}{4}$.

27. Add $\frac{4}{13}+\frac{8}{26}+\frac{16}{52}+\frac{32}{104}$.   *Ans.* $1\frac{3}{13}$.

28. Add $\frac{4}{15}+\frac{8}{30}+\frac{16}{60}+\frac{32}{120}$.   *Ans.* $1\frac{1}{15}$.

29. Add $\frac{5}{16}+\frac{10}{32}+\frac{20}{64}+\frac{40}{128}$.   *Ans.* $1\frac{1}{4}$.

30. Add $\frac{2}{17}+\frac{4}{34}+\frac{8}{68}+\frac{16}{136}+\frac{32}{272}$.   *Ans.* $\frac{10}{17}$.

31. Add $\frac{2}{19}+\frac{4}{38}+\frac{8}{76}+\frac{16}{152}+\frac{32}{304}+\frac{64}{608}$.   *Ans.* $\frac{12}{19}$.

32. Add $\frac{4}{12}+\frac{16}{48}+\frac{32}{96}+\frac{72}{192}+\frac{144}{384}$.   *Ans.* $2\frac{1}{3}$.

33. Add $\frac{1}{2}+\frac{3}{4}$.   *Ans.* $1\frac{1}{4}$.

34. Add $\frac{3}{4}+\frac{5}{6}+\frac{6}{7}$.   *Ans.* $2\frac{11}{24}$.

35. Add $\frac{3}{4}+\frac{6}{7}+\frac{7}{12}$.   *Ans.* $2\frac{4}{21}$.

36. Add $\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}$.   *Ans.* $\frac{319}{420}$.

37. Add $\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$.   *Ans.* $3\frac{101}{420}$.

38. Add $\frac{3}{4}+\frac{3}{5}+\frac{5}{6}+\frac{3}{7}$.   *Ans.* $2\frac{257}{420}$.

39. Add $\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}$.   *Ans.* $\frac{1879}{2520}$.

40. Add $\frac{4}{5}+\frac{5}{6}+\frac{6}{7}+\frac{7}{8}+\frac{8}{9}$.   *Ans.* $4\frac{641}{2520}$.

41. Add $\frac{3}{5}+\frac{5}{6}+\frac{4}{7}+\frac{5}{8}+\frac{8}{9}$.   *Ans.* $3\frac{1307}{2520}$.

42. Add $\frac{2}{5}+\frac{5}{6}+\frac{3}{7}+\frac{3}{8}+\frac{6}{9}$.   *Ans.* $2\frac{1483}{2520}$.

43. Add $\frac{5}{6}+\frac{6}{7}+\frac{7}{8}+\frac{8}{9}+\frac{9}{10}+\frac{10}{11}$. *Ans.* $5\frac{7303}{27720}$.

44. Add $\frac{2}{6}+\frac{4}{7}+\frac{6}{8}+\frac{2}{9}+\frac{7}{10}+\frac{3}{11}$. *Ans.* $3\frac{7769}{27720}$.

45. Add $\frac{1}{6}+\frac{5}{7}+\frac{3}{8}+\frac{7}{9}+\frac{3}{10}+\frac{10}{11}$. *Ans.* $3\frac{6731}{27720}$.

46. Add $\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\frac{1}{12}+\frac{1}{13}$. *Ans.* $\frac{283111}{360360}$.

47. Add $\frac{2}{7}+\frac{5}{8}+\frac{6}{9}+\frac{7}{10}+\frac{3}{11}+\frac{6}{12}+\frac{12}{13}$.

$\qquad\qquad\qquad\qquad\qquad$ *Ans.* $3\frac{230627}{360360}$.

48. Add $\frac{3}{7}+\frac{7}{8}+\frac{2}{9}+\frac{3}{10}+\frac{8}{11}+\frac{7}{12}+\frac{10}{13}$.

$\qquad\qquad\qquad\qquad\qquad$ *Ans.* $3\frac{326353}{360360}$.

49. Add $\frac{5}{7}+\frac{7}{8}+\frac{4}{9}+\frac{9}{10}+\frac{2}{11}+\frac{11}{12}+\frac{6}{13}$.

$\qquad\qquad\qquad\qquad\qquad$ *Ans.* $5\frac{17569}{360360}$.

50. Add $\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\frac{1}{12}+\frac{1}{13}+\frac{1}{14}+\frac{1}{15}$.

$\qquad\qquad\qquad\qquad\qquad$ *Ans.* $\frac{261385}{360360}$.

51. Add $\frac{7}{8}+\frac{8}{9}+\frac{9}{10}+\frac{10}{11}+\frac{11}{12}+\frac{12}{13}+\frac{13}{14}$.

$\qquad\qquad\qquad\qquad\qquad$ *Ans.* $7\frac{16793}{120120}$.

52. Add $\frac{6}{8}+\frac{7}{9}+\frac{8}{10}+\frac{9}{11}+\frac{10}{12}+\frac{11}{13}+\frac{12}{14}+\frac{13}{15}$.

$\qquad\qquad\qquad\qquad\qquad$ *Ans.* $6\frac{197330}{360360}$.

53. Add $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$. *Ans.* $2\frac{3}{4}$.

54. Add $\frac{5}{6}+\frac{7}{12}+\frac{13}{18}+\frac{25}{36}$. *Ans.* $2\frac{5}{6}$.

55. Add $\frac{3}{8}+\frac{5}{10}+\frac{15}{25}$. *Ans.* $1\frac{13}{30}$.

56. Add $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$. *Ans.* $1\frac{9}{20}$.

57. Add $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}$. *Ans.* $1\frac{13}{140}$.

58. Add $\frac{1}{2}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}$. *Ans.* $1\frac{813}{2520}$.

59. Add $\frac{1}{4}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}$. *Ans.* $1\frac{237}{2520}$.

60. Add $\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{11}$. *Ans.* $1\frac{5197}{27720}$.

61. Add $\frac{1}{3}+\frac{1}{2}+\frac{3}{5}$. *Ans.* $1\frac{13}{30}$.

62. Add $\frac{1}{4}+\frac{3}{5}+\frac{7}{8}+\frac{5}{8}$. *Ans.* $2\frac{221}{30}$.

63. Add $\frac{5}{6}+\frac{1}{2}+\frac{2}{3}+\frac{4}{5}$. *Ans.* $2\frac{4}{5}$.

64. Add $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\frac{1}{12}$ $+\frac{1}{13}$. *Ans.*

LESSON XI.

225. To add mixed numbers.

RULE I.

I. *Add the whole numbers.*

II. *Add the fractional parts.*

III. *Add the sum of the fractional parts to the sum of the whole numbers for the answer.*

226. Or:

RULE II.

Reduce the mixed numbers to improper fractions, and add the improper fractions thus formed, as in addition of fractions.

1. Add $2\frac{1}{2} + 2\frac{1}{5}$.   *Ans.* $4\frac{7}{10}$.
2. Add $3\frac{1}{3} + 3\frac{1}{4}$.   *Ans.* $6\frac{7}{12}$.
3. Add $4\frac{1}{4} + 4\frac{1}{5}$.   *Ans.* $8\frac{9}{20}$.
4. Add $5\frac{1}{5} + 5\frac{1}{6}$.   *Ans.* $10\frac{11}{30}$.
5. Add $6\frac{1}{4} + 6\frac{1}{7} + 7\frac{1}{8}$.   *Ans.* $19\frac{73}{168}$.
6. Add $7\frac{2}{7} + 7\frac{3}{7} + 7\frac{4}{7}$.   *Ans.* $22\frac{2}{7}$.
7. Add $8\frac{1}{7} + 8\frac{1}{9} + 8\frac{1}{10}$.   *Ans.* $24\frac{223}{630}$.
8. Add $8\frac{4}{7} + 8\frac{5}{9} + 8\frac{3}{10}$.   *Ans.* $25\frac{319}{630}$.
9. Add $9\frac{2}{5} + 9\frac{4}{3} + 9\frac{5}{6} + 9\frac{4}{7}$.   *Ans.* $39\frac{33}{210}$.
10. Add $10\frac{1}{4} + 10\frac{3}{8} + 10\frac{4}{11}$.   *Ans.* $31\frac{47}{220}$.
11. Add $11\frac{1}{3} + 11\frac{7}{4} + 11\frac{5}{8}$.   *Ans.* $34\frac{5}{6}$.
12. Add $12\frac{2}{3} + 13\frac{1}{2} + 13\frac{3}{4} + 18\frac{1}{6}$.   *Ans.* $58\frac{1}{12}$.
13. Add $13\frac{2}{7} + 14\frac{5}{6} + 21\frac{1}{14} + 22\frac{1}{18}$.   *Ans.* $70\frac{22}{126}$.
14. Add $14\frac{2}{5} + 15\frac{3}{8} + 19\frac{1}{10} + 12\frac{1}{16}$.   *Ans.* $60\frac{15}{80}$.
15. Add $15\frac{1}{3} + 16\frac{1}{4} + 18\frac{1}{4} + 19\frac{1}{12}$.   *Ans.* $63\frac{21}{33}$.
16. Add $16\frac{2}{9} + 17\frac{3}{10} + 18\frac{7}{18} + 19\frac{21}{30}$.   *Ans.* $71\frac{55}{90}$.
17. Add $17\frac{2}{3} + 18\frac{5}{6} + 19\frac{10}{13} + 24\frac{3}{26}$.   *Ans.* $80\frac{9}{13}$.

18. Add $18\frac{1}{2} + 19\frac{1}{3} + 20\frac{1}{4} + 21\frac{1}{5} + 22\frac{1}{6}$. *Ans.* $101\frac{27}{60}$.

19. Add $19\frac{1}{3} + 20\frac{1}{2} + 21\frac{1}{4} + 25\frac{1}{6} + 26\frac{4}{7} + 18\frac{2}{8}$.

Ans. $131\frac{47}{252}$.

20. Add $20\frac{2}{3} + 30\frac{3}{4} + 40\frac{4}{5} + 50\frac{5}{6} + 60\frac{6}{7} + 70\frac{7}{8}$.

Ans. $274\frac{557}{840}$.

LESSON XII.

21. A grocer bought 4 firkins of butter, weighing respectively $24\frac{1}{2}$, $25\frac{1}{3}$, $26\frac{1}{4}$, $27\frac{1}{5}$ pounds. How many pounds in the 4 firkins?　*Ans.* $103\frac{17}{60}$.

22. A person traveled in one day $25\frac{1}{4}$ miles, $26\frac{2}{3}$ in the next, $24\frac{1}{2}$ in the third, and $27\frac{1}{8}$ in the fourth. How many miles did he travel in 4 days?

Ans. $103\frac{13}{24}$.

23. Jas. Brown owes $2\frac{5}{8}$ dollars to one man, $6\frac{2}{3}$ dollars to a second, and $24\frac{1}{4}$ dollars to a third. How much does he owe to the three men?　*Ans.* $\$33\frac{5}{24}$.

24. A merchant sold 6 hams. The weight of them was as follows: $10\frac{1}{2}$, $12\frac{1}{4}$, $13\frac{1}{8}$, $9\frac{2}{3}$, $15\frac{5}{6}$, $16\frac{3}{7}$ pounds. How many pounds did they weigh?

Ans. $77\frac{3}{8}$.

25. An auctioneer sold 4 farms, containing the following number of acres: $17\frac{1}{4}$, $25\frac{3}{4}$, $46\frac{7}{15}$, $64\frac{7}{13}$ acres. How many acres were in the four farms together; and how much did they cost, at one dollar an acre?　*Ans.*

26. A man sold three horses for the following sums of money: the first horse for $22\frac{1}{4}$ dollars, the second horse for $30\frac{1}{3}$ dollars, and the third horse for $25\frac{1}{4}$ dollars. How much did he get for the three horses?　*Ans.*

LESSON XIII.

227. When whole numbers and mixed numbers are found in the same problem.

RULE I.

Add the whole numbers to those of the mixed numbers.

Add the fractions of the mixed numbers.

Add the sum of the fractions of the mixed numbers to the sum of the whole numbers. The sum will be the answer required.

228. Or:

RULE II.

Consider the denominator of the whole number as 1. Reduce the mixed numbers to improper fractions, and add.

229. Note.—Since the denominator of the whole number is 1, multiply the numerator of the whole number (which is itself the whole number) by the common multiple for the new denominator.

EXAMPLES.

1. Add 2, $3\frac{1}{2}$, $4\frac{1}{4}$.

OPERATION.

Consider the denominator of the whole number 2 as 1, and reduce the mixed numbers to improper fractions. We then have $\frac{2}{1} + \frac{7}{2} + \frac{17}{4}$, the common denominator of which is 4.

Write the 4 beneath the line, as below:

$$\frac{8+14+17}{4} = \frac{39}{4} = 9\tfrac{3}{4} \quad Ans.$$

Begin the division of the multiple by the denominators of the given fractions, and multiply the denominators; thus: 1 in 4, 4 times; 4 times 2 are 8, the first numerator. 2 into 4, 2 times; 2 times 7 are 14, the second numerator. 4 in 4, 1 time; once 17 is 17, the last numerator. Add the numerators, $8+14+17=39$, and write it above the denominator, 4; as, $\tfrac{39}{4}$. Reducing $\tfrac{39}{4}$ to a mixed number gives, 4 into 39, 9 times and $\tfrac{3}{4}$ over. The answer will be $9\tfrac{3}{4}$.

2. Add $2+2\tfrac{1}{2}+2\tfrac{2}{3}$.	*Ans.* $7\tfrac{1}{6}$.
3. Add $3+4+2\tfrac{1}{2}$.	*Ans.* $9\tfrac{1}{2}$.
4. Add $4+5\tfrac{1}{2}+6\tfrac{2}{5}$.	*Ans.* $15\tfrac{9}{10}$.
5. Add $5\tfrac{3}{8}+5+5\tfrac{4}{3}$.	*Ans.* $15\tfrac{5}{12}$.
6. Add $6\tfrac{2}{3}+4+8\tfrac{1}{8}+20$.	*Ans.* $38\tfrac{7}{8}$.
7. Add $7\tfrac{5}{6}+7\tfrac{6}{7}+7\tfrac{7}{8}+4$.	*Ans.* $27\tfrac{95}{168}$.
8. Add $8\tfrac{3}{4}+8\tfrac{4}{5}+18\tfrac{5}{6}+19$.	*Ans.* $55\tfrac{23}{60}$.
9. Add $9\tfrac{1}{4}+10\tfrac{2}{5}+11\tfrac{3}{4}+14$.	*Ans.* $45\tfrac{2}{5}$.
10. Add $10\tfrac{1}{6}+11\tfrac{5}{7}+21+22$.	*Ans.* $64\tfrac{37}{42}$.
11. Add $11\tfrac{2}{3}+5\tfrac{4}{9}+18+22$.	*Ans.* $57\tfrac{1}{9}$.
12. Add $12\tfrac{1}{9}+13\tfrac{5}{18}+16+80$.	*Ans.* $121\tfrac{7}{18}$.
13. Add $13\tfrac{2}{3}+14\tfrac{5}{6}+18\tfrac{1}{12}+14$.	*Ans.* $60\tfrac{11}{36}$.
14. Add $14\tfrac{6}{7}+18+81+9\tfrac{4}{21}$.	*Ans.* $123\tfrac{1}{21}$.
15. Add $22\tfrac{1}{8}+19\tfrac{7}{4}+14\tfrac{3}{4}+18$.	*Ans.* $74\tfrac{3}{4}$.
16. Add $141\tfrac{2}{8}+248\tfrac{6}{24}+188$.	*Ans.* $577\tfrac{3}{12}$.
17. Add $2263\tfrac{1}{3}+1497\tfrac{2}{3}+100$.	*Ans.* 3861.
18. Add $1999\tfrac{1}{18}+876\tfrac{17}{18}+10+12$.	*Ans.* 2898.

17

SUBTRACTION OF FRACTIONS.

LESSON XIV.

230. SUBTRACTION OF FRACTIONS is the process of subtracting one fraction from another.

CASE I.

When the fractions have the same denominator.

RULE.

231. *Subtract one numerator from the other, and write the difference over the common denominator.*

EXAMPLES.

1. From $\frac{4}{5}$ take $\frac{3}{5}$.   *Ans.* $\frac{1}{5}$.

OPERATION.

232. The denominators being the same and common to both, subtract the numerator 3 from the numerator 4, and write the difference, 1, over the common denominator, 5. The fraction thus formed is $\frac{1}{5}$.

2. From $\frac{5}{7}$ take $\frac{2}{7}$.   *Ans.* $\frac{3}{7}$.
3. From $\frac{9}{19}$ take $\frac{5}{19}$.   *Ans.* $\frac{4}{19}$.
4. From $\frac{13}{21}$ take $\frac{12}{21}$.   *Ans.* $\frac{1}{21}$.
5. From $\frac{17}{36}$ take $\frac{14}{36}$.   *Ans.* $\frac{3}{36}$ or $\frac{1}{12}$.
6. From $\frac{88}{99}$ take $\frac{20}{99}$.   *Ans.* $\frac{68}{99}$.
7. From $\frac{141}{200}$ take $\frac{41}{200}$.   *Ans.* $\frac{100}{200}$ or $\frac{1}{2}$.
8. From $\frac{840}{981}$ take $\frac{64}{981}$.   *Ans.* $\frac{576}{981}$.
9. From $\frac{724}{941}$ take $\frac{124}{941}$.   *Ans.* $\frac{600}{941}$.

10. From $\frac{267}{1872}$ take $\frac{167}{1872}$.   *Ans.* $\frac{100}{1872}$.

11. From $\frac{627}{527}$ take $\frac{352}{527}$.   *Ans.* $\frac{275}{527}$.

12. From $\frac{333}{900}$ take $\frac{241}{900}$.   *Ans.* $\frac{92}{900}$.

13. From $\frac{2222}{3333}$ take $\frac{333}{3333}$.   *Ans.* $\frac{1889}{3333}$.

14. From $\frac{1234}{5678}$ take $\frac{123}{5678}$.   *Ans.* $\frac{1111}{5678}$.

LESSON XV.

CASE II.

233. When there are two fractions whose nu-merators are 1, and whose denominators are not common.

RULE.

Multiply the denominators together for the new denominator.

Subtract the denominators, and write the difference over the new denominator.   Reduce, if possible. The fraction thus formed will be the answer required.

EXAMPLES.

1. From $\frac{1}{4}$ take $\frac{1}{8}$.   *Ans.* $\frac{1}{8}$.

OPERATION.

Multiply the denominators 8 and 4: 8 times 4 are 32, the new denominator.   Subtract the denominators: 4 from 8 leaves 4, the new numerator.   The fraction is now $\frac{4}{32}$, which reduced, by dividing both numerator and denominator by 4, 4)$\frac{4}{32}=\frac{1}{8}$, the required answer.

2. From $\frac{1}{8}$ take $\frac{1}{9}$.   *Ans.* $\frac{1}{72}$.

3. From $\frac{1}{9}$ take $\frac{1}{10}$.   *Ans.* $\frac{1}{90}$.

4. From $\frac{1}{10}$ take $\frac{1}{11}$.   *Ans.* $\frac{1}{110}$.

5. From $\frac{1}{11}$ take $\frac{1}{12}$.   *Ans.* $\frac{1}{132}$.

6. From $\frac{1}{12}$ take $\frac{1}{13}$.   *Ans.* $\frac{1}{156}$.

7. From $\frac{1}{13}$ take $\frac{1}{14}$.   *Ans.* $\frac{1}{182}$.

8. From $\frac{1}{14}$ take $\frac{1}{15}$.   *Ans.* $\frac{1}{210}$.

9. From $\frac{1}{15}$ take $\frac{1}{16}$.   *Ans.* $\frac{1}{240}$.

10. From $\frac{1}{16}$ take $\frac{1}{17}$.   *Ans.* $\frac{1}{272}$.

11. From $\frac{1}{17}$ take $\frac{1}{18}$.   *Ans.* $\frac{1}{306}$.

LESSON XVI.

CASE III.

234. When the numerators are other figures than 1.

RULE.

Find the least common multiple of the denominators.

Find the numerators by dividing the common multiple by the denominator and multiplying the numerators by the quotient. Subtract the numerators, and write their difference over the common multiple. Reduce, if possible.

EXAMPLES.

1. From $\frac{5}{6}$ take $\frac{5}{8}$.

OPERATION.

$$\frac{20 - 15}{24} = \frac{5}{24}$$

The common multiple between 6 and 8 is 24, the new denominator. Divide 24 by 6, one of the

numerators; the quotient is 4. Multiply the numerator 5 by 4; the product is 20. Write 20 above the line. Divide the multiple 24 by 8; the quotient is **3.** Multiplying the numerator 5 by 3 gives 15. Write 15 as above. Subtract the numerator 15 from the numerator 20, and write the difference over the common multiple.

2. From $\frac{3}{4}$ take $\frac{1}{8}$.   *Ans.* $\frac{5}{8}$.

3. From $\frac{7}{8}$ take $\frac{2}{16}$.   *Ans.* $\frac{3}{4}$.

4. From $\frac{9}{10}$ take $\frac{12}{30}$.   *Ans.* $\frac{1}{2}$.

5. From $\frac{10}{11}$ take $\frac{2}{14}$.   *Ans.* $\frac{9}{11}$.

6. From $\frac{11}{12}$ take $\frac{6}{13}$.   *Ans.* $\frac{71}{156}$.

7. From $\frac{12}{13}$ take $\frac{5}{8}$.   *Ans.* $\frac{31}{104}$.

8. From $\frac{13}{14}$ take $\frac{3}{6}$.   *Ans.* $\frac{9}{21}$.

9. From $\frac{14}{15}$ take $\frac{3}{18}$.   *Ans.* $\frac{23}{30}$.

10. From $\frac{15}{16}$ take $\frac{4}{19}$.   *Ans.* $\frac{221}{304}$.

11. From $\frac{16}{17}$ take $\frac{6}{12}$.   *Ans.* $\frac{107}{204}$.

12. From $\frac{17}{18}$ take $\frac{4}{13}$.   *Ans.* $\frac{149}{234}$.

13. From $\frac{18}{19}$ take $\frac{5}{14}$.   *Ans.* $\frac{157}{266}$.

14. From $\frac{19}{20}$ take $\frac{2}{12}$.   *Ans.* $\frac{47}{60}$.

15. From $\frac{20}{21}$ take $\frac{6}{18}$.   *Ans.* $\frac{13}{21}$.

16. From $\frac{21}{22}$ take $\frac{7}{14}$.   *Ans.* $\frac{5}{11}$.

17. From $\frac{22}{23}$ take $\frac{11}{26}$.   *Ans.* $\frac{312}{598}$.

18. From $\frac{23}{24}$ take $\frac{6}{8}$.   *Ans.* $\frac{1}{4}$.

19. From $\frac{24}{25}$ take $\frac{3}{15}$.   *Ans.* $\frac{19}{25}$.

20. From $\frac{25}{26}$ take $\frac{4}{13}$.   *Ans.* $\frac{17}{26}$.

21. From $\frac{26}{27}$ take $\frac{6}{9}$.   *Ans.* $\frac{20}{27}$.

17*

LESSON XVII.

CASE IV.

235. When one mixed number is to be subtracted from another.

RULE.

Reduce the mixed numbers to improper fractions. Find the least common multiple of the denominators. Divide the least common multiple by the denominator, and multiply the numerator of the improper fraction by the quotient. Subtract the numerators, and write the difference over the common multiple. Reduce, if possible.

EXAMPLES.

1. From $12\frac{5}{8}$ take $5\frac{5}{6}$.

OPERATION.

Reducing the mixed numbers to improper fractions, gives $\frac{101}{8}$ and $\frac{29}{6}$. The least common multiple between 8 and 6 is 24, the new denominator.

$$\frac{303 - 116}{24}$$

8 into 24, 3 times; 3 times 101 gives 303, the new numerator. 6 into 24, 4 times; 4 times 29 gives 116, the other new numerator. Subtracting 116 from 303 leaves 187. Writing 187 over the least common multiple, 24, gives $\frac{187}{24}$ for the required fraction. Reducing $\frac{187}{24}$ to a mixed number gives $7\frac{19}{24}$, the required answer.

2. From $12\frac{2}{3}$ take $4\frac{1}{5}$.　　　　　*Ans.* $8\frac{7}{15}$.

3. From $13\frac{3}{4}$ take $5\frac{2}{3}$.　　　　　*Ans.* $8\frac{1}{12}$.

4. From $14\frac{4}{5}$ take $6\frac{7}{8}$.　　　　　*Ans.* $7\frac{37}{40}$.

5. From $15\frac{5}{6}$ take $7\frac{9}{10}$.　　　　　*Ans.* $7\frac{14}{15}$.

6. From $16\frac{6}{7}$ take $8\frac{1}{9}$.　　　　　*Ans.* $8\frac{32}{115}$.

7. From $17\frac{7}{8}$ take $9\frac{5}{6}$.　　　　　*Ans.* $8\frac{1}{24}$.

8. From $18\frac{8}{9}$ take $10\frac{4}{7}$.　　　　　*Ans.* $8\frac{2}{63}$.

9. From $19\frac{9}{10}$ take $11\frac{7}{8}$.　　　　　*Ans.* $8\frac{1}{40}$.

10. From $20\frac{10}{11}$ take $12\frac{8}{9}$.　　　　　*Ans.* $8\frac{2}{99}$.

11. From $21\frac{11}{12}$ take $13\frac{9}{10}$.　　　　　*Ans.* $8\frac{1}{60}$.

12. From $22\frac{12}{13}$ take $14\frac{10}{11}$.　　　　　*Ans.* $8\frac{2}{143}$.

13. From $23\frac{13}{14}$ take $15\frac{11}{12}$.　　　　　*Ans.* $8\frac{1}{84}$.

14. From $24\frac{14}{15}$ take $16\frac{12}{13}$.　　　　　*Ans.* $8\frac{2}{195}$.

15. From $25\frac{15}{16}$ take $17\frac{13}{14}$.　　　　　*Ans.* $8\frac{1}{112}$.

16. From $26\frac{16}{17}$ take $18\frac{14}{15}$.　　　　　*Ans.* $8\frac{2}{255}$.

17. From $27\frac{17}{18}$ take $19\frac{15}{16}$.　　　　　*Ans.* $8\frac{1}{144}$.

18. From $28\frac{18}{19}$ take $20\frac{16}{17}$.　　　　　*Ans.* $8\frac{2}{323}$.

19. From $29\frac{19}{20}$ take $21\frac{17}{18}$.　　　　　*Ans.* $8\frac{1}{180}$.

20. From $30\frac{20}{21}$ take $22\frac{18}{19}$.　　　　　*Ans.* $8\frac{31}{589}$.

21. From $32\frac{31}{32}$ take $23\frac{19}{20}$.　　　　　*Ans.* $9\frac{3}{160}$.

LESSON XVIII.

CASE V.

236. When a mixed number is to be subtracted from a whole number.

RULE.

Write the whole number above the mixed number, as in simple subtraction, with the fractional part a little to the right and not under the whole number.

Borrow 1 *from the whole number, which is equal to as many of the same denomination as the fraction below as would make a whole one.*

Subtract the fractional parts, and write the difference over the common denominator.

Add 1 *to the first figure of the subtrahend and subtract the figures as in simple subtraction.*

EXAMPLES.

1. From 12 subtract $2\frac{3}{8}$.

Borrow 1 from 12, which is equal to $\frac{8}{8}$. Subtracting $\frac{3}{8}$ from $\frac{8}{8}$ leaves $\frac{5}{8}$ for the fractional part. Add 1 to 2, making 3. 3 from 12 leaves 9. The answer required is $9\frac{5}{8}$.

OPERATION.

$$12$$
$$2\frac{3}{8}$$
$$\overline{}$$
$$9\frac{5}{8} \quad Ans.$$

Or:

237. *Subtract the numerator of the fraction from the denominator, and write the difference over the denominator.*

Add 1 *to the first figure of the subtrahend, and subtract as in simple numbers.*

EXAMPLE.

Subtract the numerator 3 from the denominator 8, and the remainder will be 5. Write the 5 above the 8, as in the example. Add 1 to the figure 2, making it 3. 3 from 12 leaves 9. The answer is $9\frac{5}{8}$.

OPERATION.

$$12$$
$$2\frac{3}{8}$$
$$\overline{}$$
$$9\frac{5}{8}$$

Or:

238. Consider the denominator of the whole number to be 1; and the sum is, from $1\frac{2}{1}$ subtract $2\frac{3}{8}$. Reducing $2\frac{3}{8}$ to an improper fraction gives $1\frac{9}{8}$. The sum now stands, From $1\frac{2}{1}$ take $1\frac{9}{8}$.

$$\frac{96 - 19}{8}$$

The common multiple is 8. 1 into 8, 8 times. Multiplying the numerator 12 by 8 gives 96 for the new numerator. 8 into 8, once; once 19 is 19, the other numerator. Subtracting 19 from 96 leaves 77, which written over the numerator 8 gives $\frac{77}{8}$. Reducing $\frac{77}{8}$ to a mixed number gives (8 into 77, 9 times and 5 remaining), $9\frac{5}{8}$, the required answer.

2. From 18 take $12\frac{2}{3}$.	Ans. $5\frac{1}{3}$.
3. From 19 take $13\frac{2}{7}$.	Ans. $5\frac{5}{7}$.
4. From 20 take $14\frac{5}{8}$.	Ans. $5\frac{4}{8}$.
5. From 21 take $15\frac{9}{10}$.	Ans. $5\frac{1}{10}$.
6. From 22 take $16\frac{10}{11}$.	Ans. $5\frac{1}{11}$.
7. From 23 take $17\frac{18}{23}$.	Ans. $5\frac{5}{23}$.
8. From 28 take $12\frac{2}{7}$.	Ans. $15\frac{5}{7}$.
9. From 30 take $13\frac{5}{15}$.	Ans. $16\frac{1}{15}$.
10. From 36 take $14\frac{18}{19}$.	Ans. $21\frac{1}{19}$.
11. From 42 take $5\frac{21}{23}$.	Ans. $36\frac{2}{23}$.
12. From 89 take $16\frac{67}{84}$.	Ans. $72\frac{17}{84}$.
13. From 34 take $17\frac{72}{81}$.	Ans. $16\frac{9}{81}$.
14. From 98 take $18\frac{21}{92}$.	Ans. $79\frac{71}{92}$.
15. From 240 take $19\frac{18}{41}$.	Ans. $220\frac{18}{41}$.
16. From 380 take $20\frac{23}{24}$.	Ans. $359\frac{1}{24}$.

17. From 760 take $21\frac{23}{27}$.   *Ans.* $738\frac{4}{27}$.
18. From 123 take $22\frac{15}{16}$.   *Ans.* $100\frac{13}{16}$.
19. From 345 take $100\frac{24}{31}$.   *Ans.* $244\frac{23}{31}$.
20. From 6789 take $2000\frac{135}{250}$.   *Ans.* $4788\frac{3}{5}$.
21. From 101112 take $1897\frac{200}{321}$.   *Ans.* $99214\frac{23}{8}$.

LESSON XIX.

CASE VI.

239. When a whole number is to be subtracted from a mixed number.

RULE.

Write the fraction below the line to the right of the remainder.   The remainder is found by subtracting as in simple numbers.

EXAMPLES.

1. From $14\frac{1}{8}$ take 5.

Take the fractional part of the mixed number, and write it below the line as in the example.

Subtract 5 from 14, leaving 9, the answer is $9\frac{1}{8}$.

OPERATION.

$14\frac{1}{8}$
5
———
$9\frac{1}{8}$ *Ans.*

2. From $14\frac{5}{6}$ take 12.   *Ans.* $2\frac{5}{6}$.
3. From $13\frac{1}{4}$ take 2.   *Ans.* $11\frac{1}{4}$.
4. From $18\frac{3}{8}$ take 8.   *Ans.* $10\frac{3}{8}$.
5. From $22\frac{5}{8}$ take 10.   *Ans.* $12\frac{5}{8}$.
6. From $46\frac{5}{8}$ take 21.   *Ans.* $25\frac{5}{8}$.
7. From $98\frac{13}{20}$ take 64.   *Ans.* $34\frac{13}{20}$.

8. From $100\frac{20}{33}$ take 25.   *Ans.* $75\frac{19}{33}$.
9. From $286\frac{12}{24}$ take 89.   *Ans.* $197\frac{3}{4}$.
10. From $300\frac{24}{34}$ take 98.   *Ans.* $202\frac{24}{34}$.
11. From $496\frac{22}{101}$ take 248.   *Ans.* $248\frac{22}{101}$.

LESSON XX.

CASE VII.

240. When one mixed number is to be subtracted from another, the fractional part of the subtrahend being greater than the fractional part of the minuend.

RULE I.

Reduce the mixed numbers to improper fractions, and subtract as in subtraction of fractions.

Or:

RULE II.

241. *To the fractional part of the minuend add 1 whole one. Reduce the 1 whole one and the fraction to a mixed number. You then have a proper fraction to be subtracted from an improper one.*

Subtract the fractional parts as in subtraction of fractions.

Add 1 to the 1st figure of the dividend, and subtract as in simple numbers.

EXAMPLES.

1. From $12\frac{1}{3}$ take $10\frac{1}{2}$.

Reducing to improper fractions gives

$$\frac{37}{3} \text{ and } \frac{21}{2} = \frac{74 - 63}{6} = \frac{11}{6} = 1\frac{5}{6}$$

The least common multiple is 6.

The numerators are 74 and 63.

Subtracting 63 from 74 leaves 11, which written over the multiple 6, gives $\frac{11}{6}$ for the required fraction. Reducing $\frac{11}{6}$ to a mixed, gives $1\frac{5}{6}$, the required answer.

Or:

OPERATION.

$12\frac{1}{3}$
$10\frac{1}{2}$
———

1. From $12\frac{1}{3}$ take $10\frac{1}{2}$.

$\frac{1}{2}$ being larger than $\frac{1}{3}$, it cannot be subtracted. Borrow 1 whole one from the 12, making $\frac{3}{3}$, together with $\frac{1}{3}$ already in the minuend, would make $\frac{4}{3}$. The sum now would read, from $11\frac{4}{3}$ take $10\frac{1}{2}$.

$\frac{4}{3}$ is equal to $\frac{8}{6}$, and $\frac{1}{2}$ is equal to $\frac{3}{6}$. Write the 8 a little to the right of the $\frac{4}{3}$, and write the 3 a little to right of the $\frac{1}{2}$.

OPERATION.

$11\frac{4}{3} = 8$
$10\frac{1}{2} = 3$
———

The denominator 6 is understood to be written under the 8 and 3. Subtracting 3 from 8 leaves 5, that is, $\frac{5}{6}$. Write the $\frac{5}{6}$ under the fractional parts, and subtract the 10 from the 11. 10 from 11 leaves 1.

The required answer is $1\frac{5}{6}$.

2. From $2\frac{1}{4}$ take $1\frac{2}{3}$.   *Ans.* $\frac{7}{8}$.

3. From $2\frac{2}{3}$ take $1\frac{5}{6}$.   *Ans.* $\frac{5}{6}$.

4. From $3\frac{3}{4}$ take $1\frac{7}{8}$.   *Ans.* $1\frac{7}{8}$.

5. From $3\frac{1}{4}$ tade $1\frac{3}{8}$.   *Ans.* $1\frac{7}{8}$.

6. From $4\frac{1}{5}$ take $2\frac{3}{10}$.   *Ans.* $1\frac{9}{10}$.

7. From $4\frac{2}{5}$ take $3\frac{8}{10}$.   *Ans.* $\frac{3}{5}$.

8. From $4\frac{3}{5}$ take $1\frac{12}{15}$.   *Ans.* $2\frac{4}{5}$.

9. From $5\frac{1}{6}$ take $2\frac{4}{6}$.   *Ans.* $2\frac{1}{2}$.

10. From $5\frac{2}{5}$ take $3\frac{3}{4}$.   *Ans.* $1\frac{13}{20}$.

11. From $5\frac{3}{5}$ take $4\frac{7}{10}$.   *Ans.* $\frac{9}{10}$.

12. From $6\frac{1}{4}$ take $5\frac{2}{4}$.   *Ans.* $\frac{3}{4}$.

13. From $6\frac{2}{4}$ take $1\frac{3}{4}$.   *Ans.* $4\frac{3}{4}$.

14. From $6\frac{3}{7}$ take $1\frac{5}{8}$.   *Ans.* $4\frac{34}{63}$.

15. From $7\frac{1}{8}$ take $3\frac{3}{8}$.   *Ans.* $3\frac{3}{4}$.

16. From $7\frac{3}{8}$ take $4\frac{7}{8}$.   *Ans.* $2\frac{1}{2}$.

17. From $7\frac{5}{8}$ take $3\frac{11}{16}$.   *Ans.* $3\frac{15}{16}$.

18. From $8\frac{1}{9}$ take $5\frac{3}{18}$.   *Ans.* $2\frac{17}{18}$.

19. From $8\frac{2}{9}$ take $6\frac{6}{18}$.   *Ans.* $1\frac{8}{9}$.

20. From $8\frac{4}{9}$ take $7\frac{9}{18}$.   *Ans.* $\frac{17}{18}$.

21. From $9\frac{1}{10}$ take $4\frac{9}{20}$.   *Ans.* $4\frac{13}{20}$.

22. From $9\frac{3}{10}$ take $5\frac{7}{20}$.   *Ans.* $3\frac{19}{20}$.

23. From $9\frac{7}{10}$ take $6\frac{15}{20}$.   *Ans.* $2\frac{19}{20}$.

24. From $12\frac{1}{5}$ take $7\frac{2}{5}$.   *Ans.* $4\frac{4}{5}$.

25. From $13\frac{1}{8}$ take $8\frac{3}{8}$.   *Ans.* $4\frac{3}{4}$.

26. From $21\frac{1}{12}$ take $9\frac{3}{12}$.   *Ans.* $11\frac{5}{6}$.

27. From $31\frac{1}{14}$ take $12\frac{2}{14}$.   *Ans.* $18\frac{83}{84}$.

28. From $14\frac{1}{15}$ take $11\frac{2}{15}$.   *Ans.* $2\frac{13}{15}$.

29. From $15\frac{1}{20}$ take $12\frac{3}{20}$.   *Ans.* $2\frac{19}{20}$.

30. From $41\frac{21}{40}$ take $21\frac{22}{40}$.   *Ans.* $19\frac{39}{40}$.

31. From $51\frac{21}{42}$ take $30\frac{22}{42}$.   *Ans.* $20\frac{41}{42}$.

18

MULTIPLICATION OF FRACTIONS.

LESSON XXI.

PRINCIPLES OF CANCELLING.

242. From what we have already seen, we know that dividing the numerator and denominator by the same number does not alter the value of the fraction. The form alone is changed, not the value.

243. In multiplication of fractions we may consider the several fractions of which a sum is composed as one compound fraction, because the word "of" means multiplication. Now, if we divide the numerator of any one of those fractions by any number, and divide the denominator of any of the other fractions by the same number, the value of the multiplication is not changed. Thus,

$$\frac{2}{\overset{\displaystyle 3}{\underset{\displaystyle 1}{}}} \times \frac{\overset{\displaystyle 2}{6}}{7}$$

If we divide the numerator (6) of the second fraction by 3, and divide the denominator (3) of the first fraction by 3, the value of the multiplication is not changed. We get 2 for the numerator of the second fraction, and 1 for the denominator of the 1st fraction. These remarks apply to any number of fractions.

$$\frac{\cancel{2}}{\cancel{3}} \times \frac{\overset{\cancel{2}}{\cancel{6}}}{\cancel{7}} \times \frac{\overset{\cancel{7}}{\cancel{14}}}{\cancel{8}} = $$

Divide (3) denominator and (6) numerator by 3. Divide (2) numerator and (8) denominator by 2. Dixide (14) numerator and (4) denominator (quotient) by 2. Divide (7) denominator and (7) numerator (quotient) by 7. Divide (2) numerator (quotient) and (2) denominator (quotient) by 2. There remain but 1's for the numerator, and 1's for the denominator. That is, all the factors of the numerators are common and equal to those of the denominators.

<div align="center">PROOF.</div>

The factors of the numerators are as follows:

$$2 = 2 \qquad 14 = 2 \times 7$$
$$6 = 2 \times 3$$

The factors of the denominators are:

$$3 = 3 \qquad 8 = 2 \times 2 \times 2$$
$$7 = 7$$

Writing the factors of the numerators and denominators as one fraction, with the sign of multiplication between them, we have

$$\frac{2 \times 2 \times 3 \times 2 \times 7}{3 \times 7 \times 2 \times 2 \times 2}$$

Cancelling the factors common to both, gives

$$\frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{7}}}{\underset{1}{\cancel{3}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}}}$$

1's for the numerator, and 1's for the denominator.

244. MULTIPLICATION OF FRACTIONS.

CASE I.

GENERAL RULE.

Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator, after having cancelled all the factors common to both.

245. When a fraction is to be multiplied by a whole number:

RULE.

Multiply the whole number by the numerator, and divide the product by the denominator. If any remainder, write it in the form of a fraction.

EXAMPLE.

Multiply $\frac{2}{3}$ by 8.

Multiply the 8 by the denominator 2, and write the result over the denominator in the form of an improper fraction, and reduce the improper fraction to a mixed number.

OPERATION.

$$\frac{2 \times 8}{3} = \frac{16}{3} = 5\frac{1}{3}$$

EXERCISES.

1. Multiply $\frac{1}{2} \times 4$.   *Ans.* 2.
2. Multiply $\frac{2}{3} \times 6$.   *Ans.* 4.
3. Multiply $\frac{3}{4} \times 8$.   *Ans.* 6.
4. Multiply $\frac{4}{5} \times 10$.   *Ans.* 8.
5. Multiply $\frac{5}{6} \times 12$.   *Ans.* 10.
6. Multiply $\frac{6}{7} \times 14$.   *Ans.* 12.
7. Multiply $\frac{7}{8} \times 23$.   *Ans.* $20\frac{1}{8}$.
8. Multiyly $\frac{8}{9} \times 98$.   *Ans.* $87\frac{1}{9}$.
9. Multiply $\frac{9}{10} \times 18$.   *Ans.* $16\frac{1}{5}$.
10. Multiply $\frac{10}{11} \times 24$.   *Ans.* $21\frac{9}{11}$.
11. Multiply $\frac{11}{12} \times 36$.   *Ans.* 33.
12. Multiply $\frac{12}{13} \times 39$.   *Ans.* 36.
13. Multiply $\frac{13}{14} \times 98$.   *Ans.* 91.
14. Multiply $\frac{14}{15} \times 72$.   *Ans.* $67\frac{1}{5}$.
15. Multiply $\frac{15}{16} \times 100$.   *Ans.* $93\frac{3}{4}$.
16. Multiply $\frac{16}{17} \times 156$.   *Ans.* $146\frac{14}{17}$.

LESSON XXII.

246. When a mixed number is to be multiplied by a whole number:

RULE.

Reduce the mixed number to an improper fraction. Multiply the numerator thus found by the whole number and divide the result by the denominator.

EXAMPLE.

Multiply $2\frac{1}{3}$ by 8.

$$\frac{3}{7} \times \frac{8}{1} = \frac{7 \times 8}{3} = \frac{56}{3} = 18\frac{2}{3}.$$

18*

Reducing 2¼ to an improper fraction gives ⁷⁄₃. The problem is, as it now stands : Multiply ⁷⁄₃ by 8. Multiply the numerator, 7, by the whole number, 8, and divide the product (56) by the denominator, 3. If any remainder, write it in the form of a fraction.

<div align="center">EXERCISES.</div>

1.	Multiply 2¼ by 18.	*Ans.* 40½.
2.	Multiply 3⅕ by 20.	*Ans.* 64.
3.	Multiply 4⅚ by 22.	*Ans.* 106⅓.
4.	Multiply 6⅞ by 24.	*Ans.* 165.
5.	Multiply 9⅖ by 40.	*Ans.* 377⅞.
6.	Multiply 10¹³⁄₁₅ by 30.	*Ans.* 327⁹⁄₁₃.
7.	Multiply 11²¹⁄₂₄ by 20.	*Ans.* 233⅓.
8.	Multiply 21¹⁸⁄₁₉ by 12.	*Ans.* 263⁷⁄₁₉.
9.	Multiply 62⅔ by 8.	*Ans.* 501⅓.
10.	Multiply 18¾ by 10.	*Ans.* 187½.
11.	Multiply 31⅘ by 11.	*Ans.* 349⅘.
12.	Multiply 46⅚ by 12.	*Ans.* 562.
13.	Multiply 92⁴⁄₉ by 13.	*Ans.* 1207¼.
14.	Multiply 100⅞ by 14, 15, 17, 18, 19, 12, 21.	
15.	Multiply 131⁹⁹⁄₁₀₀ by 22, 23, 24, 25, 26, 27, 28.	

<div align="center">LESSON XXIII.</div>

247. To multiply a whole number by a fraction.

<div align="center">RULE.</div>

Multiply the whole number by the numerator of the fraction and divide the product by the denominator of the fraction.

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EXAMPLE.

Multiply 21 by $\frac{4}{5}$.

OPERATION.

$$\frac{21 \times 4}{5} = \frac{84}{5} = 16\frac{4}{5}$$

EXERCISES.

1. Multiply 10 by $\frac{1}{2}$.   Ans. 5.
2. Multiply 20 by $\frac{2}{3}$.   Ans. 13$\frac{1}{3}$.
3. Multiply 24 by $\frac{3}{4}$.   Ans. 18.
4. Multiply 28 by $\frac{4}{5}$.   Ans. 22$\frac{2}{5}$.
5. Multiply 32 by $\frac{5}{6}$.   Ans. 26$\frac{2}{3}$.
6. Multiply 36 by $\frac{6}{7}$.   Ans. 30$\frac{6}{7}$.
7. Multiply 40 by $\frac{7}{8}$.   Ans. 35.
8. Multiply 44 by $\frac{8}{9}$.   Ans. 39$\frac{1}{9}$.
9. Multiply 48 by $\frac{9}{10}$.   Ans. 43$\frac{1}{5}$.
10. Multiply 52 by $\frac{10}{11}$.   Ans. 47$\frac{3}{11}$.
11. Multiply 56 by $\frac{11}{12}$.   Ans. 51$\frac{1}{3}$.
12. Multiply 60 by $\frac{12}{13}$.   Ans. 55$\frac{5}{13}$.
13. Multiply 64 by $\frac{13}{14}$.   Ans. 59$\frac{3}{7}$.
14. Multiply 68 by $\frac{14}{15}$.   Ans. 63$\frac{7}{15}$.
15. Multiply 72 by $\frac{15}{18}$.   Ans. 30.
16. Multiply 74 by $\frac{17}{34}$.   Ans. 37.
17. Multiply 78 by $\frac{18}{39}$.   Ans. 36.
18. Multiply 82 by $\frac{1}{2}$.   Ans. 41.
19. Multiply 86 by $\frac{2}{3}$.   Ans. 57$\frac{1}{3}$.
20. Multiply 100 by $\frac{2}{25}$.   Ans. 8.

LESSON XXIV.

248. To multiply one simple fraction by another.

RULE.

Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator, after having cancelled all the factors common to both.

EXAMPLE.

$$\text{Multiply } \tfrac{1}{2} \times \tfrac{2}{3} \times \tfrac{5}{7} = \frac{1 \times 2 \times 5}{2 \times 3 \times 7} = \frac{5}{21} \; Ans.$$

By cancelling the figure 2 in the numerator and denominator, there remains but 1 and 5 in the numerator, and 3 and 7 in the denominator. Multiplying the 1 by the 5, gives 5 for the numerator of the answer, and by multiplying the 3 and the 7, we get 21 for the denominator. The answer is, then, $\tfrac{5}{21}$.

EXERCISES.

1. Multiply $\tfrac{1}{2}$ by $\tfrac{2}{3}$ of $\tfrac{3}{8}$.　　　　　*Ans.* $\tfrac{1}{8}$.
2. Multiply $\tfrac{2}{3}$ by $\tfrac{9}{12}$ of $\tfrac{12}{9}$.　　　　*Ans.* $\tfrac{2}{3}$.
3. Multiply $\tfrac{5}{6}$ by $\tfrac{13}{25}$ of $\tfrac{25}{13}$.　　　*Ans.* $\tfrac{5}{6}$.
4. Multiply $\tfrac{7}{8}$ by $\tfrac{5}{6}$ of $\tfrac{6}{5}$.　　　　*Ans.* $\tfrac{7}{8}$.
5. Multiply $\tfrac{8}{9}$ by $\tfrac{6}{7}$ of $\tfrac{7}{6}$.　　　　*Ans.* $\tfrac{8}{9}$.
6. Multiply $\tfrac{9}{10}$ by $\tfrac{9}{18}$ of $\tfrac{18}{9}$.　　*Ans.* $\tfrac{9}{10}$.
7. Multiply $\tfrac{10}{11}$ by $\tfrac{22}{30}$ of $\tfrac{30}{44}$.　*Ans.* $\tfrac{5}{11}$.

8. Multiply $\frac{11}{12}$ by $\frac{22}{33}$ of $\frac{11}{20}$.   *Ans.* $\frac{11}{36}$.

9. Multiply $1\frac{2}{3}$ by $\frac{18}{30}$ of $3\frac{9}{12}$.   *Ans.* $\frac{9}{15}$.

10. Multiply $\frac{13}{14}$ by $\frac{28}{58}$ of $\frac{78}{78}$.   *Ans.* $1\frac{13}{14}$.

11. Multiply $1\frac{4}{15}$ by $\frac{28}{58}$ of $\frac{38}{58}$.   *Ans.* $\frac{7}{15}$.

12. Multiply $1\frac{6}{15}$ by $\frac{39}{52}$ of $\frac{40}{50}$.   *Ans.* $\frac{28}{64}$.

13. Multiply $1\frac{8}{15}\times\frac{38}{24}\times\frac{180}{100}\times\frac{50}{90}$.   *Ans.* 1.

14. Multiply $\frac{36}{12}\times\frac{24}{48}\times\frac{10}{9}\times\frac{4}{5}$.   *Ans.* 1.

15. Multiply $\frac{14}{25}\times\frac{56}{28}\times\frac{180}{200}\times\frac{100}{90}$.   *Ans.* 1.

16. Multiply $\frac{18}{36}\times\frac{76}{171}\times\frac{4}{3}\times\frac{3}{12}$.   *Ans.* $\frac{2}{27}$.

17. Multiply $\frac{20}{40}\times\frac{80}{160}\times\frac{3}{2}\times\frac{4}{6}$.   *Ans.* $\frac{1}{4}$.

18. Multiply $\frac{22}{33}\times\frac{33}{44}\times\frac{6}{1}\times\frac{1}{4}$.   *Ans.* $\frac{1}{2}$.

19. Multiply $\frac{26}{65}\times\frac{104}{138}\times\frac{18}{20}\times\frac{20}{18}$.   *Ans.* $\frac{2}{7}$.

20. Multiply $\frac{18}{36}\times\frac{36}{162}\times\frac{87}{40}\times\frac{48}{87}$.   *Ans.* $\frac{1}{9}$.

21. Multiply $\frac{92}{184}\times\frac{184}{368}\times\frac{3}{5}\times\frac{5}{6}$.   *Ans.* $\frac{1}{4}$.

22. Multiply $\frac{87}{174}\times\frac{248}{435}\times\frac{5}{2}\times\frac{4}{3}$.   *Ans.* $\frac{2}{5}$.

23. Multiply $\frac{73}{146}\times\frac{228}{228}\times\frac{15}{1}\times\frac{10}{30}$.   *Ans.* $3\frac{23}{114}$.

24. Multiply $\frac{98}{196}\times\frac{196}{392}\times\frac{9}{7}\times\frac{14}{18}$.   *Ans.* $\frac{1}{4}$.

25. Multiply $\frac{100}{200}\times\frac{400}{600}\times\frac{11}{1}\times\frac{12}{22}$.   *Ans.* $\frac{1}{3}$.

26. Multiply $\frac{24}{216}\times\frac{108}{12}\times\frac{217}{1}\times\frac{12}{24}$.   *Ans.* $108\frac{1}{2}$.

27. Multiply $\frac{18}{162}\times\frac{81}{36}\times\frac{72}{18}\times\frac{18}{144}$.   *Ans.* $\frac{1}{8}$.

28. Multiply $\frac{24}{64}\times\frac{192}{48}\times\frac{12}{8}\times\frac{8}{24}$.   *Ans.* $\frac{1}{2}$.

29. Multiply $\frac{33}{132}\times\frac{132}{66}\times\frac{16}{1}\times\frac{8}{16}$.   *Ans.* 4.

30. Multiply $\frac{18}{162}\times\frac{324}{1818}\times\frac{1}{12}\times\frac{12}{36}$.   *Ans.* $\frac{1}{1818}$.

LESSON XXV.

249. To multiply one mixed number by another.

RULE.

Reduce the mixed numbers to improper fractions, and multiply the numerators together for a new nu-

numerator, and multiply the denominators together
for a new denominator, after having cancelled the
factors common to both numerator and denominator.

$$\text{Multiply } 2\tfrac{1}{2} \text{ by } 3\tfrac{1}{3} = \frac{5}{\cancel{2}} \times \frac{\overset{5}{\cancel{10}}}{3} = \frac{25}{3}$$

1st. Reduce the mixed numbers to improper frac-
tions.   2d. Cancel the denominator 2, and the nu-
merator 10, by dividing both by 2.   3d. Multiply the
numerator 5, and the quotient 5 together, and write
the product over the denominator 3, in the form of
an improper fraction.   Reduce the improper fraction
to a mixed number, by dividing the numerator by
the denominator, and if there is a remainder, write it
in the form of a fraction.

  1. Multiply $1\tfrac{1}{2}$ by $2\tfrac{2}{3}$.   *Ans.* 4.
  2. Multiply $2\tfrac{2}{3}$ by $3\tfrac{3}{4}$.   *Ans.* 10.
  3. Multiply $3\tfrac{3}{4}$ by $4\tfrac{4}{5}$.   *Ans.* 18.
  4. Multiply $4\tfrac{4}{5}$ by $5\tfrac{5}{6}$.   *Ans.* 28.
  5. Multiply $5\tfrac{5}{6}$ by $6\tfrac{6}{7}$.   *Ans.* 40.
  6. Multiply $6\tfrac{6}{7}$ by $7\tfrac{7}{8}$.   *Ans.* 54.
  7. Multiply $7\tfrac{7}{8}$ by $8\tfrac{8}{9}$.   *Ans.* 70.
  8. Multiply $8\tfrac{8}{9}$ by $9\tfrac{9}{10}$.   *Ans.* 88.
  9. Multiply $9\tfrac{9}{10}$ by $10\tfrac{10}{11}$.   *Ans.* 108.
10. Multiply $10\tfrac{10}{11}$ by $11\tfrac{11}{12}$.   *Ans.* 130.
11. Multiply $11\tfrac{11}{12}$ by $12\tfrac{12}{13}$.   *Ans.* 154.

12. Multiply $12\frac{4\cdot9}{1\cdot3}$ by $13\frac{1\cdot3}{1\cdot4}$.   *Ans.* 180.

13. Multiply $13\frac{1\cdot3}{1\cdot4}$ by $14\frac{1\cdot4}{1\cdot5}$.   *Ans.* 208.

14. Multiply $14\frac{1\cdot4}{1\cdot5}$ by $15\frac{1\cdot5}{1\cdot6}$.   *Ans.* 238.

15. Multiply $15\frac{1\cdot6}{1\cdot8}$ by $16\frac{1\cdot6}{1\cdot7}$.   *Ans.* 270.

16. Multiply $16\frac{1\cdot6}{1\cdot7}$ by $17\frac{7}{18}$.   *Ans.* 304.

17. Multiply $17\frac{7}{18}$ by $18\frac{1\cdot8}{1\cdot9}$.   *Ans.* 340.

18. Multiply $20\frac{1}{2}$ by $10\frac{1}{4}$.   *Ans.* $211\frac{5}{8}$.

19. Multiply $30\frac{2}{3}$ by $15\frac{1}{4}$.   *Ans.* $467\frac{2}{3}$.

20. Multiply $40\frac{3}{4}$ by $20\frac{1}{4}$.   *Ans.* $821\frac{1\cdot1}{2\cdot4}$.

21. Multiply $50\frac{5}{6}$ by $25\frac{4}{7}$.   *Ans.* $1299\frac{3\cdot7}{4\cdot2}$.

22. Multiply $60\frac{4}{7}$ by $30\frac{3}{8}$.   *Ans.* $1837\frac{1}{2}$.

23. Multiply $70\frac{3}{8}$ by $35\frac{5}{10}$.   *Ans.* $2516\frac{5}{8}$.

24. Multiply $80\frac{3}{10}$ by $40\frac{6}{11}$.   *Ans.* $3255\frac{4}{5}$.

25. Multiply $90\frac{5}{11}$ by $45\frac{8}{12}$.   *Ans.* $4130\frac{2\cdot5}{3\cdot3}$.

26. Multiply $100\frac{6}{12}$ by $50\frac{7}{14}$.   *Ans.* $5075\frac{1}{4}$.

27. Multiply $110\frac{7}{14}$ by $55\frac{8}{16}$.   *Ans.* $6132\frac{3}{4}$.

28. Multiply $112\frac{8}{16}$ by $56\frac{1}{4}$.   *Ans.* $6356\frac{1}{4}$.

29. Multiply $113\frac{9}{18}$ by $13\frac{1\cdot0}{2\cdot0}$.   *Ans.* $1532\frac{1}{4}$.

30. Multiply $114\frac{1\cdot0}{2\cdot0}$ by $57\frac{5}{10}$ *Ans.* $6583\frac{3}{4}$.

DIVISION OF FRACTIONS.

LESSON XXVI.

CASE I.

250. To divide a whole number by a fraction.

RULE.

Invert the terms of the divisor and multiply.

<center>EXAMPLE.</center>

Divide 4 by $\frac{2}{3}$.

<center>OPERATION.</center>

$$4 \div \frac{2}{3} + \overset{2}{\cancel{4}} \times \frac{3}{\cancel{2}} = 6, \ \cdot Ans.$$

<center>EXERCISES.</center>

1. Divide 2 by $\frac{1}{3}$.	Ans. 6.
2. Divide 4 by $\frac{1}{5}$.	Ans. 20.
3. Divide 6 by $\frac{1}{7}$.	Ans. 42.
4. Divide 8 by $\frac{1}{8}$.	Ans. 64.
5. Divide 10 by $\frac{2}{3}$.	Ans. 15.
6. Divide 12 by $\frac{4}{9}$.	Ans. 27.
7. Divide 14 by $\frac{9}{12}$.	Ans. $18\frac{2}{3}$.
8. Divide 16 by $\frac{5}{7}$.	Ans. $22\frac{2}{5}$.
9. Divide 18 by $\frac{7}{13}$.	Ans. $33\frac{3}{7}$.
10. Divide 20 by $\frac{13}{14}$.	Ans. $21\frac{7}{13}$.
11. Divide 22 by $\frac{14}{15}$.	Ans. $23\frac{4}{7}$.
12. Divide 24 by $\frac{15}{16}$.	Ans. $25\frac{3}{5}$.
13. Divide 26 by $\frac{14}{15}$.	Ans. $27\frac{6}{7}$.
14. Divide 36 by $\frac{21}{23}$.	Ans. $37\frac{5}{7}$.
15. Divide 46 by $\frac{22}{23}$.	Ans. $48\frac{1}{11}$.
16. Divide 56 by $\frac{23}{24}$.	Ans. $58\frac{10}{23}$.
17. Divide 66 by $\frac{33}{18}$.	Ans. 36.
18. Divide 76 by $\frac{96}{100}$.	Ans. $79\frac{1}{6}$.
19. Divide 86 by $\frac{20}{40}$.	Ans. 172.
20. Divide 96 by $\frac{1}{18}$.	Ans. 1728.
21. Divide 106 by $\frac{5}{18}$.	Ans. $381\frac{3}{5}$.
22. Divide 206 by $\frac{9}{18}$.	Ans. 412.
23. Divide 306 by $\frac{2}{3}$.	Ans. 459.
24. Divide 406 by $\frac{8}{9}$.	Ans. $456\frac{3}{4}$.

LESSON XXVII.

CASE II.

251. To divide a fraction by a whole number.

RULE.

Consider the whole number to be the denominator of a fraction whose numerator is 1. Then multiply as in multiplication of fractions.

EXAMPLE.

Divide $\frac{2}{3}$ by 4.

OPERATION.

$$\frac{2}{3} \div 4 = \frac{\cancel{2}}{3} \times \frac{1}{\cancel{4}_{2}} = \frac{1}{6}, \quad Ans.$$

EXERCISES.

1. Divide $\frac{2}{5}$ by 12.   *Ans.* $\frac{1}{18}$.
2. Divide $\frac{3}{4}$ by 3.   *Ans.* $\frac{1}{4}$.
3. Divide $\frac{4}{5}$ by 14.   *Ans.* $\frac{2}{35}$.
4. Divide $\frac{5}{6}$ by 5.   *Ans.* $\frac{1}{6}$.
5. Divide $\frac{6}{7}$ by 16.   *Ans.* $\frac{3}{56}$.
6. Divide $\frac{7}{8}$ by 7.   *Ans.* $\frac{1}{8}$.
7. Divide $\frac{8}{9}$ by 18.   *Ans.* $\frac{4}{81}$.
8. Divide $\frac{9}{10}$ by 9.   *Ans.* $\frac{1}{10}$.
9. Divide $\frac{10}{11}$ by 20.   *Ans.* $\frac{1}{22}$.
10. Divide $\frac{11}{12}$ by 11.   *Ans.* $\frac{1}{12}$.
11. Divide $\frac{12}{13}$ by 22.   *Ans.* $\frac{6}{143}$.
12. Divide $\frac{13}{14}$ by 13.   *Ans.* $\frac{1}{14}$.
13. Divide $\frac{14}{15}$ by 24.   *Ans.* $\frac{7}{180}$.

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14. Divide 1⅝ by 15.   *Ans.* $\frac{1}{16}$.

15. Divide 1⅘ by 26.   *Ans.* $\frac{8}{221}$.

16. Divide 1⁷⁄₉ by 17.   *Ans.* $\frac{1}{18}$.

17. Divide 1⅚ by 28.   *Ans.* $\frac{9}{266}$.

18. Divide 1⁹⁄₂₀ by 19.   *Ans.* $\frac{1}{20}$.

19. Divide ³⁴⁄₂₁ by 40.   *Ans.* $\frac{1}{13}$.

20. Divide 3⅟₂₂ by 22.   *Ans.* $\frac{1}{23}$.

21. Divide 2⅘ by 42.   *Ans.* $\frac{11}{483}$.

LESSON XXVIII.

CASE III.

252. To divide one simple fraction by another.

RULE.

Invert the terms of the divisor, and multiply as in multiplication of fractions.

EXAMPLE.

1. Divide ⅘ by ⅚ $\frac{4}{5} : \frac{5}{6} = \frac{4}{5} \times \frac{6}{5} = \frac{24}{25}$ *Ans.*

Invert the terms of the divisor ⅚, and multiply.

2. Divide ½ by ²⁰⁄₂₁.   *Ans.* ²¹⁄₄₀.

3. Divide ⅔ by ¹⁹⁄₂₀.   *Ans.* ⁴⁰⁄₅₇.

4. Divide ¾ by ¹⁸⁄₁₉.   *Ans.* ¹⁹⁄₂₄.

5. Divide ⅘ by ¹⁷⁄₁₈.   *Ans.* ⁷²⁄₈₅.

6. Divide ⅚ by ¹⁶⁄₁₇.   *Ans.* ⁸⁵⁄₉₆.

7. Divide ⁶⁄₇ by ¹⁵⁄₁₆.   *Ans.* ³²⁄₃₅.

8. Divide ⁷⁄₈ by ¹⁴⁄₁₇.   *Ans.* 1¹¹⁄₁₂₈.

9. Divide ⅞ by ¹⁹⁄₁₈.   *Ans.* ¹⁶⁄₁₉.

10. Divide $\frac{9}{10}$ by $1\frac{3}{5}$.   *Ans.* $\frac{27}{28}$.
11. Divide $1\frac{9}{11}$ by $1\frac{3}{4}$.   *Ans.* $1\frac{19}{44}$.
12. Divide $1\frac{1}{2}$ by $1\frac{2}{5}$.   *Ans.* $1\frac{13}{14}$.
13. Divide $1\frac{3}{8}$ by $1\frac{1}{5}$.   *Ans.* $1\frac{1}{143}$.
14. Divide $1\frac{3}{4}$ by $1\frac{9}{11}$.   *Ans.* $1\frac{2}{140}$.
15. Divide $1\frac{4}{5}$ by $\frac{9}{10}$.   *Ans.* $1\frac{1}{27}$.
16. Divide $1\frac{5}{8}$ by $\frac{3}{8}$.   *Ans.* $1\frac{7}{128}$.
17. Divide $1\frac{9}{4}$ by $\frac{7}{8}$.   *Ans.* $1\frac{9}{119}$.
18. Divide $1\frac{7}{8}$ by $\frac{4}{7}$.   *Ans.* $1\frac{11}{108}$.
19. Divide $1\frac{8}{9}$ by $\frac{5}{8}$.   *Ans.* $1\frac{4}{9}$.
20. Divide $1\frac{9}{10}$ by $\frac{1}{4}$.   *Ans.* $1\frac{3}{10}$.
21. Divide $\frac{20}{21}$ by $\frac{3}{4}$.   *Ans.* $1\frac{17}{63}$.

LESSON XXIX.

CASE IV.

253. To divide one mixed number by another.

RULE.

Reduce the mixed numbers to improper fractions.
Invert the terms of the divisor and multiply.

EXAMPLES.

1. Divide $2\frac{1}{2}$ by $3\frac{1}{3}$.

$$2\frac{1}{2} \div 3\frac{1}{3} = \frac{5}{2} \;:\; \frac{10}{3} = \frac{\cancel{5}}{2} \times \frac{3}{\cancel{10}} = \frac{3}{4}$$

Reducing $2\frac{1}{2}$ and $3\frac{1}{3}$ to improper fractions gives $\frac{5}{2}$ and $\frac{10}{3}$. Inverting $\frac{10}{3}$, cancelling and multiplying, gives $\frac{3}{4}$.

2. Divide $2\frac{1}{2}$ by $20\frac{1}{2}$.   *Ans.* $\frac{5}{41}$.

3. Divide $3\frac{1}{3}$ by $19\frac{1}{3}$.   *Ans.* $\frac{5}{29}$.

4. Divide $4\frac{3}{4}$ by $18\frac{3}{4}$.   *Ans.* $\frac{19}{75}$.

5. Divide $5\frac{4}{5}$ by $17\frac{4}{5}$.   *Ans.* $\frac{29}{89}$.

6. Divide $6\frac{1}{7}$ by $16\frac{1}{7}$.   *Ans.* $\frac{43}{113}$.

7. Divide $7\frac{5}{8}$ by $15\frac{5}{8}$.   *Ans.* $\frac{61}{125}$.

8. Divide $8\frac{1}{9}$ by $14\frac{1}{9}$.   *Ans.* $\frac{73}{127}$.

9. Divide $9\frac{3}{10}$ by $13\frac{3}{10}$.   *Ans.* $\frac{93}{133}$.

10. Divide $10\frac{4}{11}$ by $12\frac{4}{11}$.   *Ans.* $1\frac{14}{136}$.

11. Divide $11\frac{5}{12}$ by $11\frac{5}{12}$.   *Ans.* 1.

12. Divide $12\frac{1}{13}$ by $10\frac{1}{13}$.   *Ans.* $1\frac{16}{131}$.

13. Divide $13\frac{7}{14}$ by $9\frac{7}{14}$.   *Ans.* $1\frac{56}{133}$.

14. Divide $14\frac{5}{15}$ by $8\frac{5}{15}$.   *Ans.* $1\frac{18}{25}$.

15. Divide $15\frac{6}{16}$ by $7\frac{6}{16}$.   *Ans.* $2\frac{5}{59}$.

16. Divide $16\frac{3}{18}$ by $6\frac{3}{18}$.   *Ans.* $2\frac{67}{111}$.

17. Divide $17\frac{4}{19}$ by $5\frac{4}{19}$.   *Ans.* $3\frac{10}{33}$.

18. Divide $18\frac{20}{21}$ by $4\frac{20}{21}$.   *Ans.* $3\frac{43}{52}$.

19. Divide $19\frac{20}{30}$ by $3\frac{20}{30}$.   *Ans.* $5\frac{4}{11}$.

20. Divide $20\frac{5}{8}$ by $2\frac{1}{2}$.   *Ans.* $8\frac{1}{4}$.

COMPLEX FRACTIONS.

LESSON XXX.

254. A complex fraction is one which has a fraction for its numerator, or a fraction for its denominator, or a fraction for both, Thus, $\frac{1}{\frac{2}{3}}$, $\frac{\frac{2}{3}}{4}$, $\frac{\frac{2}{3}}{\frac{5}{6}}$ are complex fractions.

ADDITION OF COMPLEX FRACTIONS.

EXAMPLE.

Add $\dfrac{\frac{2}{3}}{\frac{4}{5}} + \dfrac{\frac{1}{2}}{\frac{3}{4}} = \left(\dfrac{\cancel{2}}{3} \times \dfrac{5}{\cancel{4}}\right) + \left(\dfrac{1}{\cancel{2}} \times \dfrac{\overset{2}{\cancel{4}}}{3}\right) = \dfrac{5}{6} \times \dfrac{2}{3} =$

$\dfrac{5+4}{6} = \dfrac{9}{6} = 1\frac{1}{2}.$

The first complex fraction is read $\frac{2}{3}$ divided by $\frac{4}{5}$, the second $\frac{1}{2}$ divided by $\frac{3}{4}$; from which we have two problems in division. By performing the first division according to the rule of division, we have $\dfrac{\cancel{2}}{3} \times \dfrac{5}{\cancel{4}}$ or $\dfrac{5}{6}$. Applying the rule of division to the second complex fraction, we have $\dfrac{1}{\cancel{2}} \times \dfrac{\overset{2}{\cancel{4}}}{3}$ or $\dfrac{2}{3}$. The question is now reduced to $\frac{5}{6} + \frac{2}{3}$. According to the rule of addition of fractions, we find $\dfrac{5+4}{6}$ or $\dfrac{9}{6}$, which, reduced to a mixed number, gives $1\frac{3}{6}$, or $1\frac{1}{2}$ for the answer.

1. Add $\left(\dfrac{\frac{1}{12}}{\frac{3}{4}}\right)$ and $\left(\dfrac{\frac{2}{3}}{\frac{4}{5}}\right)$ *Ans.* $1\frac{7}{18}$.

2. Add $\left(\dfrac{\frac{3}{14}}{\frac{4}{5}}\right)$ and $\left(\dfrac{\frac{6}{7}}{\frac{9}{10}}\right)$.   *Ans.* $1\frac{37}{168}$.

3. Add $\left(\dfrac{\frac{4}{15}}{\frac{5}{6}}\right)$ and $\left(\dfrac{\frac{8}{9}}{\frac{10}{11}}\right)$.   *Ans.* $1\frac{233}{825}$.

4. Add $\left(\dfrac{\frac{5}{6}}{\frac{6}{17}}\right)$ and $\left(\dfrac{1\frac{1}{12}}{\frac{5}{8}}\right)$. *Ans.* $3\frac{142}{180}$.

5. Add $\left(\dfrac{\frac{6}{7}}{\frac{7}{13}}\right)$ and $\left(\dfrac{\frac{9}{10}}{\frac{4}{5}}\right)$. *Ans.* $3\frac{122}{332}$.

6. Add $\left(\dfrac{\frac{7}{8}}{\frac{8}{19}}\right)$ and $\left(\dfrac{\frac{8}{9}}{\frac{9}{11}}\right)$. *Ans.* $3\frac{853}{5184}$.

7. Add $\left(\dfrac{\frac{8}{19}}{\frac{9}{10}}\right)$ and $\left(\dfrac{1\frac{6}{8}}{1\frac{2}{48}}\right)$. *Ans.* $8\frac{80}{171}$.

LESSON XXXI.

255. SUBTRACTION OF COMPLEX FRACTIONS.

EXAMPLE.

From $\dfrac{4\frac{2}{3}}{\frac{1}{2}}$ take $\dfrac{1\frac{2}{3}}{\frac{5}{8}}$.

First reduce the mixed numbers to improper fractions. From $\dfrac{\frac{14}{3}}{\frac{1}{2}}$ take $\dfrac{\frac{5}{3}}{\frac{5}{8}}$. Place the complex fractions in the form of division of fractions: $(\frac{14}{3}\div\frac{1}{2})-(\frac{5}{3}\div\frac{5}{8})$. Inverting the terms according to the rule of division of fractions, and cancelling: $\left(\dfrac{14}{3}\times\dfrac{2}{1}\right) - \left(\dfrac{\cancel{5}}{3}\times\dfrac{8}{\cancel{5}}\right)$. Multiply as in multiplication: $\dfrac{28}{3} - \dfrac{8}{3}$. Perform the subtraction according to the rule of subtraction: $\dfrac{28-8}{3} = \dfrac{20}{3} = 6\frac{2}{3}$. *Ans.*

1. From $\dfrac{16}{1\frac{1}{2}}$ take $\dfrac{2\frac{1}{2}}{3\frac{1}{3}}$. $Ans.\ 9\frac{11}{12}$.

2. From $\dfrac{14\frac{1}{2}}{2\frac{1}{4}}$ take $\dfrac{5\frac{1}{8}}{4\frac{1}{6}}$. $Ans.\ 5\frac{103}{900}$.

3. From $\dfrac{18\frac{5}{6}}{3\frac{1}{2}}$ take $\dfrac{9\frac{2}{3}}{8\frac{1}{6}}$. $Ans.\ 4\frac{145}{546}$.

4. From $\dfrac{28\frac{1}{2}}{12\frac{1}{4}}$ take $\dfrac{9\frac{3}{4}}{12\frac{1}{2}}$. $Ans.\ 1\frac{1388}{2450}$.

5. From $\dfrac{13\frac{5}{8}}{4\frac{1}{2}}$ take $\dfrac{10\frac{1}{4}}{20\frac{1}{2}}$. $Ans.\ 2\frac{3005}{5796}$.

6. From $\dfrac{98\frac{2}{3}}{2\frac{1}{8}}$ take $\dfrac{22\frac{1}{4}}{8\frac{1}{4}}$. $Ans.\ 43\frac{83}{102}$.

7. From $\dfrac{64\frac{5}{8}}{\frac{2}{3}}$ take $\dfrac{14\frac{1}{6}}{5\frac{1}{3}}$. $Ans.\ 94\frac{9}{32}$.

8. From $\dfrac{18\frac{9}{10}}{5\frac{1}{4}}$ take $\dfrac{16\frac{1}{4}}{8\frac{1}{9}}$. $Ans.\ 1\frac{871}{1460}$.

LESSON XXXII.

256. MULTIPLICATION OF COMPLEX FRACTIONS.

EXAMPLE.

Multiply $\dfrac{1\frac{1}{3}}{2\frac{1}{4}}$ by $\dfrac{8\frac{1}{6}}{4\frac{1}{3}}$.

Reduced to improper fractions gives $\dfrac{\frac{4}{3}}{\frac{9}{4}}$ by $\dfrac{\frac{49}{6}}{\frac{13}{3}}$.

Inverting the denominators gives $\frac{4}{3} \times \frac{4}{9}$ by $\frac{49}{6} \times \frac{3}{13}$.

Since the word by, in this sum means multiplication, substitute the sign of multiplication in its place, and the fraction will be a continued multiplication; thus,

$\dfrac{4}{3} \times \dfrac{4}{9} \times \dfrac{49}{6} \times \dfrac{3}{13}$. Multiplying the numerators

and denominators gives $\frac{784}{702}$; which, being reduced,

gives $1\frac{41}{351}$ for the answer.

1. Multiply $\dfrac{16}{1\frac{1}{2}}$ by $\dfrac{3\frac{1}{3}}{2\frac{1}{2}}$.   *Ans.* $14\frac{2}{9}$.

2. Multiply $\dfrac{14\frac{1}{2}}{2\frac{1}{4}}$ by $\dfrac{4\frac{1}{6}}{5\frac{1}{8}}$.   *Ans.* $5\frac{265}{1107}$.

3. Multiply $\dfrac{18\frac{5}{6}}{2\frac{1}{4}}$ by $\dfrac{8\frac{4}{6}}{9\frac{2}{3}}$.   *Ans.* $7\frac{335}{783}$.

4. Multiply $\dfrac{28\frac{1}{2}}{12\frac{1}{4}}$ by $\dfrac{12\frac{1}{2}}{9\frac{3}{4}}$.   *Ans.* $2\frac{656}{637}$.

5. Multiply $\dfrac{13\frac{5}{8}}{4\frac{1}{2}}$ by $\dfrac{20\frac{1}{2}}{10\frac{1}{4}}$.   *Ans.* $5\frac{2787}{2052}$.

6. Multiply $\dfrac{98\frac{2}{5}}{2\frac{1}{8}}$ by $\dfrac{8\frac{1}{2}}{22\frac{1}{4}}$.   *Ans.* $17\frac{147}{267}$.

7. Multiply $\dfrac{64\frac{5}{8}}{\frac{2}{3}}$ by $\dfrac{5\frac{1}{3}}{14\frac{1}{4}}$.   *Ans.* $36\frac{42}{55}$.

8. Multiply $\dfrac{18\frac{9}{10}}{5\frac{1}{4}}$ by $\dfrac{8\frac{1}{9}}{16\frac{1}{4}}$.   *Ans.* $1\frac{252}{325}$.

LESSON XXXIII.

257. DIVISION OF COMPLEX FRACTIONS.

EXAMPLE.

Divide $\dfrac{2\frac{1}{4}}{3\frac{1}{4}}$ by $\dfrac{2\frac{1}{5}}{4\frac{1}{5}}$.

Reduced to improper fractions gives $\dfrac{\frac{9}{4}}{\frac{13}{4}}$ by $\dfrac{\frac{11}{5}}{\frac{21}{5}}$.
Placed in the form of division gives $(\frac{9}{4} \div \frac{13}{4})$ by $(\frac{11}{5} \div \frac{21}{5})$. Inverting the divisors gives $(\frac{9}{4} \times \frac{4}{13})$ by $(\frac{11}{5} \times \frac{5}{21})$. The last complex fraction reduced $(\frac{11}{5} \times \frac{5}{21})$ being the divisor of the given sum, invert according to the rule, in which case all fractions of the dividend are inverted, and the sum is $\dfrac{9}{4} \times \dfrac{4}{13} \times \dfrac{2}{5} \times \dfrac{21}{5}$. Cancelled and multiplied gives $\frac{378}{325}$. Reducing gives $1\frac{53}{325}$ for the answer.

1. Divide $\dfrac{9\frac{1}{2}}{31}$ by $\dfrac{1\frac{1}{3}}{3\frac{1}{2}}$.   *Ans.* $\frac{399}{496}$.

2. Divide $\dfrac{\frac{7}{11}}{\frac{4}{3}}$ by $\dfrac{1\frac{1}{4}}{9\frac{1}{2}}$.   *Ans.* $3\frac{69}{110}$.

3. Divide $\dfrac{\frac{5}{8}}{1\frac{1}{8}}$ by $\dfrac{1\frac{5}{8}}{11\frac{5}{8}}$.   *Ans.* $3\frac{3}{104}$.

4. Divide $\dfrac{6\frac{1}{3}}{11\frac{6}{11}}$ by $\dfrac{\frac{40}{527}}{\frac{40}{527}}$.   *Ans.* $\frac{899}{881}$.

5. Divide $\dfrac{8\frac{9}{11}}{6\frac{4}{5}}$ by $\dfrac{6\frac{1}{3}}{6\frac{6}{11}}$.   *Ans.* $1\frac{27643}{66611}$.

6. Divide $\dfrac{74\frac{1}{2}}{83\frac{1}{2}}$ by $\dfrac{21\frac{1}{2}}{34\frac{1}{3}}$. $\qquad$ *Ans.* $1\frac{9151}{21543}$.

7. Divide $\dfrac{2\frac{1}{2}}{5\frac{1}{4}}$ by $\dfrac{11\frac{2}{4}}{14\frac{3}{4}}$. $\qquad$ *Ans.* $\frac{590}{1113}$.

8. Divide $\dfrac{2\frac{4}{5}}{9\frac{1}{2}}$ by $\dfrac{2\frac{2}{5}}{4\frac{1}{2}}$. $\qquad$ *Ans.* $\frac{55}{114}$.

9. Divide $\dfrac{\frac{27}{36}}{\frac{9}{5}}$ by $\dfrac{\frac{14}{18}}{\frac{3}{4}}$. $\qquad$ *Ans.* $\frac{135}{392}$.

10. Divide $\dfrac{\frac{28}{65}}{\frac{11}{13}}$ by $\dfrac{\frac{16}{17}}{\frac{4}{5}}$. $\qquad$ *Ans.* $\frac{221}{550}$.

11. Divide $\dfrac{\frac{25}{33}}{\frac{100}{891}}$ by $\dfrac{\frac{28}{27}}{\frac{13}{36}}$. $\qquad$ *Ans.* $2\frac{121}{304}$.

12. Divide $\dfrac{\frac{15}{45}}{\frac{45}{392}}$ by $\dfrac{\frac{156}{78}}{\frac{12}{13}}$. $\qquad$ *Ans.* $1\frac{3}{13}$.

13. Divide $\dfrac{\frac{1108}{386}}{1\frac{36}{38}}$ by $\dfrac{3\frac{1}{3}}{4}$. $\qquad$ *Ans.* $\frac{2058}{27335}$.

14. Divide $\dfrac{1\frac{4}{5}}{1\frac{6}{7}}$ by $\dfrac{4\frac{9}{2}}{\frac{7}{4}}$. $\qquad$ *Ans* $\frac{9}{130}$.

15. Divide $\dfrac{2\frac{20}{3}}{3\frac{2}{3}}$ by $\dfrac{1\frac{9}{8}}{24}$. $\qquad$ *Ans.* $23\frac{185}{209}$.

16. Divide $\dfrac{\frac{12}{46}}{\frac{6}{23}}$ by $\dfrac{32\frac{1}{2}}{11\frac{1}{6}}$. $\qquad$ *Ans.* $\frac{736}{2275}$.

17. Divide $\dfrac{\frac{28}{36}}{\frac{17}{19}}$ by $\dfrac{\frac{1}{2}}{\frac{2}{3}}$. $\qquad$ *Ans.* $1\frac{73}{459}$.

18. Divide $\dfrac{\frac{52}{62}}{\frac{4}{5}}$ by $\dfrac{\frac{2}{3}}{\frac{3}{4}}$. $\qquad$ *Ans.* $2\frac{89}{248}$.

19. Divide $\dfrac{1\frac{1}{2}}{1\frac{3}{5}}$ by $\dfrac{\frac{3}{4}}{\frac{4}{5}}$. $\qquad$ *Ans.* $\frac{20}{27}$.

20. Divide $\dfrac{\frac{13}{15}}{\frac{12}{14}}$ by $\dfrac{\frac{4}{5}}{\frac{5}{6}}$. *Ans.* $\frac{65}{216}$.

21. Divide $\dfrac{\frac{3}{7}}{\frac{26}{35}}$ by $\dfrac{\frac{6}{6}}{\frac{6}{7}}$. *Ans.* $\frac{524}{637}$.

22. Divide $\dfrac{\frac{4}{27}}{\frac{6}{8}}$ by $\dfrac{\frac{6}{7}}{\frac{7}{8}}$. *Ans.* $\frac{49}{864}$.

23. Divide $\dfrac{\frac{11}{3}}{\frac{15}{14}}$ by $\dfrac{\frac{7}{8}}{\frac{8}{9}}$. *Ans.* $3\frac{103}{105}$.

LESSON XXXIV.

258. Brackets are used for the purpose of enclosing a set of numbers which must be considered separately from all others in a problem. Thus: $(1+2+3)-4$ means that 1, 2, and 3 must be added together before the figure 4 is to be subtracted from them.

EXAMPLES.

1. Find the sum of $(2+3+4)-(5)=9-5=4$.

2. Find the sum of $(4-2)\times3=2\times3=6$.

3. Find the sum of $(5-2)\times3)-4=(3\times3)-4=9-4=5$.

4. Find the value of $(\frac{1}{2}+\frac{2}{3}+\frac{3}{4})-(\frac{1}{8}+\frac{1}{4})$. *Ans.* $1\frac{19}{120}$.

5. Find the value of $(\frac{1}{3}+\frac{1}{4}+\frac{5}{6})\times(\frac{1}{2}+\frac{2}{3})$. *Ans.* $2\frac{53}{180}$.

6. Find the value of $(\frac{1}{2}\times\frac{3}{4})\div(\frac{2}{3}\times\frac{1}{4})$. *Ans.* $\frac{45}{64}$.

7. Find the value of $(\frac{1}{8} \div \frac{3}{4}) \times (\frac{5}{6} \div \frac{1}{4})$.   *Ans.* $\frac{5}{9}$.

8. Find the value of $(\frac{9}{10} \times \frac{5}{6}) + (\frac{5}{8} - \frac{1}{3})$.   *Ans.* $1\frac{19}{12}$.

9. Find the value of $(\frac{1}{12} \div \frac{2}{4}) - (\frac{4}{5} - \frac{1}{9})$.   *Ans.*

10. Find the value of $(\frac{4}{5} - \frac{1}{3}) \div (\frac{5}{4} + \frac{3}{4})$.   *Ans.* $\frac{23}{144}$.

11. Find the value of $(\frac{4}{9} \div \frac{5}{8}) \times (\frac{1}{3} + \frac{1}{4})$.   *Ans.* $\frac{27}{45}$.

12. Find the value of $(\frac{1}{3} + \frac{3}{4}) - (\frac{1}{3} + \frac{1}{6})$.   *Ans.* $\frac{1}{24}$.

13. Find the value of $(\frac{5}{4} - \frac{1}{5}) + (\frac{1}{8} \div \frac{2}{5})$.   *Ans.* $\frac{47}{45}$.

14. Find the value of $(\frac{5}{8} + \frac{6}{5}) - (\frac{1}{3} - \frac{1}{8})$.   *Ans.* $2\frac{35}{48}$.

15. Find the value of $(\frac{1}{2} \times \frac{2}{3} \times \frac{6}{5}) - \frac{1}{4}$.   *Ans.* $\frac{3}{20}$.

16. Find the value of $(12 - 1) \times (\frac{1}{3} \times \frac{1}{4})$.   *Ans.* $\frac{11}{12}$.

17. Find the value of $(3 + 4) \times (\frac{1}{3} - \frac{1}{5})$.   *Ans.* $6\frac{7}{4}\frac{1}{4}$.

18. Find the value of $(\frac{1}{3} - \frac{1}{5}) \div (\frac{1}{4} \times \frac{1}{6})$.   *Ans.* 12.

19. Find the value of $(\frac{4}{5} \times \frac{1}{3}) \div (\frac{5}{6} \times \frac{2}{3})$.   *Ans.* $\frac{3}{10}$.

20. Find the value of $(\frac{1}{5} + \frac{1}{4}) \times (\frac{1}{3} \div \frac{1}{4})$.   *Ans.* $\frac{1}{5}\frac{3}{5}$.

21. Find the value of $(\frac{2}{3} + \frac{3}{4}) \div (\frac{1}{3} \div \frac{1}{4})$.   *Ans.*

22. Find the value of $(\frac{33}{44} \div \frac{44}{66}) \div (\frac{1}{3} \times \frac{1}{3} \times \frac{72}{88})$.

     Ans. 82.

LESSON XXXV.

23. Find the product of

$$\left(\frac{\frac{1}{2} + \frac{2}{3} + \frac{3}{4}}{\frac{1}{8} + \frac{1}{16} + \frac{3}{24}}\right) \times \frac{(\frac{2}{3} \times \frac{4}{5} \times \frac{90}{24})}{(\frac{1}{8} + \frac{1}{4}) \div (\frac{1}{3} \times \frac{1}{6})}$$

     Ans. $\frac{1472}{1841}$.

24. What is the sum of

$$\left(\frac{\left(\frac{1}{6}+\frac{4}{18}+\frac{1}{3}\right)}{\left(\frac{2}{3}\times\frac{4}{5}\times\frac{20}{24}\right)}+\frac{\left(\frac{1}{2}+\frac{1}{4}+\frac{5}{6}\right)}{\left(\frac{1}{3}-\frac{1}{4}\right)\times\left(\frac{1}{4}-\frac{1}{16}\right)}\right)\times3$$

Ans. 420½.

25. Divide

$$\left[\frac{\frac{2\frac{2}{5}}{3}\times\frac{5\frac{1}{3}}{2\frac{1}{2}}\times\frac{\frac{3}{4}}{6\frac{2}{3}}}{\frac{4\frac{1}{2}}{7\frac{1}{5}}\times\frac{6\frac{1}{4}}{3\frac{1}{5}}\times\frac{8\frac{1}{6}}{7\frac{1}{3}}}\right] \div \left[\frac{\frac{27\frac{1}{2}}{37\frac{4}{5}}\times\frac{9\frac{5}{6}}{2\frac{1}{5}}\times\frac{81\frac{5}{11}}{128\frac{1}{2}}}{\frac{87\frac{2}{5}}{98\frac{1}{5}}\times\frac{3}{4}\times\frac{7\frac{1}{2}}{11\frac{1}{4}}\times\frac{5\frac{3}{4}}{8\frac{1}{4}}}\right]$$

Ans. $\frac{39064}{401841}$.

26. Find the answer for

$$\frac{\left(\frac{7}{11}\times\frac{4}{5}\times\frac{5}{3}\times1\frac{1}{8}\right)}{\left(\frac{12}{60}\times\frac{300}{30}\times\frac{3}{6}\times\frac{6}{3}\times1\frac{2}{9}\right)} \div \frac{\left(\frac{3}{4}\times\frac{6}{35}\times\frac{4}{5}\times\frac{3}{8}\right)}{\left(5\frac{3}{5}\div3\frac{1}{2}\right)}$$

Ans. $15\frac{3115}{5045}$.

27. Find the answer for

$$\left[\frac{\left(\frac{1\frac{1}{2}}{\frac{4}{5}}\times\frac{\frac{3}{4}}{1\frac{5}{12}}\times\frac{\frac{7}{15}}{\frac{5}{21}}\times\frac{\frac{7}{10}}{\frac{2}{7}}\times\frac{5\frac{1}{3}}{13\frac{1}{2}}\times\frac{2\frac{1}{3}}{5\frac{1}{2}}\right)}{\left(\frac{2\frac{1}{2}}{6\frac{1}{4}}\times\frac{39\frac{1}{16}}{6\frac{1}{4}}\times\frac{12\frac{1}{4}}{3\frac{1}{2}}\times\frac{7\frac{7}{10}}{3\frac{1}{2}}\times\frac{30\frac{1}{4}}{5\frac{1}{3}}\right)}\times\frac{\left(\frac{272\frac{1}{4}}{16\frac{1}{2}}\times\frac{20\frac{1}{4}}{4\frac{1}{2}}\times\frac{56\frac{1}{4}}{7\frac{1}{2}}\times\frac{8\frac{3}{4}}{3\frac{1}{2}}\times\frac{2\frac{1}{3}}{\frac{2}{3}}\right)}{\left(\frac{26}{8\frac{1}{2}}\times\frac{18\frac{1}{8}}{9\frac{1}{4}}\times\frac{9\frac{1}{2}}{3\frac{1}{4}}\times\frac{8\frac{3}{4}}{2\frac{1}{6}}\times\frac{7\frac{8}{15}}{\frac{5}{6}}\right)}\right]\div\frac{1}{4}\times\frac{1}{3}$$

Ans. $\frac{37383125}{41261316}$.

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CHAPTER XI.

LESSON I.

259. THE RECIPROCAL of any number is unity (1) divided by the number.

The reciprocal of a fraction is unity divided by the fraction which is the same as inverting the terms of the fractions.

EXAMPLES.

The reciprocals of 2, 3, 4, 5, etc., are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, etc.

The reciprocals of $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$, are $\frac{4}{3}, \frac{5}{4}, \frac{6}{5}, \frac{7}{6}$.

To find the reciprocal of a fraction divide 1 by the fraction and the terms will be inverted.

Thus $1 \div \frac{5}{6} = 1 \times \frac{6}{5} = \frac{6}{5}$, the reciprocal of $\frac{5}{6}$.

1. What are the reciprocals of 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40?

2. What are the reciprocals of $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}$, $\frac{9}{10}, \frac{10}{11}, \frac{11}{12}, \frac{12}{13}, \frac{13}{14}, \frac{14}{15}, \frac{15}{16}$?

3. When a fraction is a mixed number, reduce to an improper fraction and invert the terms. What are the reciprocals of $1\frac{1}{2}, 2\frac{2}{3}, 3\frac{3}{4}, 4\frac{4}{5}, 5\frac{5}{6}, 6\frac{6}{7}, 7\frac{7}{8}, 8\frac{8}{9}$, $9\frac{9}{10}, 10\frac{10}{11}, 11\frac{11}{12}, 12\frac{12}{13}, 13\frac{13}{14}$?

4. What is the reciprocal of $\frac{1}{2}$ of $\frac{5}{8}$. $\frac{1}{2} \times \frac{5}{8} = \frac{5}{16}$. Inverted gives $\frac{16}{5}$, *Ans.*

5. What are the reciprocals of $\frac{1}{3}$ of $4\frac{1}{2}$? $\frac{5}{6}$ of $\frac{8}{9}$? $1\frac{1}{2}$ of $2\frac{1}{3}$? $\frac{4}{5}$ of $\frac{9}{12}$? $1\frac{3}{4}$ of $\frac{32}{36}$? $\frac{24}{132}$ of $1\frac{3}{8}$?

CHAPTER XII.

DECIMALS.

LESSON I.

260. If an apple be divided into 10 equal parts, each part would be one tenth of the whole apple. Again, if one of the tenths be divided into 10 equal parts, each one of the parts is called one hundredth of the apple.

261. The above expressions are written two ways; either in the form of a fraction having a denominator, as $\frac{1}{10}$, $\frac{1}{100}$, or in the form of a fraction whose denominator is understood, as .1, .01. The last expressions are called decimal fractions. From the above it will be seen that the denominator of a decimal fraction is 10 or any number of 10's.

262. Decimal fractions are generally written without the denominator, the period or decimal point serving to show where the numerator begins. All the figures to the right of the decimal point make the numerator, those to the left are whole numbers; thus, 45.123 is read, 45 whole ones and 123 thousandths. Therefore the denominator of a decimal fraction is 1 with as many 0's as there are figures to the right of the decimal point. Every cipher written between the decimal figures and the decimal

point decreases the value ten fold. Thus .4 is 4 tenths, .04 is 4 hundredths, .004 is 4 thousandths, etc., etc., etc. Ciphers placed to the right of any figure after the decimal point do not change the value of the fraction; thus, 4 tenths, 40 hundredths, 400 thousandths, etc., etc., are of the same value.

263. The first place to the right of the decimal point is called tenths, 2d, hundredths, 3d, thousandths, etc., etc.

<div align="center">TABLE.</div>

Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.	Decimal point.	Tenths.	Hundredths.	Thousandths.	Tens of thousandths.	Hundreds of thousandths.	Millionths.	Etc., etc.
7	6	5	4	3	2	1	.	1	2	3	4	5	6	

<div align="center">Whole numbers.   Decimals.</div>

264. A mixed decimal number is a whole number and a fraction; as, 2.5, two and five tenths.

A complex decimal fraction is a decimal fraction and a common fraction; thus, $.2\frac{1}{3}$, two and one third tenths.

<div align="center">NUMERATION.</div>

265. To read decimal numbers:

<div align="center">RULE.</div>

Read the decimal figures as if they were whole numbers, and add the name of the right hand figure.

Thus, read .2046.

The number of places is 4. The denominator is then ten thousandths. It is read two thousand and forty-six ten thousandths.

Read the following :

.5 .8 .7 .3 .9 .4 .6 .2 .1 .10 .11 .12
.13 .14 .16 .28 .92 .43 .105 .111 .246
.384 .975 .287 .693 .108 .1012 .1001 .2003
.3976 .1876 .1869 .42976 .40976 .40076
.40006 .98764 .764921 .300010 .401010
.870001 .287641.

LESSON II.

NOTATION.

266. To write decimals.

Write the decimal as if it were a whole number. Supply the vacant spaces with ciphers, and be careful to have as many figures in the numerator as there are 0's in the denominator. Never omit the period on the left of the decimal figures.

EXAMPLE.

267. Write in figures one hundred and one thousandths. In thousands there are three 0's, there must be three figures in the numerator. One hundred and one is composed of three figures. Write one hundred and one as if it was a whole number, and place the decimal point to the left of it. Thus, .101

1. Write the following : five tenths, ten hund-

redths, one hundred and five ten thousandths, one
thousand and one hundreds of thousandths.   Three
hundred thousand and ten millionths.   Forty thou-
sand nine hundred and seventy-six one hundred
thousandths.

<div align="center">

LESSON III.

268. ADDITION OF DECIMALS.

RULE.

</div>

*Write the numbers under each other, so that the
figures of the same value fall under each other.   Add
as in whole numbers.*

<div align="center">

EXAMPLES.

</div>

1. Add 2.461, 31.96 and 841.2581.

	OPERATION.
Write the numbers so that the figures of the same decimal place will fall one under the other. Add and point off as many figures, beginning at the right, as there are decimal figures in the number having the greatest number of decimals.	2.461 31.96 841.2581 ――――― 875.6791

2. Add 1234.56, 123.456, and 12.3456.

<div align="right">

Ans. 1370.3616.

</div>

3. Add 2.34567, 234.567, and 2345.67.

<div align="right">

Ans. 2582.58267.

</div>

4. Add 34567.8, 345.678, and 345.678.

<div align="right">

Ans. 35259.156.

</div>

5. Add 45.6789, 45678.9, and 45678.9.

<div align="right">

Ans. 91403.4789.

</div>

6. Add 56.7891, 567.891, and 5678.91.

Ans. 6303.5901.

7. Add 6789.10, 678.910, and 67.8910.

Ans. 7535.901.

8. Add 789.11, 7891.1, and 78911.

Ans. 87591.21.

9. Add 89.101, 8.9101, and .89101.

Ans. 98.90211.

10. Add 912.34, 912.34, and 9.1234.

Ans. 1833.8034.

11. Add 1.2345, 1234.5, and 123.45.

Ans. 1359.1845.

12. Add 1.3456, .13456, and 1.3456.

Ans. 2.592576.

13. Add 5.4321, 543.21, and 54.321.

Ans. 602.9631.

14. Add 65.432, 654.32, and 6543.2.

Ans. 7262.952.

15. Add 7.6543, 7654.3, and 765.43.

Ans. 8427.3843.

16. Add 87.654, 876.54, and 87654.

Ans. 88618.194.

LESSON IV.

269. SUBTRACTION OF DECIMALS.

RULE.

Write the subtrahend under the minuend, so that the decimals of the same order will be found under each other, and subtract as in whole numbers.

EXAMPLES.

1. From 25.1304 take 8.24.

OPERATION.

Supply the vacant spaces with 0's, and subtract as in whole numbers.

$$\begin{array}{r} 25.1304 \\ 8.2400 \\ \hline 16.9904 \end{array}$$

2. From 102.03 take 12.345.   *Ans.* 89.685.

3. From 12.030 take 2.3045.   *Ans.* 9.7255.

4. From 1230.4 take 234.05.   *Ans.* 996.35.

5. From 123.40 take 20.405.   *Ans.* 102.995.

6. From 23.040 take 12.040.   *Ans.* 11.00.

7. From 340.50 take 60.872.   *Ans.* 279.628.

8. From 4050.5 take 968.43.   *Ans.* 3082.07.

9. From 60.704 take 12.3042.   *Ans.* 48.3998.

10. From 123.45 take 102.0302.   *Ans.* 21.4198.

11. From 6780.9 take 187.643.   *Ans.* 6593.257.

12. From 901.011 take 24.680.   *Ans.* 876.331.

13. From 12.3042 take 1.8764.   *Ans.* 10.4278.

14. From 8976.41 take 102.36.   *Ans.* 8874.05.

15. From 8090.706 take 1809.2.   *Ans.* 6281.506.

16. From 24.6809 take 2.8096.   *Ans.* 21.8713.

17. From 123807.2 take 6.3082. *Ans.* 123800.8918.

18. From 2380.70 take 60.302.   *Ans.* 2320.398.

19. From 642.890 take 630.89.   *Ans.* 12.00.

20. From 389.7602 take 60.987.   *Ans.* 328.7732.

LESSON V.

270. MULTIPLICATION OF DECIMALS

Multiply 13.24 by 2.13.

	OPERATION.
Multiply as in whole numbers. From the product cut off as many figures, counting from right to left, as there are decimal figures in both multiplier and multiplicand. The figures to the left of the decimal point represent whole numbers and those to the right represent the fractional part.	13.24
	2.13
	39 72
	1 32 4
	26 48
	28.20 12

EXAMPLES.

1. Multiply 12.3 by 12.   *Ans.* 147.6.
2. Multiply 12.34 by 12.3.   *Ans.* 151.782.
3. Multiply 1.234 by 32.1.   *Ans.* 39.6114.
4. Multiply 43.21 by 2.13.   *Ans.* 92.0373.
5. Multiply 34.12 by 3.12.   *Ans.* 106.4544.
6. Multiply 876.4 by 5.15.   *Ans.* 4513.46.
7. Multiply 786.4 by 5.51.   *Ans.* 4333.064.
8. Multiply 7.864 by 15.5.   *Ans.* 12.1892.
9. Multiply 68.74 by 1.24.   *Ans.* 85.2376.
10. Multiply 87.64 by .214.   *Ans.* 18.75496.
11. Multiply 21.87 by .313.   *Ans.* 6.84531.
12. Multiply 187.2 by .331.   *Ans.* 61.9632.
13. Multiply 18.76 by .248.   *Ans.* 4.65248.
14. Multiply 76.98 by 21.83.   *Ans.* 1680.4734.
15. Multiply 187.64 by 28.96.   *Ans.* 5434.0544.

16. Multiply 9898.98 by .1111. *Ans.* 1099.776678.

17. Multiply 8989.89 by 2.222. *Ans.* 19975.55558.

18. Multiply 7643.924 by 121.212.

Ans. 926535.315888.

LESSON VI.

19. Multiply .0121 by 4.1.

271. Multiply as in whole num-
bers. From the product cut off as
many figures, counting from right to
left, as there are decimals in both
multiplier and multiplicand. If there
is not as many figures in the product
as decimals in both multiplier and
multiplicand, prefix the required num-
ber of ciphers.

OPERATION.

.0121
4.1
———
121
484
———
.04961

EXAMPLES.

20. Multiply 1.234 by .02.   *Ans.* .02468.

21. Multiply 1.324 by .021.   *Ans.* .027804.

22. Multiply 2.134 by .0213.   *Ans.* .0454542.

23. Multiply 4.132 by .0287.   *Ans.* .1185884.

24. Multiply 3.142 by .003.   *Ans.* .009426.

25. Multiply 4.312 by .008.   *Ans.* 034496.

26. Multiply 6.492 by .0092.   *Ans.* .0597264.

27. Multiply 2.946 by .0028.   *Ans.* .0082488.

28. Multiply 9.246 by .0093.   *Ans.* .0859878.

29. Multiply 9.642 by .087.   *Ans.* .838854.

30. Multiply 8.287 by .021.   *Ans.* .174027.

31. Multiply 2.878 by .082.   *Ans.* .235996.
32. Multiply 8.782 by .002.   *Ans.* .017564.
33. Multiply 2.788 by .0003.   *Ans.* .0008364.
34. Multiply 9.673 by .0121.   *Ans.* .1170433.
35. Multiply 7.693 by .0128.   *Ans.* .0984704.
36. Multiply 8.428 by .0212.   *Ans.* .1786736.
37. Multiply 8.367 by .0218.   *Ans.* .1824006.
38. Multiply 2.892 by .0812.   *Ans.* .2348304.

LESSON VII.

272. Note.—Multiplying any decimal by 10, 100, 1000, etc., removes the decimal point 1, 2, 3, etc., periods to the left. If there be not enough of places, annex ciphers.

EXAMPLES.

39. Multiply 24.68 by 10.   *Ans.* 246.8.
40. Multiply 46.28 by 100.   *Ans.* 4628.
41. Multiply 86.42 by 1000.   *Ans.* 86420.
42. Multiply 24.86 by 10.   *Ans.* 248.6.
43. Multiply 98.743 by 100.   *Ans.* 9874.3.
44. Multiply 34.789 by 100.   *Ans.* 3478.9.
45. Multiply 4.3789 by 1000.   *Ans.* 4378.9.
46. Multiply 74.389 by 100.   *Ans.* 7438.9.
47. Multiply 8.7439 by 1000.   *Ans.* 8743.9.
48. Multiply 8974.3 by 10.   *Ans.* 89743.
49. Multiply 78.943 by 100.   *Ans.* 7894.3.
50. Multiply 47.893 by 1000.   *Ans.* 47893.
51. Multiply 218.726 by 10.   *Ans.* 2187.26.
52. Multiply 281.721 by 100.   *Ans.* 2817.21.
53. Multiply 821.217 by 100.   *Ans.* 82121.7.

54. Multiply 281.721 by 100.   *Ans.* 28172.1.

55. Multiply 67.434 by 100.   *Ans.* 674.34.

56. Multiply 8972.3 by 100.   *Ans.* 897230.

57. Multiply 67.348 by 1000.   *Ans.* 67348.

58. Multiply 32.798 by 1000.   *Ans.* 32798.

59. Multiply 2972.43 by 10.   *Ans.* 29724.3.

60. Multiply 8.64398 by 100000.   *Ans.* 864398.

61. Multiply 9.87248 by 100000.   *Ans.* 987248.

62. Multiply 9.876432 by 1000000. *Ans.* 9876432.

LESSON VIII.

273. DIVISION OF DECIMALS.

Divide 25.4 by 2.41.

In the divisor there are 2 decimal figures ; in the dividend but 1. Make the number of decimal figures equal in both, by supplying the deficiency with 0's, as in the example, and divide as in whole numbers. The quotient thus found will be the whole number. When there is a remainder, annex a cipher, after writing the decimal point in the quotient to the right of the whole number, and continue the division as far as you think proper.

OPERATION.

$$2.41)25.40(10.539+$$
$$\quad 24.1$$
$$\overline{\quad\quad 1300}$$
$$\quad\quad 1205$$
$$\overline{\quad\quad\quad 950}$$
$$\quad\quad\quad 723$$
$$\overline{\quad\quad\quad 2270}$$
$$\quad\quad\quad 2169$$
$$\overline{\quad\quad\quad 101}$$

EXAMPLES.

1. Divide 864.32 by 12.345.   *Ans.* $70.013\frac{1203}{2469}$.
2. Divide 264.38 by 12.345.   *Ans.* $21.415\frac{2385}{2469}$.
3. Divide 197.64 by 12.345.   *Ans.* $16.009\frac{593}{823}$.
4. Divide 419.76 by 54.321.   *Ans.* $7.727\frac{7211}{18107}$.
5. Divide 64.197 by 87.643.   *Ans.* $.742\frac{65894}{87643}$.
6. Divide 76.419 by 22.876.   *Ans.* $3.340\frac{3290}{5719}$.
7. Divide 647.91 by 98.724.   *Ans.* $6.562\frac{6826}{8227}$.
8. Divide 28.435 by 22.472.   *Ans.* $1.260\frac{45}{5618}$.
9. Divide 82.345 by 1.212.   *Ans.* $67.941\frac{127}{323}$.
10. Divide 32.854 by 13.13.   *Ans.* $2.501\frac{687}{1313}$.
11. Divide 43.285 by 14.14.   *Ans.* $3.061\frac{120}{707}$.
12. Divide 54.328 by 15.15.   *Ans.* $3.586\frac{2}{303}$.
13. Divide 85.432 by 16.16.   *Ans.* $5.286\frac{331}{404}$.
14. Divide 679.482 by 17.17.   *Ans.* $39.515\frac{9457}{17170}$.
15. Divide 728.4365 by 28.28.   *Ans.* $25.758\frac{13}{1414}$.

LESSON IX.

274. Divide .12 by 3.4.

In the dividend we have two decimals, and in the divisor but one. Annex an 0 to the divisor. It now becomes 3.40. The decimal places being even, we now divide as in whole numbers. 340 into 12, 0 times, 0 whole ones. Annex a cipher, making 120 tenths. 340 into 120 tenths, 0 tenths. An-

OPERATION.

$$3.40).1200(0.035+$$
$$\underline{1020}$$
$$1800$$
$$1700$$
$$\underline{}$$
$$100$$

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nex another cipher, making it hundredths (1200 hundredths). 340 into 1200, 3 hundred times and 180 hundredths over. Add an 0 to the remainder, making it 1800 thousandths. 340 into 1800, 5 thousand times and 100 over, etc., etc., etc.

16. Divide 2.46 by 11.11.	*Ans.* .221$\frac{469}{1111}$.
17. Divide 4.62 by 22.22.	*Ans.* .207$\frac{1023}{1111}$.
18. Divide 6.42 by 33.33.	*Ans.* .192$\frac{2064}{3333}$.
19. Divide 4.26 by 44.44.	*Ans.* .095$\frac{3832}{4444}$.
20. Divide 6.24 by 55.55.	*Ans.* .112$\frac{1840}{5550}$.
21. Divide 9.87 by 66.66.	*Ans.* .148$\frac{1532}{6666}$.
22. Divide 3.48 by 7.777.	*Ans.* .044$\frac{5812}{7777}$.
23. Divide 9.82 by 88.88.	*Ans.* .110$\frac{4320}{8888}$.
24. Divide 7.64 by 9.999.	*Ans.* .764$\frac{764}{9999}$.
25. Divide 9.81 by 10.111.	*Ans.* .970$\frac{3500}{10111}$.
26. Divide 24.61 by 112.22.	*Ans.* .219$\frac{1691}{5611}$.
27. Divide 8.97 by 12.333.	*Ans.* .727$\frac{1303}{4111}$.
28. Divide 3.82 by 14.1414.	*Ans.* .270$\frac{9110}{70707}$.
29. Divide 9.43 by 15.1617.	*Ans.* 621$\frac{145843}{151617}$.
30. Divide 8.64 by 12.345.	*Ans.* .699$\frac{123}{823}$.

CHAPTER XIII.

LESSON I.

275. REDUCTION OF COMMON FRACTIONS TO DE-
CIMALS.

GENERAL RULE.

Divide the numerator by the denominator.

1. Reduce $\frac{7}{8}$ to a decimal fraction.

8 into 7 units, 0 times, or 0 units. Annexing an 0 to the 7 makes it tens. 8 into 70 tens, 8 tenths times. 8 times 8 are 64, and 6 remaining. Annexing an 0 to the 6, makes it 60 hundredths. 8 into 60, 7 times, and 4 hundredths over. Annexing an 0 to the 4 makes it 40 thousandths. 8 into 40, 5 times, and 0 remaining. If the decimal figure repeats, write the remainder in the form of a fraction. Thus,

OPERATION.

$$8)70(0.875$$
$$\underline{64}$$
$$\begin{array}{r} 60 \\ 56 \\ \hline \end{array}$$
$$\begin{array}{r} 40 \\ 40 \\ \hline 0 \end{array}$$

2. Reduce $\frac{2}{3}$ to a decimal fraction.

By dividing, we find 2 remaining at every subtraction. Carry the decimal to 2 or 3 figures, and write the remainder in the form of a fraction. Before dividing, reduce the fractions to their lowest terms, if not so given.

OPERATION.

$$3)20(0.66\tfrac{2}{3}$$
$$\underline{18}$$
$$\begin{array}{r} 20 \\ 18 \\ \hline \frac{2}{3} \end{array}$$

nex another cipher, making it
hundredths).   340 into 1?
and 180 hundredths ove terms, gives
mainder, making it
1800, 5 thousand ation is improper,
etc.

OPERATION.
4)30(0.75
28
——
20
20
——
0

16. Divid

17. Di

18. ~

1~   Reduce 4/4 to a decimal fraction.

into 5, once, or 1 whole one.
Write 1, and place the point to the
right of it. To the remainder annex
an 0, and divide as before.

OPERATION.
4)5(1.25
4
—
10
8
——
20

276. If the given fraction is a mixed number:

1st. *Reduce to an improper fraction, and divide.*

Or:

2d. *Find the value of the fractional part, and
write it to the right of the whole number with the deci-
mal point between the whole number and the decimal
fraction.*

Thus:

5. Reduce 2¾ to a decimal fraction.

OPERATION.

$$4 \,) \, 11 \, (\, 2.75$$
$$8$$
$$\overline{}$$
$$30$$
$$28$$
$$\overline{}$$
$$20$$
$$20$$
$$\overline{}$$
$$0$$

ıst. Reduced to an improper fraction gives $\frac{11}{4}$. Divide as above.

Or:

277. 2d. Find the value of $\frac{3}{4}$ by the general rule. By the operation we find it to be .75. Write .75 to the right of the whole number 2, and we find the same answer as in the first case, viz., 2.75.

OPERATION.

$$4 \,) \, 30 \, (\, 0.75$$
$$28$$
$$\overline{}$$
$$20$$

278. If a complex fraction is given, reduce it to a simple fraction and divide as before.

279. Before dividing, all fractions must be reduced to their simplest forms.

EXAMPLES.

1. Reduce $\frac{1}{2}$ to a decimal fraction.   *Ans.* 0.5.
2. Reduce $\frac{1}{3}$ to a decimal fraction.   *Ans.* 0.333$\frac{1}{3}$.
3. Reduce $\frac{1}{4}$ to a decimal fraction.   *Ans.* 0.25.
4. Reduce $\frac{1}{5}$ to a decimal fraction.   *Ans.* 0.2.
5. Reduce $\frac{1}{6}$ to a decimal fraction.   *Ans.* 0.166$\frac{2}{3}$.
6. Reduce $\frac{1}{7}$ to a decimal fraction.   *Ans.* 0.142$\frac{6}{7}$.
7. Reduce $\frac{1}{8}$ to a decimal fraction.   *Ans.* 0.125.

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8. Reduce $\frac{1}{9}$ to a decimal fraction. *Ans.* 0.111$\frac{1}{9}$.

9. Reduce $\frac{1}{10}$ to a decimal fraction.　　*Ans.* 0.1.

10. Reduce $\frac{2}{3}$ to a decimal fraction. *Ans.* 0.666$\frac{2}{3}$.

11. Reduce $\frac{3}{4}$ to a decimal fraction.　　*Ans.* 0.75·

12. Reduce $\frac{4}{5}$ to a decimal fraction.　　*Ans.* 0.8.

13. Reduce $\frac{5}{6}$ to a decimal fraction. *Ans.* 0.833$\frac{1}{3}$.

14. Reduce $\frac{6}{7}$ to a decimal fraction. *Ans.* 0.857$\frac{1}{7}$.

15. Reduce $\frac{7}{8}$ to a decimal fraction.　*Ans.* 0.875.

16. Reduce $\frac{8}{9}$ to a decimal fraction.　*Ans.* 0.888$\frac{8}{9}$.

17. Reduce $\frac{9}{10}$ to a decimal fraction.　　*Ans.* 0.9.

18. Reduce $\frac{1}{2} + \frac{1}{3}$ to a decimal fraction.

Ans. 0.833$\frac{1}{3}$.

19. Reduce $\frac{1}{2} + \frac{2}{3}$ to a decimal fraction.

Ans. 1.166$\frac{2}{3}$.

20. Reduce $\frac{1}{3} + \frac{3}{4}$ to a decimal fraction.

Ans. 1.083$\frac{1}{3}$·

21. Reduce $\frac{1}{8} + \frac{9}{10}$ to a decimal fraction.

Ans. 1.025.

22. Reduce $\frac{1}{9} - \frac{1}{10}$ to a decimal fraction.

Ans. .0111$\frac{1}{9}$.

23. Reduce $\frac{5}{9} - \frac{4}{10}$ to a decimal fraction.

Ans. .155$\frac{5}{9}$.

24. Reduce $\frac{2}{3} - \frac{1}{6}$ to a decimal fraction.　*Ans.* .5.

25. Reduce $\frac{5}{6} - \frac{1}{12}$ to a decimal fraction. *Ans.* .75.

26. Reduce $\frac{1}{2} \times \frac{2}{3}$ to a decimal fraction.

Ans. .333$\frac{1}{3}$.

27. Reduce $\frac{1}{2} \times \frac{4}{8}$ to a decimal fraction. *Ans.* .25.

28. Reduce $\frac{1}{8} \times \frac{9}{10}$ to a decimal fraction.

Ans. .1125.

29. Reduce $\frac{1}{10} \times \frac{8}{11}$ to a decimal fraction.

Ans. .0727$\frac{3}{11}$.

30. Reduce $\frac{1}{12} \times \frac{24}{30}$ to a decimal fraction.

Ans. .066$\frac{2}{3}$.

31. Reduce $\frac{1}{2} \div \frac{2}{3}$ to a decimal fraction. *Ans.* .75.

32. Reduce $\frac{1}{8} \div \frac{1}{4}$ to a decimal fraction.

Ans. 444$\frac{4}{9}$.

33. Reduce $\frac{1}{8} \div \frac{5}{8}$ to a decimal fraction. *Ans.* .32.

34. Reduce $\frac{1}{8} \div \frac{2}{3}$ to a decimal fraction.

Ans. .1875.

35. Reduce $(\frac{1}{2} + \frac{1}{3}) \times (\frac{1}{3} - \frac{1}{6})$ to a decimal fraction.

Ans. .925$\frac{25}{27}$.

36. Reduce $(\frac{1}{8} \div \frac{1}{8}) + (\frac{1}{8} \div \frac{1}{10})$ to a decimal fraction.

Ans. 2.236$\frac{1}{9}$.

37. Reduce $\left(\dfrac{\frac{4}{5}}{\frac{5}{6}}\right) \div \frac{1}{2}$ to a decimal fraction.

Ans. 1.92.

38. Reduce $(\frac{3}{8} \div \frac{4}{7}) + \frac{1}{3}$ to a decimal fraction.

Ans. .819$\frac{4}{9}$.

LESSON II.

280. REDUCTION OF DECIMALS TO COMMON FRACTIONS.

RULE.

Write the figures of the given number for the numerator.

Write the figure 1 followed by as many 0's as there are decimal figures in the given number for the denominator.

Reduce the fraction thus formed to its lowest terms.

EXAMPLE.

Reduce .75 to a common fraction.

In this sum there are 2 decimal figures. Write the given number, 75, for the numerator, and for the | OPERATION.

$$\tfrac{75}{100} = \tfrac{3}{4}$$

denominator write 1 followed by two 0's. Reducing the fraction thus formed gives $\tfrac{3}{4}$ for the answer.

1. Reduce .5 to a simple fraction.   *Ans.* $\tfrac{1}{2}$.
2. Reduce .05 to a simple fraction.   *Ans.* $\tfrac{1}{20}$.
3. Reduce .2 to a simple fraction.   *Ans.* $\tfrac{1}{5}$.
4. Reduce .02 to a simple fraction.   *Ans.* $\tfrac{1}{50}$.
5. Reduce .25 to a simple fraction.   *Ans.* $\tfrac{1}{4}$.
6. Reduce .025 to a simple fraction.   *Ans.* $\tfrac{1}{40}$.
7. Reduce .175 to a simple fraction.   *Ans.* $\tfrac{7}{40}$.
8. Reduce .0175 to a simple fraction.   *Ans.* $\tfrac{7}{400}$.
9. Reduce .24 to a simple fraction.   *Ans.* $\tfrac{6}{25}$.
10. Reduce .024 to a simple fraction.   *Ans.* $\tfrac{3}{125}$.
11. Reduce .85 to a simple fraction.   *Ans.* $\tfrac{17}{20}$.
12. Reduce .085 to a simple fraction.   *Ans.* $\tfrac{17}{200}$.
13. Reduce 0.125 to a simple fraction.   *Ans.* $\tfrac{1}{8}$.
14. Reduce .0125 to a simple fraction.   *Ans.* $\tfrac{1}{80}$.
15. Reduce 0.1 to a simple fraction.   *Ans.* $\tfrac{1}{10}$.
16. Reduce .01 to a simple fraction.   *Ans.* $\tfrac{1}{100}$.
17. Reduce .75 to a simple fraction.   *Ans.* $\tfrac{3}{4}$.
18. Reduce .075 to a simple fraction.   *Ans.* $\tfrac{3}{40}$.
19. Reduce .8 to a simple fraction.   *Ans.* $\tfrac{4}{5}$.
20. Reduce .08 to a simple fraction.   *Ans.* $\tfrac{2}{25}$.
21. Reduce .875 to a simple fraction.   *Ans.* $\tfrac{7}{8}$.
22. Reduce .0875 to a simple fraction.   *Ans.* $\tfrac{7}{80}$.

23. Reduce .9 to a simple fraction. *Ans.* $\frac{9}{10}$.

24. Reduce .09 to a simple fraction. *Ans.* $\frac{9}{100}$.

25. Reduce .1125 to a simple fraction. *Ans.* $\frac{9}{80}$.

26. Reduce .01125 to a simple fraction. *Ans.* $\frac{9}{800}$.

27. Reduce 0.32 to a simple fraction. *Ans.* $\frac{8}{25}$.

28. Reduce .032 to a simple fraction. *Ans.* $\frac{4}{125}$.

29. Reduce .1875 to a simple fraction. *Ans.* $\frac{3}{16}$.

30. Reduce .01875 to a simple fraction. *Ans.* $\frac{3}{160}$.

31. Reduce .92 to a simple fraction. *Ans.* $\frac{23}{25}$.

32. Reduce .092 to a simple fraction. *Ans.* $\frac{23}{250}$.

33. Reduce .6 to a simple fraction. *Ans.* $\frac{3}{5}$.

34. Reduce .66 to a simple fraction. *Ans.* $\frac{33}{50}$.

35. Reduce .06 to a simple fraction. *Ans.* $\frac{3}{50}$.

36. Reduce .066 to a simple fraction. *Ans.* $\frac{33}{500}$.

37. Reduce .486 to a simple fraction. *Ans.* $\frac{243}{500}$.

38. Reduce .648 to a simple fraction. *Ans.* $\frac{81}{125}$.

39. Reduce .325 to a simple fraction. *Ans.* $\frac{13}{40}$.

LESSON III.

281. When there is an irreducible fraction at the end of the decimal.

Reduce .22$\frac{2}{9}$ to a simple fraction.

Multiply the decimal by the denominator, and to the result add the numerator, as in reduction of mixed numbers to improper fractions. Under the

result write the denominator followed by as many 0's as there are decimal figures in the given number.

$$(9 \times 22) + 2 = 200. \quad \frac{2\emptyset\emptyset}{9\emptyset\emptyset}. \quad \text{Reduced gives } \tfrac{2}{9}.$$

Reduce the fraction to its lowest terms.

RULE.

Reduce the decimal as in mixed numbers, and for the denominator, write the denominator of the fractional part followed by as many 0's as there are decimal figures in the given number.

Reduce the fraction thus formed to its lowest terms.

EXAMPLES.

1. Reduce .33⅓ to simple fractions.   *Ans.* ⅓.
2. Reduce .166⅔ to simple fractions.   *Ans.* ⅙.
3. Reduce .142⁶⁄₇ to simple fractions.   *Ans.* ⅐.
4. Reduce .111⅑ to simple fractions.   *Ans.* ⅑.
5. Reduce .66⅔ to simple fractions.   *Ans.* ⅔.
6. Reduce .833⅓ to simple fractions.   *Ans.* ⅚.
7. Reduce .857⅐ to simple fractions.   *Ans.* ⁶⁄₇.
8. Reduce .888⁸⁄₉ to simple fractions.   *Ans.* ⅘.
9. Reduce .106⅔ to a common fraction. *Ans.* ⁸⁄₇₅.
10. Reduce .083⅓ to a common fraction. *Ans.* ¹⁄₁₂.
11. Reduce .0111⅑ to a common fraction.

Ans. ¹⁄₉₀.
12. Reduce .155⁵⁄₉ to a common fraction. *Ans.* ⁷⁄₄₅.
13. Reduce .33⅔ to a common fraction. *Ans.* ¹⁰¹⁄₃₀₀.
14. Reduce .253⅓ to a common fraction. *Ans.* ¹⁹⁄₇₅.

15. Reduce $.82\frac{1}{4}$ to a common fraction. *Ans.* $\frac{247}{300}$.

16. Reduce $.0727\frac{3}{11}$ to a common fraction.

$$\textit{Ans. } \frac{4}{55}.$$

17. Reduce $.0606\frac{2}{3}$ to a common fraction.

$$\textit{Ans. } \frac{91}{1500}.$$

18. Reduce $.44\frac{4}{5}$ to a common fraction. *Ans.* $\frac{4}{9}$.

19. Reduce $.925\frac{25}{37}$ to a common fraction. *Ans.* $\frac{25}{27}$.

20. Reduce $.236\frac{1}{4}$ to a common fraction. *Ans.* $\frac{17}{72}$.

21. Reduce $.819\frac{1}{4}$ to a common fraction. *Ans.* $\frac{9}{11}$.

22. Reduce $.047\frac{1}{3}$ to a common fraction.

$$\textit{Ans. } \frac{71}{1500}.$$

23. Reduce $.28\frac{7}{8}$ to a common fraction. *Ans.* $\frac{259}{900}$.

24. Reduce $.33\frac{3}{5}$ to a common fraction. *Ans.* $\frac{61}{180}$.

25. Reduce $.01\frac{1}{4}$ to a common fraction. *Ans.* $\frac{1}{90}$.

26. Reduce $.46\frac{1}{3}\frac{2}{3}$ to a common fraction. *Ans.* $\frac{61}{130}$.

27. Reduce $.28\frac{4}{7}$ to a common fraction. *Ans.* $\frac{2}{7}$.

28. Reduce $.28\frac{1}{27}$ to a common fraction.

$$\textit{Ans. } \frac{757}{2700}.$$

29. Reduce $.33\frac{2}{3}$ to a common fraction. *Ans.* $\frac{299}{900}$.

30. Reduce $.48\frac{1}{4}$ to a common fraction. *Ans.* $\frac{29}{60}$.

31. Reduce $.26\frac{1}{6}$ to a common fraction. *Ans.* $\frac{47}{180}$.

32. Reduce $.18\frac{3}{4}$ to a common fraction. *Ans.* $\frac{11}{60}$.

33. Reduce $.16\frac{4}{11}$ to a common fraction. *Ans.* $\frac{9}{55}$.

34. Reduce $.283\frac{1}{4}$ to a common fraction. *Ans.* $\frac{17}{60}$.

35. Reduce $.24\frac{3}{5}$ to a common fraction. *Ans.* $\frac{55}{225}$.

36. Reduce $.2\frac{3}{5}$ to a common fraction. *Ans.* $\frac{13}{45}$.

37. Reduce $.1\frac{1}{3}$ to a common fraction. *Ans.* $\frac{2}{15}$.

CHAPTER XIII.

REPETENDS.

LESSON I.

282. It very often happens, when reducing a common fraction to a decimal, that the same figures will repeat in the quotient as long as the division is carried on. In order to show what figures repeat, a dot (.) is placed over them. In some instances, several sets of figures repeat. The dot is placed on the first and last; thus, $2\dot{4}3\dot{2}4\dot{3}2\dot{4}\dot{3}$. In a quotient like the one given, but 3 figures are used, because the other figures of the dividend are the same, since they are but a repetition of the numbers between the dots.

283. Whenever a dot is placed over a decimal figure, or over two figures between which there are other figures, it shows that the figure is repeated indefinitely. In the second case it shows that the figures over which the dot is placed, together with the figures between them, repeat in the order in which they are written. Sometimes a number like the following occurs: $24\dot{8}$. It shows that the figure 8 alone repeats.

284. All figures between the dots and under the dot repeat, and none others. In this example

(.3̇), the figure 3 repeats. In this example, 2̇46̇, the number 246 repeats, in the order in which the figures are written.

In such an expression as the following, .26̇89̇, the figures 2 and 6 do not repeat. The 8 and 9 repeat in the order in which they are written.

Which of the following figures repeat:

2̇, .3̇, .6̇, .8̇, .7̇, .5̇, .28̇, .49̇, .67̇, .37̇, .443̇, .684̇, .772̇, .846̇, .2406̇, .8927̇, .2248̇, .3976̇, .286̇, .874̇, .982̇, .647̇, .241̇.

LESSON II.

285. When the decimal repeats and that the given number has not the fractional part at the end of the decimal number. Thus:

Reduce .46̇ to a simple fraction.

Write the number 46 for the numerator, and for the denominator write as many 9's as there are decimal figures in the given number. Reduce the fraction thus formed to its lowest terms.

OPERATION.

$\frac{46}{99}$ *Ans.*

EXAMPLES.

1. Reduce .3̇ to a common fraction.   *Ans.* $\frac{1}{3}$.

2. Reduce .03̇, to a common fraction.   *Ans.* $\frac{1}{33}$.

3. Reduce .47̇ to a common fraction.   *Ans.* $\frac{47}{99}$.

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4. Reduce .2̇4̇ to a common fraction.   *Ans.* $\frac{8}{33}$.

5. Reduce .3̇6̇ to a common fraction.   *Ans.* $\frac{4}{11}$.

6. Reduce .8̇2̇ to a common fraction.   *Ans.* $\frac{82}{99}$.

7. Reduce .3̇4̇ to a common fraction.   *Ans.* $\frac{34}{99}$.

8. Reduce .2̇9̇ to a common fraction.   *Ans.* $\frac{29}{99}$.

9. Reduce .2̇7̇ to a common fraction.   *Ans.* $\frac{3}{11}$.

10. Reduce .6̇6̇ to a common fraction.   *Ans.* $\frac{2}{3}$.

11. Reduce .1̇8̇ to a common fraction.   *Ans.* $\frac{2}{11}$.

12. Reduce .3̇6̇ to a common fraction.   *Ans.* $\frac{4}{11}$.

13. Reduce .6̇3̇ to a common fraction.   *Ans.* $\frac{7}{11}$.

14. Reduce .4̇5̇ to a common fraction.   *Ans.* $\frac{5}{11}$.

15. Reduce .5̇4̇ to a common fraction.   *Ans.* $\frac{6}{11}$.

16. Reduce .9̇ to a common fraction.   *Ans.* 1.

17. Reduce .8̇1̇ to a common fraction.   *Ans.* $\frac{9}{11}$.

18. Reduce .3̇6̇0̇ to a common fraction. *Ans.* $\frac{40}{111}$.

19. Reduce .1̇2̇ to a common fraction.   *Ans.* $\frac{4}{33}$.

20. Reduce .1̇8̇0̇ to a common fraction. *Ans.* $\frac{20}{111}$.

LESSON III.

286. When there is a finite part before the repetend. Thus:

Reduce .46̇ to a simple fraction.

Make a mixed number of it, with 9 for the denominator of the repetend; thus, $4\frac{6}{9}$. Divide $4\frac{6}{9}$ by 1 followed by as many 0's as there are decimal figures

in the given number, after having reduced it to a mixed number, which would be in this case but one 0. The fraction would now appear thus: $\frac{4\frac{8}{9}}{10}$, or $4\frac{8}{9} \div 10$. Reduced to its simplest form would give $(\frac{44}{9} \times \frac{1}{10} = \frac{44}{90} = \frac{22}{45})$ $\frac{22}{45}$ for the required fraction.

PROOF.

1. Reduce $\frac{22}{45}$ to a decimal.

$$45\ )\ 210\ (\ 0.4\dot{6}\dot{6}$$

$$\begin{array}{r} 180 \\ \hline 300 \\ 270 \\ \hline 300 \\ 270 \\ \hline 30 \end{array}$$

EXAMPLES.

1. Reduce $.2\dot{4}$ to a common fraction.   *Ans.* $\frac{11}{45}$.

2. Reduce $.8\dot{6}$ to a common fraction.   *Ans.* $\frac{13}{15}$.

3. Reduce $.3\dot{8}$ to a common fraction.   *Ans.* $\frac{7}{18}$.

4. Reduce $.7\dot{3}$ to a common fraction.   *Ans.* $\frac{11}{15}$.

5. Reduce $.2\dot{6}$ to a common fraction.   *Ans.* $\frac{4}{15}$.

6. Reduce $.6\dot{2}$ to a common fraction.   *Ans.* $\frac{28}{45}$.

7. Reduce $.8\dot{1}$ to a common fraction.   *Ans.* $\frac{73}{90}$.

8. Reduce $.9\dot{7}$ to a common fraction.   *Ans.* $\frac{44}{45}$.

9. Reduce $.3\dot{6}$ to a common fraction.   *Ans.* $\frac{11}{30}$.

10. Reduce .2̇8 to a common fraction.   *Ans.* $\frac{13}{45}$.

11. Reduce .86̇4 to a common fraction. *Ans.* $\frac{428}{495}$.

12. Reduce .2̇8 to a common fraction.   *Ans.* $\frac{13}{45}$.

13. Reduce .8̇2̇ to a common fraction.   *Ans.* $\frac{37}{45}$.

14. Reduce .7̇6̇2̇ to a common fraction. *Ans.* $\frac{151}{198}$.

15. Reduce .98̇7̇3̇ to a common fraction. *Ans.* $\frac{548}{555}$.

16. Reduce .8̇76̇ to a common fraction. *Ans.* $\frac{434}{495}$.

17. Reduce .283̇ to a common fraction.   *Ans.* $\frac{17}{60}$.

18. Reduce .246̇ to a common fraction.   *Ans.* $\frac{37}{150}$.

19. Reduce .6̇1̇ to a common fraction.   *Ans.* $\frac{11}{18}$.

20. Reduce .861̇ to a common fraction.   *Ans.* $\frac{31}{36}$.

21. Reduce .248̇ to a common fraction.   *Ans.* $\frac{56}{225}$.

22. Reduce .726̇ to a common fraction.   *Ans.* $\frac{109}{150}$.

23. Reduce .278̇ to a common fraction.   *Ans.* $\frac{251}{900}$.

24. Reduce .286̇ to a common fraction.   *Ans.* $\frac{43}{150}$.

25. Reduce .928̇ to a common fraction.   *Ans.* $\frac{209}{225}$.

26. Reduce .8̇74̇ to a common fraction.   *Ans.* $\frac{433}{495}$.

27. Reduce .6̇39̇ to a common fraction.   *Ans.* $\frac{211}{330}$.

28. Reduce .2876̇ to a common fraction.

Ans. $\frac{863}{3000}$.

29. Reduce .249̇ to a common fraction.   *Ans.* $\frac{247}{990}$.

30. Reduce .876 to a common fraction.   *Ans.* $\frac{863}{300}$.

31. Reduce .881̇ to a common fraction.   *Ans.* $\frac{793}{900}$.

32. Reduce .896 to a common fraction.   *Ans.* $\frac{863}{300}$.

33. Reduce .284̇ to a common fraction.   *Ans.* $\frac{64}{225}$.

34. Reduce .383̇ to a common fraction.   *Ans.* $\frac{23}{60}$.

35. Reduce .649̇1̇ to a common fraction.

Ans. $\frac{6427}{9900}$.

36. Reduce .643̇ to a common fraction.   *Ans.* $\frac{193}{300}$.

37. Reduce .686̇ to a common fraction.   *Ans.* $\frac{103}{150}$.

38. Reduce .283̇3̇ to a common fraction.   *Ans.* $\frac{17}{60}$.

39. Reduce .866̇6̇ to a common fraction.   *Ans.* $\frac{13}{15}$.

PROBLEMS INVOLVING PRECEDING RULES AND PRINCIPLES.

LESSON IV.

287. 1. I bought 250000 bricks, at 12½ dollars per thousand.   I retailed ¼ of them to I. Smith at 14 dollars per thousand, ½ of the remainder to J. Jones at 15 dollars per thousand, and the balance to Robert Peters at 12 dollars per thousand.   Did I gain or lose, and how much ?   *Ans.* Lost $78.75.

2. If a horse eat ⅓ bushel of oats in one day, ¼ in another, and $\frac{1}{12}$ in another, how many bushels did he eat?   *Ans.* $\frac{7}{12}$ bushels.

3. Sam. Browne bought two hhds. of sugar, each weighing 2000 pounds, at 4 cents a pound.   How much profit would he make by selling it at 4½ cents per pound ?   *Ans.* $20.

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4. A man bought two pieces of cotton, each piece containing 82¼ yards. He sold 24⅝ yards. How many yards had he left, and what was it worth at 16 cents a yard?   *Ans.* $22.38.

5. A grocer bought 2 boxes soap, at 8 dollars apiece; 2 bbls. molasses, 40 gallons each, at 1 dollar per gallon. How much change did he receive from the merchant in exchange for a 100 dollar bill?

Ans. $4

6. A merchant bought two pieces of cloth, 20⅛ yards each, and divided it between 8 persons. How many yards did each one receive?   *Ans.* $5\frac{1}{32}$ yds.

7. From ½ of a dollar subtract ⅛ of a dollar.

Ans. 37½ cents.

8. I buy sugar for ⅛ of a dollar, and I sell it for ¼ of a dollar. What is my profit?   *Ans.* 12½ cts.

9. What is the difference between ⅛ of a dollar and $\frac{1}{16}$ of a dollar?   *Ans.* 6¼ cts.

10. From 250 acres of land I gave 24⅝ acres. How many had I left?   *Ans.* 225⅜ A.

11. Divide 12½ dollars among 100 boys.

Ans. 12½ cts.

12. Ninety cords wood cost me 270 dollars· How much is each cord worth?   *Ans.* $3.

13. I bought 90 cords of wood at $4½ a cord. What had I to pay?   *Ans.* $405.

14. A teacher bought 3 dozen spellers at 12½ cents a piece. What did he have to pay?

Ans. $4.50

15. A merchant bought 24 cords of wood at $8 a cord, and sold ⅓ of it at 9 dollars a cord. For the balance he got 64 dollars. How much did he sell the balance per cord? *Ans.* $4.

16. I bought a cask of wine containing 50 gallons, at 1 dollar a gallon. I sold it to a grocer for 150 dollars. How much per gallon did the grocer pay for it? *Ans.* $3.

17. A train has to travel 240 miles from one station to another. How long will it take the train to go that distance if it should travel 30½ miles an hour? *Ans.* 7 hrs. 52 min.

18. If I sell 24000 bricks at 12 dollars a thousand, and buy the same number of bricks at $1.20 per hundred, how much do I gain? *Ans.*

19. A boy, after losing ¼ of his kite-string, bought 20 yards of cord, and added it to what he had left. He now finds that he has but ⅝ of what he had at first. How many feet of string had he at first? *Ans.* 240 yards.

20. I own ⅞ of a ship valued at $77,000. If I sell ¾ of my share, how much will I receive, and how much will I still own?

        Ans. Sold $19,250. Own $6,416.66⅔.

21. A and B are 150 miles apart. A travels 2 miles an hour, and B travels 4 miles an hour. When will they meet, and how many miles will each one have travelled? *Ans.* A 50 miles, B 100 miles.

22. A merchant begins business with $20,000. The first year his capital increased ½ the original

amount; the second year he lost $2500. How much has he left? *Ans.* $27500.

23. Memphis is about 880 miles from New Orleans by rail. If a train was 24 hours in making that distance, how many miles an hour would that be?

Ans. 36⅔ miles.

24. A boy gave ½ of his money to a beggar, ¼ for a grammar, and ⅛ for a reader, and now he has 20 cents left. How much had he at first? *Ans.* $1.60.

25. A pole is ¼ of its length in the mud, ½ in water, and 40 feet above the water. How long is the pole? *Ans.* 160 feet.

26. From one dollar take one mill.

Ans. .999 mills.

27. What will 2 hhds. cost, each hogshead weighing 460 pounds at 10½ cents a pound. *Ans.* $96.60.

28. I bought 20 hhds. wine, 63 gallons each, at 22¼ cents per gallon. How much had I to pay?

Ans. $280.35.

29. I employ 14 men for 10½ days, for $2.25 per day. How much is my expense? *Ans.* $330.75.

30. I sold to Johnson & Co. 720 loads of brick; each load contained 500 bricks. How much had Johnson & Co. to pay at $14.50 per thousand?

Ans. $5220.

31. When building my stables, I required, to cover them, 600 bundles of shingles; each bundle contained 250. How many shingles did I buy, and what had I to pay for them at $3.25 per thousand?

Ans. $487.50.

32. A mason requires 350000 bricks to build a wall. He receives 600 loads of 500 bricks each. How many more does he want, and what would he have to pay for them at $13 a thousand ? *Ans.* $650.

33. A dressmaker bought 24 yards of silk at 33⅓ cents a yard. How much had she to pay ? *Ans.* $8.

34. A boy after losing ½ of his marbles at play, finds that he has 20 left. How many had he at first ?   *Ans.* 40 marbles.

35. After losing ¼ of his fortune, a man has $25000 left. How much had he at first ?

                              Ans. $33333.33⅓.

36. Seven eighths of the steamer Pocahontas is valued at $70000. James Brown owns ⅓ of her, and he sells ½ of ⅔ of his share to Smith. How much does Smith own ? How much has Brown left ?

                              Ans. $17777.77⁷⁄₉.

37. If in a school ⅓ of the boys study grammar, ½ of the balance study arithmetic, and the remainder study history, what part study history ?   *Ans.* ⅓.

38. A and B own 280 sheep, worth $5 apiece. A owns ¼ and B the remainder. How many sheep does each one own, and how much is each one's share valued at ?   *Ans.* A, 70 ; B, 210.

39. A man begins business with $1000 and doubles his capital every year for 4 years. How much capital has he at the end of the fourth year?

                              Ans. $16000.

40. A can do a piece of work in 2 days and B can do it in 3 days. How long will it take both of them to do it, working together ?   *Ans.* 1⅕ day.

41. After spending ⅛ of my income every year, I find that I save $1200. What is my income?

Ans. $1500.

42. A man divides $20000 between his 4 sons as follows: A gets ¼ of it, B gets ¼ of the remainder, C gets ¼ of what is now left, and D gets the balance. What is the share of each?

Ans. A, $5000; B, $3750; C, $2812.50; D, $8437.50.

CHAPTER XIV.

DENOMINATE NUMBERS.

LESSON I.

288. A *compound* or *denominate number* is a collection of units of several denominations; thus, £2 3s. 4d. 2 far. is a compound number.

289. A compound or denominate number may be changed to a different denomination without altering its value.

290. There are two ways of changing the denomination, called *reduction ascending* and *reduction descending.*

291. Reduction ascending is the process of reducing from a lower to a higher denomination, as pence to pounds, etc.

This operation is performed by division.

292. Reduction descending is the process of reducing from a higher to a lower denomination, as pounds to farthings.

This operation is performed by multiplication.

ENGLISH MONEY.

293. English money is the currency of England and her dependencies.

294. It is used in buying and selling in some of the Middle and Eastern States, but it is almost obsolete.

The following is the

TABLE.

4 farthings (*far.*) make	1 penny,	abb. *d.*
12 pence	" 1 shilling,	" *s.*
20 shillings	" 1 pound,	" £
21 shillings	" 1 guinea,	" *G.*
5 shillings	" 1 crown,	" *Cr.*

1 £	is worth $4.84		in U. S. money.	
1 s.	"	0.24⅕	"	"
1 d.	"	0.02001+	"	"
1 far.	"	0.005004+	" ·	"

EXERCISES.

295. 1. In £4 how many farthings?

OPERATION.

$$4 \times 12 \times 20 \times 4 = 3840 \text{ farthings.}$$

In such examples as the above, begin at farthings, and multiply by all the figures until you arrive at

the required point. In the above example it is required to go from farthings to pounds, then multiply all the figures from farthings to pounds, and since the question is £4, you will require 4 times as many as are found in the table. If the question was to reduce from pence to pounds, we would begin at pence and multiply until we came to pounds; thus, begin at pence, 12 pence × 20 shillings = £1.

This remark applies to all of the tables of denominate numbers, therefore it is useless to repeat it again.

2. In 1 pound how many farthings? 2? 3? 4? 5? 6? 7? 9? 10? 11? 13? 14?

3. In 1 shilling how many farthings? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13?

4. In 1 penny how many farthings? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13?

5. In 1 pound how many pennies? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13?

6. In 1 shilling how many pennies? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13? 14?

7. In 1 pound how many shillings? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

Reduction Ascending.

8. In 960 farthings how many pounds? 1920? 2880? 3840? 4800? 5760? 5320?

9. In 48 farthings how many shillings? 96? 192? 384? 768? 1536? 3072?

10. In 4 farthings how many pennies? 8? 16? 32? 64? 128? 256? 512? 1024?

11. In 240 pennies how many pounds? 480? 960? 1920? 3840? 7680? 15360?

12. In 12 pennies how many shillings? 24? 48? 96? 192? 384? 568? 1136?

LESSON III.

296. TROY WEIGHT

is used in weighing gold, silver, and diamonds, etc.

TABLE.

24 grains (grs.) make 1 pennyweight, abb. dwt.
20 pennyweights " 1 ounce, " oz.
12 ounces " 1 pound, " lb.

Reduction Descending.

1. In 1 pound how many grains? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

2. In 1 ounce how many grains? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

3. In 1 pennyweight how many grains? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

4. In 1 pound how many pennyweights? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

5. In 1 ounce how many pennyweights? 2? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

6. In 1 pound how many ounces? 2? 3? . 4? 5? 6? 7? 8? 9? 10? 11? 12?

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Reduction Ascending.

7. In 5760 grains how many pounds? 11520?
17280? 23040? 28800? 74880?

8. In 480 grains how many ounces? 960?
1440? 1920? 11040? 12960?

9. In 24 grains how many pennyweights? 48?
96? 192? 384? 768? 1536?

10. In 240 pennyweights how many pounds?
480? 960? 1920? 3840? 7680? 15360?

11. In 20 pennyweights how many ounces?
40? 80? 160? 320? 640? 1280? 2560? 5120?

12. In 12 ounces how many pounds? 24? 48?
96? 192? 384? 768? 1536? 3072? 6144?

LESSON IV.

297. APOTHECARIES' WEIGHT
is used in mixing medicines.

TABLE.

20 grains (gr.) make	1 scruple,	sc. or ℈
3 scruples "	1 dram,	dr. or ʒ
8 drams "	1 ounce,	oz. or ℥
12 ounces "	1 pound,	lb. or ℔.

Reduction Descending.

1. In 4 scruples how many grains? 6? 12?

2. In 1 pound how many grains? 2? 3? 4?

3. In 1 ounce how many grains? 2? 3? 4?
5?

4. In one dram how many grains? 2 ? 3 ? 4 ?
5 ?

Reduction Ascending.

5. In 5760 grains how many pounds? 11520 ?
23040 ? 46080 ? 92160 ? 184320 ?

6. In 480 grains how many ounces? 960 ?
1920 ? 3840 ? 7680 ? 15360 ? 30720 ?

7. In 60 grains how many drams? 120 ? 240 ?
480 ? 960 ? 1920 ? 3840 ? 7680 ?

8. In 20 grains how many scruples? 40 ? 80 ?
160 ? 320 ? 640 ? 1280 ? 2560 ?

LESSON V.

298. AVOIRDUPOIS WEIGHT

is used in weighing every kind of goods having
weight, such as groceries, grain, iron, copper, etc.

TABLE.

16 drams (*dr.*)	make 1 ounce,		*oz.*
16 ounces	" 1 pound,		*lb.*
25 pounds	" 1 quarter,		*qr.*
4 quarters	" 1 hundred weight,	*cwt.*
20 hundred weight	" 1 ton,		*T.*

EXERCISES.
Reduction Descending.

1. In 1 ton how many drams? 2 ? 3 ? 4 ?
2. In 1 cwt. how many drams? 2 ? 3 ? 4 ?
3. In 1 quarter how many drams? 2 ? 3 ? 4 ?

4. In 1 pound how many drams? 2? 3? 4?
5. In 1 ounce how many drams? 2? 3? 4?
6. In 1 ton how many ounces? 2? 3? 4? 5?
7. In 1 cwt. how many ounces? 2? 3? 4? 5?
8. In 1 qr. how many ounces? 2? 3? 4? 5?
9. In 1 pound how many ounces? 2? 3? 4? 5?
10. In 1 ton how many pounds? 2? 3? 4? 5?
11. In 1 cwt. how many pounds? 2? 3? 4? 5?
12. In 1 quarter how many pounds? 2? 3? 4? 5?
13. In 1 ton how many quarters? 2? 3? 4? 5?
14. In 1 cwt. how many quarters? 2? 3? 4? 5?

Reduction Ascending.

15. In 512000 drams how many tons? 1024000?
16. In 25600 drams how many cwt.? 51200?
17. In 6400 drams how many quarters? 12800?
18. In 256 drams how many pounds? 2048?
19. In 320 drams how many ounces? 2880?
20. In 32000 oz. how many tons? 288000?
21. In 1600 oz. how many cwt? 11200?
22. In 400 oz. how many qr.? 1600? 3200?
23. In 16 oz. how many pounds? 160? 1280?
24. In 2000 lbs. how many tons? 18000? 36000?
25. In 100 lbs. how many cwt.? 1800? 3600?
26. In 25 lbs. how many qr.? 125? 625?
27. In 80 qr. how many tons? 640? 1280?
28. In 4 qr. how many cwt.? 16? 32? 64?
29. In 20 cwt. how many tons? 40? 80? 160?
30. In 4602000 how many tons?

LESSON VI.

299. CLOTH MEASURE

is used in measuring cloth and all other articles sold by the yard.

TABLE.

2¼ inches (*in.*) make 1 nail.			*na.*
4 nails	"	1 quarter of a yard.	*qr.*
4 quarters	"	1 yard.	*yd.*
5 quarters	"	1 Ell English.	*E. E.*
3 quarters	"	1 Ell Flemish.	*E. F.*

Reduction Descending.

1. In 1 yard how many inches ? 2 ? 3 ? 4 ? 5 ?
2. In 1 quarter how many inches ? 2 ? 3 ? 4 ? 5 ?
3. In 1 nail how many inches ? 4 ? 8 ? 12 ? 16 ?

Reduction Ascending.

4. In 36 inches how many yards ? 72 ? 144 ? 288 ?

5. In 9 inches how many quarters? 18 ? 36 ? 72 ?
6. In 2¼ in. how many nails ? 8 ? 12 ? 16 ?
7. In 16 nails how many yards ? 32 ? 64 ? 128 ?
8. In 4 nails how many quarters ? 8 ? 12 ? 24 ? 48 ?
9. In 4 quarters how many yards ? 8 ? 12 ? 36 ? 72 ?

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LESSON VII.

300. LONG MEASURE

is used in measuring distances.

TABLE.

12	inches (*in.*)	make	1 foot.	*ft.*
3	feet	"	1 yard.	*yd.*
5½	yards	"	1 rod.	*r.*
40	rods	"	1 furlong.	*fur.*
8	furlongs	"	1 mile.	*m.*
3	miles	"	1 league.	*lea.*
69¼	(nearly) miles	"	1 degree.	*deg.* or °.
360	degrees	"	1 circumference of the earth.	
				circ.

EXERCISES.

Reduction Descending.

1. In 1 mile how many feet?   2?   4?   6? 12?   14?

2. In 1 fur. how many feet?   4?   8?   12? 16?   32?

3. In 1 rod how many feet?   8?   12?   16? 32?   64?

4. In 1 yard how many feet? 9? 8? 7? 6? 5?

5. In 1 mile how many inches?   2?   3?   4? 5?   6?

6. In 1 fur. how many inches?   3?   6?   9? 12?   15?

7. In 1 rod how many inches? 6? 12? 24? 48?

8. In 1 yard how many inches?   2?   4?   8? 16?   32?

9. In 1 foot how many inches? 8? 16? 32? 64?

10. In 1 mile how many yards? 3? 6? 12? 24? 48?

11. In 1 fur. how many yards? 2? 8? 32? 128?

12. In 1 rod how many yards? 6? 36? 216?

13. In 1 mile how many rods? 8? 16? 32? 64?

14. In 1 fur. how many rods? 2? 4? 5? 6? 7?

15. In 1 circle how many inches? 2? 3? 4? 5? 6?

Reduction Ascending.

16. In 63360 in. how many miles? 126720?

17. In 7920 inches how many fur.? 23760?

18. In 198 inches how many rods? 792?

19. In 36 inches how many yards? 180?

20. In 5280 feet how many miles? 10560? 21120?

21. In 660 feet how many furlongs? 1980? 5940?

22. In 16½ feet how many rods? 66? 528?

23. In 3 feet how many yards? 27? 54? 108?

24. In 1760 yards how many miles? 3520? 7040?

25. In 220 yards how many furlongs? 440? 880? 1760?

26. In 5½ yards how many rods? 22? 44? 88?

27. In 320 rods how many miles? 640? 1280? 2560?

28. In 40 rods how many furlongs? 80? 160? 320? 640?

29. In 8 furlongs how many miles? 16? 32? 64? 128?

LESSON VIII.

301. SURVEYOR'S MEASURE

is used in measuring land.

TABLE.

8 inches (nearly)(7.92)	make	1 link.	*l.*
25 links	"	1 pole.	*p.*
100 links	"	1 chain.	*cha.*
4 poles	"	1 chain.	*cha.*
66 feet	"	1 chain.	*cha.*
10 chains	"	1 furlong.	*fur.*
80 chains	"	1 mile.	*mi.*
8 furlongs	"	1 mile.	*mi.*

EXERCISES.

Reduction Descending.

1. In 1 mile how many inches? 2? 3? 4? 5?
2. In 1 furlong how many links? 2? 4? 8? 16?
3. In 1 chain how many links? 2? 9? 13? 26?
4. In 1 furlong how many poles? 4? 8? 9? 12?

Reduction Ascending.

5. In 8000 links how many miles? 160000?
6. In 80 chains how many furlongs? 640? 1280?
7. In 80 chains how many miles? 640? 2560?
8. In 320 poles how many miles? 640? 1280?

LESSON IX.

302. SQUARE MEASURE.

Square measure is the square of Long Measure, and is used in measuring any thing which has length and breadth.

TABLE.

144	inches (*in.*) make 1 square ft.				*sq. ft.*
9	feet	"	1	" yd.	*sq. yd.*
30¼	yards	"	1	" rod.	*sq. r.*
40	rods	"	1	" rood.	*R.*
4	roods	"	1	" acre.	*A.*
640	acres	"	1 square mile.		*sq. m.*

EXERCISES.

Reduction Descending.

1. In 1 square mile how many inches? 2? 3?

2. In 1 acre how many inches? 2? 4? 6?

3. In 1 rood how many inches? 2? 3? 4? 5? 6?

4. In 1 rod how many inches? 3? 5? 6? 7? 8?

5. In 1 yard how many inches? 5? 6? 7? 8? 9?

6. In 1 foot how many inches? 10? 11? 12? 13?

7. In 1 mile how many feet? 2? 3? 4? 5? 6? 7?

8. In 1 acre how many feet? 3? 4? 5? 7? 9?

9. In 1 rood how many feet? 4? 6? 8? 10? 11?

10. In 1 rod how many feet? 3? 6? 12? 24? 5?

11. In 1 yard how many feet? 2? 3? 4? 5? 6? 7?

12. In 1 acre how many yards? 3? 4? 5? 6? 7? 8?

13. In 1 mile how many yards? 4? 5? 6? 7? 8? 9?

14. In 1 rood how many yards? 2? 3? 4? 5? 6? 7?

15. In 1 rod how many yards? 3? 5? 7? 9? 11? 13?

16. In 1 mile how many rods? 3? 4? 5? 6? 7? 8?

17. In 1 acre how many rods? 4? 5? 6? 7? 8? 9?

18. In 1 rood how many rods? 3? 8? 9? 18? 30? 32?

19. In 1 mile how many roods? 2? 3? 4? 5? 6? 7? 8? 9?

20. In 1 acre how many roods? 2? 3? 4? 5? 6? 7? 8? 9?

Reduction Ascending.

21. In 8028979200 inches how many miles?
22. In 627264000 inches how many acres?
23. In 15681600 inches how many roods?
24. In 39204000 inches how many poles?
25. In 1296000 inches how many yards?
26. In 278784000 feet how many miles?
27. In 43560000 feet how many acres?

28. In 1089000 feet how many roods?

29. In 272¼ feet how many poles?

30. In 999 feet how many yards?

31. In 3097600 yards how many miles?

32. In 48404840 yards how many acres?

33. In 12101210 yards how many roods?

34. In 1024000 poles how many acres?

35. In 160160160 poles how many acres?

36. In 40404040 poles how many roods?

37. In 25602560 roods how many miles?

38. In 40404040 roods how many acres?

39. In 640640640 acres how many miles?

40. In 40144896004014489600 inches how many miles.

LESSON X.

303. A square is a figure having 4 equal sides and 4 right angles.

2 square feet and 2 feet square are not the same. 2 feet square is a body 2 feet long and 2 feet broad, which is equal to 4 square feet. 2 square feet is ½ of 4 square feet.

Pupils always make mistakes in not being able to make any difference between such expressions as the following: 6 square feet and 6 feet square, 2 yards square and 2 square yards. Pupils would require drilling on this subject, and for that reason I have here inserted a few questions.

2 feet square.

1 sq. ft.	1 sq. ft.	
1 sq. ft.	1 sq. ft.	

2 feet square.

Explain the meaning of the following terms, and show their difference.

6 ft. square and 6 square feet.

2 ft. square and 2 square feet.

5 yds. square and 5 square yards.

2 miles square and 2 square miles.

4 miles square and 16 square miles.

LESSON XI.

304. CUBIC OR SOLID MEASURE

is used in measuring bodies having length, breadth, and thickness, such as stone, timber.

TABLE.

1728 cubic inches (cu. in.) make 1 cubic foot,   *cu. ft.*

  27 cubic feet   "   1 cubic yard, *cu. yd.*

  40 cubic feet   "   1 Ton,   *T.*

  16 cubic feet   "   1 cord foot,   *c. ft.*

   8 cord feet or   "   1 cord of wood,   *C.*

128 cubic feet   "   1 cord of wood,   *C.*

Reduction Descending and Ascending.

1. In 1 cord how many inches ? 2 ? 3 ? 4 ? 5 ?

2. In 1 cord how many feet ?   2 ?   3 ?   4 ?   5 ?

3. In 1 ton how many inches ? 2 ? 3 ?   4 ?   5 ?

4. In 1 ton how many feet ?   2 ?   3 ?   4 ?   5 ? 6 ?

5. In 221184 inches how many cords ? 442368 ? 884736 ?

6. In 69120 inches how many tons ?   128240 ? 256480 ?

7. In 46656 inches how many yards ?   93312 ?
186624 ?

8. In 1728 inches how many feet? 3456 ? 6912 ?

9. In 128 feet how many cords ?   256 ?   512 ?
1024 ?

10. In 40 feet how many tons ? 80 ? 160 ? 320 ?

11. In 27 feet how many yards ? 54 ? 108 ? 216 ?

LESSON XII.

305. A cube is a solid body having length,
breadth, and thickness.   A cube has 6 sides.
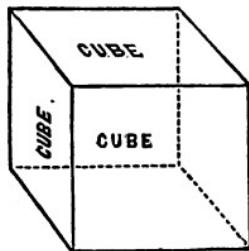
To find the solidity of a cube, multiply the
length, breadth, and height together, or, in other
words, multiply the length of one side by itself two
times.

EXAMPLES.

1. How many inches in a square block of wood,
1 foot in length, breadth, and height.

Ans. 1728 cu. in.

OPERATION.

$$12 \times 12 \times 12 = 1728.$$

24

2. What is the solidity of a piece of timber 20 feet long, 3 feet broad, and 4 feet high?

$$20 \times 3 \times 4 = 240 \text{ cubic feet.}$$

3. What is the solid contents of a cube of ice, 2 feet high?

$$2 \times 2 \times 2 = 8 \text{ cubic feet.}$$

4. What is the solidity of a square log, 40 feet long, 1 foot high, and 2 feet broad?

$$40 \times 1 \times 2 = 80 \text{ cubic feet.}$$

5. How many cubic feet of earth was removed to make a square well, 40 feet deep, and 5 feet square at the top?

$$40 \times 5 \times 5 = 1000 \text{ cubic feet.}$$

LESSON XIII.

306. WINE MEASURE

is used in measuring all kinds of liquids, except beer, milk, ale, or porter.

TABLE.

4 gills (*gi.*) make	1 pint,	abb.	*pt.*
2 pints	" 1 quart,		*qt.*
4 quarts	" 1 gallon,		*gal.*
63 gallons	" 1 hogshead,		*hdd.*
2 hogsheads	" 1 pipe,		*pi.*
2 pipes	" 1 tun,		*tun.*

Reduction Descending.

1. In 1 tun how many gills? 2? 4? 6? 8?
2. In 1 tun how many pints? 2? 3? 4? 5?

3. In 2 tuns how many quarts? 6? 7? 8? 9?

4. In 3 tuns how many gallons? 10? 11? 12? 13?

5. In 4 tuns how many pipes? 14? 15? 16? 17?

6. In 2 pipes how many gills? 3? 4? 5? 6? 7?

7. In 2 pipes how many pints? 4? 5? 6? 7? 8?

8. In 1 pipe how many quarts? 6? 7? 8? 9? 10?

9. In 2 pipes how many gallons? 24? 26? 30?

10. In 2 pipes how many hogsheads? 21? 22? 23?

11. In 1 hhd. how many gills? 2? 3? 4? 5? 6?

12. In 2 hhds. how many pints? 3? 4? 5? 6? 7?

13. In 3 hhds. how many quarts? 4? 5? 6? 7? 8?

14. In 4 hhds. how many gallons? 5? 6? 7? 8? 9?

15. In 2 galls. how many gills? 6? 7? 8? 9? 10?

16. In 3 galls. how many pints? 7? 8? 9? 10? 11?

17. In 4 galls. how many quarts? 3? 4? 5? 6?

18. In 1 quart how many gills? 2? 3? 4?. 5?

19. In 2 quarts how many pints? 2? 3? 4? 6?

20. In 2 pints how many gills? 2? 3? 4? 7?

Reduction Ascending.

1. In 16128 gills how many tuns?   32256 ?
64512 ?

2. In 4032 gills how many pipes? 8064? 16128 ?

3. In 4032 gills how many hhds. ?   8064 ?
16128 ?

4. In 320 gills how many galls. ? 1820 ? 10920 ?

5. In 72 gills how many quarts? 720 ?   14400 ?

6. In 4032 pints how many tuns?   12096 ?
36288 ?

7. In 2016 pints how many pipes? 20160 ?
141120 ?

8. In 1008 pints how many hhds. ?   2016 ?
181440 ?

9. In 64 pints how many galls. ?   128 ?   256 ?
512 ?

10. In 4 pints how many quarts? 16 ?   32 ?
64 ?   128 ?

11. In 2016 quarts how many tuns?   8064 ?
32256 ?   129024 ?

12. In 1008 quarts how many pipes ?   6048 ?
36288 ?

13. In 1512 quarts how many hhds. ?   3024 ?
6048 ?   12096 ?

14. In 44 quarts how many galls. ?   88 ?   792 ?
7128 ?

15. In 2520 galls. how many tuns?   5040 ?
10080 ?

16. In 1260 galls. how many pipes ?   2520 ?
5040 ?   10080 ?

17. In 630 galls. how many hhds.? 1260? 2520? 5040?

18. In 444 hhds. how many tuns? 888? 7004? 56032?

19. In 222 hhds. how many pipes? 666? 1332? 2664?

20. In 44 pipes how many tuns? 88? 176? 352?

21. In 20160 pints how many hhds.? 40320? 80640?

LESSON XIV.

307. BEER MEASURE

is used in measuring beer, ale, and porter. Milk is measured by the beer measure.

TABLE.

2 pints (*pt.*) make 1 quart.   *qt.*
4 quarts   "   1 gallon.   *gal.*
54 gallons   "   1 hogshead. *hhd.*

Reduction Ascending.

1. In 43200 pints how many hhds.? 8640? 17280?

2. In 88 pints how many galls.? 176? 352? 704?

3. In 880 pints how many galls.?

4. In 216 quarts how many hhds.? 432? 864? 1728?

5. In 40 quarts how many gallons? 1600? 32000?

6. In 54 galls. how many hhds.? 1080? 2160?
        24*

Reduction Descending.

7. In 1 hhd. how many pints? 2? 3? 4? 5? 6? 7?

8. In 1 gallon how many pints? 3? 4? 5? 6? 7? 8? 9?

9. In 2 quarts how many pints? 4? 5? 6? 7? 8? 9? 10?

10. In 1 hhd. how many quarts? 2? 3? 4? 5? 6? 7?

11. In 1 gallon how many quarts? 3? 4? 5? 6? 7? 8?

12. In 4 gallons how many pints? 5? 6? 7? 8? 9? 10?

NOTE.—Beer measure is becoming obsolete. Milk and malt liquors are now measured by the wine measure.

LESSON XV.

308. DRY MEASURE

is used in measuring fruit and grain. Coal and salt are also measured by Dry Measure.

TABLE.

2 pints (*pt.*) make 1 quart.	*qt.*
8 quarts " 1 peck.	*pk.*
4 pecks " 1 bushel.	*bu.*
8 bushels " 1 quarter.	*qr.*
36 bushels " 1 chaldron.	*ch.*
4 quarters " 1 load.	*L.*

Reduction Descending.

1. In 1 qr. how many pints?   2?   3?   4?
5?   6?

2. In 1 bu. how many pints?   3?   4?   5?
6?   7?

3. In 1 pk. how many pints?   7?   8?   9?
10?   11?

4. In 1 qt. how many pints?   4?   5?   6?
7?   8?

5. In 1 qr. how many quarts?   2?   3?   4?
5?   6?

6. In 1 bu. how many quarts?   7?   8?   9?
10?   11?

7. In 1 peck how many quarts?   9?   10?
11?   12?

8. In 1 qr. how many pecks?   2?   3?   4?   5?

9. In 1 bu. how many pecks?   6?   7?   8?   9?

10. In 1 qr. how many bushels? 6? 8? 10? 12?

Reduction Ascending.

11. In 2304 pints how many chaldrons?   4608?
9216?

12. In 640 pints how many bushels?   1280?
2560?

13. In 160 pints how many pecks?   320?   640?

14. In 11520   quarts how many chaldrons?
23040?   46080?

15. In 320 quarts how many bushels?   640?
1280?   2560?

16. In 80 quarts how many pecks?   160?   320?
6400?

17. In 2880 pecks how many chaldrons?   5760?
11520?

18. In 48 pecks how many bushels?   96?   192?
384?

. 19. In 720 bushels how many chaldrons?   1740?
3480?   6960?

20. In 1440 bushels how many chaldrons? 2880?
5760?

LESSON XVI.

309. TIME TABLE.

60 seconds (*sec.*)	make	1 minute,	*m.*
60 minutes	"	1 hour,	*hr.*
24 hours	"	1 day,	*d.*
7 days	"	1 week,	*w.*
52 weeks	"	1 year,	*yr.*
100 years	"	1 century,	*C.*
12 months	"	1 year,	*yr.*
30 days (sometimes 31)	"	1 month,	*mo.*

310. The Tropical year is 365 days, 5 hrs. 48
m. and 49 sec.

The Julian Year, called after Julius Cæsar, is 365
days and ¼ of a day.

The Gregorian Year, named after Pope Gregory

XIII., and which is the most commonly used by all of the nations, gives 365 days for every 3 consecutive years, and 366 days for every fourth year. As far as ascertained, this is the most correct.

A Sidereal Year is the length of time the earth takes in revolving around the sun, and is 365 days, 6 hrs. 9m. 9 sec.

311. The 12 months of a year are named :

1st. January. 2d. February. 3d. March. 4th. April. 5th. May. 6th. June. 7th. July. 8th. August. 9th. September. 10th. October. 11th. November. 12th. December.

312. The months have not the same number of days.

January has 31, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, and December 31.

313. Every leap year gives February 29 days.

314. It would be well to commit to memory the following verse, to assist in remembering the number of days in each month.

> Thirty days hath September,
> April, June, and November,
> All the rest have thirty-one,
> Except February, which alone
> Hath twenty-eight ; and this in fine,
> Till the leap year gives it 29.

315. TABLE

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE
SAME DAY OF ANY OTHER MONTH OF THE SAME YEAR.

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb	Mar.	Apr.	May.	June	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. January..	365	31	59	90	120	151	181	212	243	273	304	334
2. February .	334	365	28	59	89	120	150	181	212	242	273	303
3. March....	306	337	365	31	61	92	122	153	184	214	245	275
4. April.....	275	306	334	365	30	61	91	122	153	183	214	244
5. May......	245	276	304	335	365	31	61	92	123	153	184	214
6. June.....	214	245	273	304	334	365	30	61	92	122	153	183
7. July	184	215	243	274	304	335	365	31	62	92	123	153
8. August...	153	184	212	243	273	304	334	365	31	61	92	122
9. September	122	153	181	212	242	273	303	334	365	30	61	91
10. October ..	92	123	151	182	212	243	273	304	335	365	31	61
11. November	61	92	120	151	181	212	242	273	304	334	365	30
12. December.	31	62	90	121	151	182	212	243	274	304	335	365

316. To find the number of days from any day of one month to any day of any other month of the same year.

EXAMPLE.

1. How many days from Feb. 1st to Oct. 1st.

February is the second month. Run along the second line of figures parallel to the top of the page, until you come to the month of October. The number 242, in the vertical October column, represents the number of days from February to October.

2. How many days from Feb. 1st to Oct. 9th.

First find how many days from Feb. 1st to Oct. 1st. It is 242 days. But that is not enough of days, because we find the number of days to Oct. 1st.

To Oct. 9th would be 8 days more. Add 8 days to 242 days, and the sum is 250 for the number of days from Feb. 1st to Oct. 9th.

3. How many days from Feb. 9th to Oct. 1st.

First find from Feb. 9th to Oct. 9th, which is 242 days. But that is too many days, because we found the number of days 9th of October. The number of days too many would be the difference between 9 and 1, that is, the number of days between the 1st and 9th of October. The difference is 8 days, which must be subtracted from 242. 242—8=234 days, the number of days from Feb. 9th to Oct 1st, 1869.

EXAMPLES.

1. How many days from Jan. 1st to Dec. 10th.
                                        Ans.

2. How many days from Feb. 14th to Dec. 23d.
                                        Ans.

3. How many days from March 24th to Sept. 26th.   *Ans.*

4. How many days from April 26th to Sept. 25th.   *Ans.*

5. How many days from Jan. 10th to May 2d ?
                                        Ans.

6. How many days from Feb. 20th to Sept. 26th ?   .   *Ans.*

Reduction Descending.

1. In 1 year how many minutes ? 2 ? 3 ? 4 ? 5 ?

2. In 1 month how many minutes ? 6 ? 7 ? 8 ? 9 ?

3. In 1 day how many minutes? 10? 11? 12? 13?

4. In 1 hour how many minutes? 13? 14? 15? 16?

5. In 1 year how many hours? 2? 3? 6? 7?

6. In 1 month how many hours? 3? 8? 9? 7?

7. In 1 week how many hours? 6? 8? 10? 11?

8. In 1 day how many hours? 2? 8? 12? 14?

9. In 1 year how many days? 3? 4? 14? 5?

10. In one month how many days? 6? 7? 8? 9?

11. In one year how many months? 3? 8? 9? 10?

12. In 1 Century how many minutes?

Reduction Ascending.

13. In 315576000 seconds how many years?

14. In 6048000 seconds how many weeks?

15. In 864000 seconds how many days?

16. In 36000 seconds how many hours?

17. In 52596000 minutes how many years?

18. In 100800 minutes how many months?

19. In 144000 minutes how many days?

20. In 876600 hours how many years?

21. In 168000 hours how many months?

22. In 24000 hours how many days?
23. In 365¼ days how many years?   1461?
24. In 700 days how many weeks?   1400?
25. In 52 weeks how many years?   104?

LESSON XVII.

317. CIRCULAR MEASURE

is used in measuring circles and angles.   It is also used in reckoning Latitude and Longitude, and the revolutions of the heavenly bodies around the earth.

TABLE.

60 seconds (″)	make	1 minute,	′
60 minutes (′)	"	1 degree,	°
30 degrees	"	1 sign,	S.
12 signs	"	1 circle,	C.
360 degrees	"	1 circle,	C.

318. An angle is the opening between two lines which meet, thus:
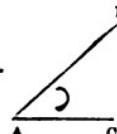
A right angle is formed by one line falling perpendicular to another.   Thus:
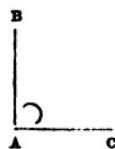
319. An acute angle is a smaller angle than a right angle.   Thus,

25
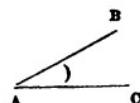
An obtuse angle is a larger angle
than a right angle.   Thus,
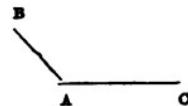
The point A, where the line meets, is called the
Vertex.

320. A Circle is a plane bounded by a curved
line, every point of which is equally distant from a
point within, called the center.

321. The Circumference is the curved line
which bounds the plane.

322. An Arc is any part of the circumference.

323. In a circle there are four right angles,
therefore a right angle is equal to one quarter of a
circle, and since a circle is divided into 360 degrees,
a right angle is equal to $\frac{1}{4}$ of 360 degrees, or 90
degrees.

324. A quadrant is $\frac{1}{4}$ of a circle.

325. The Diameter of a circle is a straight line
which divides the circle into two equal parts.

326. Two diameters crossing each other at
right angles, divide the circle into four equal parts.

327. The radius of a
circle is $\frac{1}{2}$ of the diameter,
that is, that part of the di-
ameter between the center
and the circumference.   In
the circle to the right there
are 4 radii. (Radii is the
plural of Radius.)

In any circle there are twice as many radii as
there are diameters.

328. An Arc is the measurement of an angle. Thus an arc between two lines, one perpendicular to the other, is equal to 90 degrees. The angle formed by those two lines is therefore 90 degrees.

329. The sun turns on its axis once in 24 hours, from west to east, that is, the sun passes over 15° of longitude in every hour, or 1° in every 4 minutes of time, and 1' in every 4 seconds of time. Hence, when it is noon or 12 o'clock at New Orleans, it is evening in all cities east, and morning in all cities west of that place.

Reduction Descending.

1. In 1 circle how many seconds?   2 ?   3 ?   4 ?
2. In 1 sign how many seconds ?   2 ?   3 ?   4 ?
3. In 1 degree how many seconds?   2 ?   3?   4 ?
4. In 1 minute how many seconds?   2 ?   3 ?   4 ?
5. In 1 circle how many minutes?   2 ?   3 ?   4 ?
6. In 1 sign how many minutes ?   2 ?   3 ?   4 ?
7. In 1 degree how many minutes?   2 ?   3 ?   4 ?
8. In 1 circle how many degrees ?   2 ?   3 ?   4 ?
9. In 1 sign how many degrees ?   2 ?   3 ?   4 ?
10. In 1 circle how many signs ?   2 ?   3 ?   4 ?

Reduction Ascending.

11. In 1296000 sec. how many circles ?   5184000 ?
12. In 108000 sec. how many signs ?   648000 ?
13. In 3600 sec. how many degrees ?   324000 ?
14. In 21600' how many circles ?   43200 ?
15. In 1800' how many signs ?   7200 ?
16. In 600' how many degrees ?   3000 ?

17. In 3600° how many circles? 18000?
18. In 300° how many signs? 15000?
19. In 12 signs how many circles? 4800?
20. In 120 signs how many circles? 60000?

LESSON XVIII.

330. DUODECIMALS

are compound numbers whose denominations increase or decrease in a twelvefold ratio.

It is used in measuring surfaces and solids such as paving, plastering, etc.

The denominations are foot (ft.), prime ('), second ("), third ("').

TABLE.

12''' (thirds) make 1 second.
12"   "   1 prime.
12'   "   1 foot.   *ft.*

EXERCISES.

1. In 17280''' how many feet?
2. In 1440'' how many feet?
3. In 2 feet how many seconds.
4. In 20 feet how many thirds?
5. In 20 primes how many thirds?
6. In 12 seconds how many thirds?
7. In 25 feet how many primes?
8. How many feet in 1728 primes?
9. How many primes in 144 seconds?
10. How many seconds in 25 feet?

LESSON XIX.

331. MISCELLANEOUS TABLE.

12 units	make 1 dozen.
12 doz. or 144 units	" 1 gross.
12 gross or 1728 units	" 1 great gross.
20 units	" 1 score.
56 lbs. corn shelled	" 1 bushel.
196 lbs. flour	" 1 barrel.
24 sheets paper	" 1 quire.
20 quires	" 1 ream.
2 reams	" 1 bundle.
5 bundles or 10 reams	" 1 bale.

A sheet of paper folded in 2 leaves makes a folio.

"	"	"	4	"	"	4to.
"	"	"	8	"	"	8vo.
"	"	"	12	"	"	12mo.
"	"	"	18	"	"	18mo.

LESSON XX.

332. REDUCTION DESCENDING

when there are several units of different denominations.

EXAMPLE.

Reduce 2£ 3s. 4d. 2 far. to farthings.

Begin at the highest denomination, £, and reduce it to shillings: $2 \times 20 = 40$ shillings, to which adding 3 gives 43 shillings. Reduce the 43 shillings to

25*

pence by multiplying by 12: 43 × 12 = 516. To which add 4d., making 520d. Reducing 520 pence to farthings gives 520 × 4 = 2080 farthings, to which adding 2 farthings gives 2082 farthings for the answer.

RULE.

I. *Arrange the several denominations in a horizontal line.*

II. *Multiply the highest term of the denominate number by the number of the next lower denomination required to make one of that higher one, and to the product add the term in the given quantity, which is of the same denomination as the product, and continue the work as above till the lowest given units have been added.*

1. Reduce 27 lbs. 15 dwt. to dwt.

Ans. 6495 dwt.

2. Reduce 2£ 15s. 9d. to farthings.

Ans. 2676 far.

3. Reduce 20£ 5s. to farthings.   *Ans.* 19440 far.

4. Reduce 20£ 15s. to farthings. *Ans.* 19220 far.

5. Reduce 17s. 5 far. to farthings.   *Ans.* 821 far.

6. Reduce 1 lb. 2 oz. 3 dwt. 2 gr. to grains.

Ans. 6794 grs.

7. Reduce 1 lb. 3 oz. 5 dwt. to grains.

Ans. 7320 grs.

8. Reduce 3 oz. 15 dwt. to grains.

Ans. 1560 grs.

9. Reduce 7 Ton 5 oz. to oz.   *Ans.* 224005 oz.

10. Reduce 17 Ton 15 oz. to oz.   *Ans.* 544015 oz.

11. Reduce 13 Ton 7 cwt. 3 qr. to oz.

Ans. 428400 oz.

12. Reduce 13 Ton 17 cwt. 2 qrs. to quarters.

Ans. 1110 quarters.

13. Reduce 1 qr. 10 oz. 3 dr. to drams.

Ans. 6563 drs.

14. Reduce 5 rods 3 yds. 2 ft. 6 in. to in.

Ans. 1128 in.

15. Reduce 4 rods 2 yds. 1 ft. 2 in. to in.

Ans. 878 in.

16. Reduce 22 fur. 6 rods 3 yds. 2 ft. to ft.

Ans. 14630 ft.

17. Reduce 4 mi. 6 fur. 20 r. 16 in. to in.

Ans. 101656 in.

18. Reduce 2 yds. 1 qr. 2 na. to inches.

Ans. $85\frac{1}{2}$ in.

19. Reduce 4 ft. 6 in. to inches.   *Ans.* 54 in.

20. Reduce 1 Tun 1 gall. 1 qt. to quarts.

Ans. 1013 quarts.

21. Reduce 2 Tuns 1 gall. 3 qrts. to gills.

Ans. 16184 gills.

22. Reduce 3 Tuns 2 galls. to gills.

Ans. 24256 gills.

23. Reduce 40 bbls. 18 galls. to pints.

Ans. 10224 pints.

24. Reduce 15 bu. 2 pks. to quarts.

Ans. 506 quarts.

25. Reduce 14 A. 1 R. to yards. *Ans.* 68970 yds.

26. Reduce 14 A. 2 R. to yards.   *Ans.* 58 rods.

27. Reduce 17 A. 3 R. 2 r. to inches.

                                  Ans. 111417768 in.

28. Reduce 14 A. 1 R. to feet.   *Ans.* 620730 ft.

29. Reduce 1 sq. m. 2 in. to inches.

                                  Ans. 4014489602.

30. Reduce 14 A. 3 R. to rods.   *Ans.* 59 rods.

31. Reduce 1 m. 2 fur. to yds.   *Ans.* 2200 yds.

32. Reduce 2 m. 3 fur. to ft.   *Ans.* 12540 ft.

33. Reduce 8 ft. 10 in. to in.   *Ans.* 106 in.

34. Reduce 1 yd. 2 ft. to in.   *Ans.* 60 in.

35. Reduce 2 y. 2 ft. 8 in. to in.   *Ans.* 144 in.

36. Reduce 1 mile 2 fur. to feet. •   *Ans.* 6600 ft.

37. Reduce 2 m. 3 fur. 5 rods to inches.

                                  Ans. 15147 in.

38. Reduce 5 bu. 2 pks. 1 qt. to qts. *Ans.* 177 qts.

39. Reduce 10 A. 1 R. to yards. *Ans.* 49610 yds.

40. Reduce 5 A. 1 R. 2 r. to rods. *Ans.* 842 rods.

41. Reduce 7 A. 1 ft. to inches. *Ans.* 43908624 in.

42. Reduce 8 S. 13° to seconds.

                                Ans. 910800 seconds.

43. Reduce 2 S. 3° to minutes.   *Ans.* 3780 mi.

44. Reduce 1 S. 4° 2′ 3″ to sec.   *Ans.* 122523 sec.

45. Reduce 5° 2′ 3″ to sec.   *Ans.* 18125 sec.

46. Reduce 2 da. 5 hrs. to mi.   *Ans.* 3180 mi.

47. Reduce 360 dys. 5 hrs. to mi. *Ans.* 518700 mi.

48. Reduce 5 hrs. 2 mi. to sec.   *Ans.* 18120 sec.

49. Reduce 12 hrs. 3 mi. to sec. *Ans.* 43380 sec.
50. Reduce 1 yr. 1 mo. 1 da. to mi.

Ans. 132480 mi.

51. Reduce $2 3 cents to mills. *Ans.* 2030 mills.
52. Reduce 5 fur. 2 r. 10 ft. to in. *Ans.* 40116 in.
53. Reduce 2 fur. 3 r. 8 ft. to in. *Ans.* 16530 in.
54. Reduce 1 fur. 4 r. 5 ft. to ft. *Ans.* 731 ft.
55. Reduce 1 mi. 2 fur. 3 r. to yds.

Ans. 2216½ yds.

56. Reduce 1 acre 2 R. 3 r. to rods.

Ans. 243 rods.

57. Reduce 2 acres 3 R. 4 r. to feet.

Ans. 120879 ft.

58. Reduce 1 lb. 2 oz. to grains. *Ans.* 6720 grains.
59. Reduce 2 lbs. 4 oz. 3 cwt. to cwt.

Ans. 563 cwt.

60. Reduce 3 lbs. 5 oz. 4 cwt. to grs.

Ans. 19776 grs.

61. Reduce 16 oz. 5 drm. to drms. *Ans.* 161 drms.
62. Reduce 2 Tun 1 hhd. to gills.

Ans. 18144 gills.

63. Reduce 4 Tun 1 hhd. 2 galls. to galls.

Ans. 1073 galls.

64. Reduce 7 yds. 2 ft. 3 in. to inches.

Ans. 279 in.

65. Reduce 5 ft. 6 in. to in. *Ans.* 66 in.
66. Reduce 1 sq. m. 5 a. to in.

Ans. 4045852800 in.

LESSON XXI.

333. REDUCTION ASCENDING.

Reduce 2139 farthings to £, etc.

<table>
<tr><td>Divide 2139 by 4, be-
cause 4 farthings make 1
penny :</td><td align="center">OPERATION.</td></tr>
</table>

Divide 2139 by 4, be-cause 4 farthings make 1 penny :

$$4\ )\ 2139$$
$$\overline{534} - 3$$

OPERATION.

$$4\ )\ 2139$$
$$12\ )\ \overline{534} \quad - 3 \text{ far.}$$
$$20\ )\ \overline{44} \quad - 6\text{d.}$$
$$\overline{2£} \quad - 4\text{s.}$$

Write the 3 remaining far-things a little to the right. The quotient found is pence. Divide the pence by 12, because 12 pence make 1 shilling :

$$12\ )\ 534$$
$$\overline{44} - 6$$

Write the 6 remaining pence a little to the right. The quotient found is shillings. Divide the shillings by 20, because 20 shillings make 1£.

$$20\ )\ 44$$
$$\overline{2} - 4$$

Write the remaining shillings a little to the right. The quotient is pounds sterling. The last quotient, together with the several remainders, form the compound number.

From which we derive the following

RULE.

Divide the given number by the number of units of its kind required to make one of the next higher denomination. If any remainder write it a little to the right of the quotient. Divide the quotient by the number of units of its kind required to make one of the next higher denomination. If any remainder, write it a little to the right of the quotient. Continue step by step in this way until the highest denomination required is reached. The remainders together with the last quotient is the compound number required.

1. Reduce 6495 dwt. to lbs., etc.

    Ans. 27 lbs. 15 dwt. to dwt.

2. Reduce 2676 far. to £, etc.

    Ans. 2£ 15s. 9d. to far.

3. Reduce 19440 far. to £, etc.

    Ans. 20£ 5s. to far.

4. Reduce 19920 far. to £, etc.

    Ans. 20£ 15s. to far.

5. Reduce 821 far. to £, etc.

    Ans. 17s. 5 far. to far.

6. Reduce 6794 grs. to lbs., etc.

    Ans. 1 lb. 2 oz. 3 dwt. 2 gr.

7. Reduce 7320 grs. to lbs., etc.

    Ans. 1 lb. 3 oz. 5 dwt. to grs.

8. Reduce 1560 grs. to oz., etc.

    Ans. 3 oz. 5 dwt. to grs.

9. Reduce 224005 oz. to Tons, etc.

       Ans. 7 Ton, 5 oz. to oz.

10. Reduce 544015 oz. to Tons, etc.

       Ans. 17 Ton, 15 oz. to oz.

11. Reduce 428400 oz. to Tons, etc.

     Ans. 13 Ton, 7 cwt. 3 qr. to oz.

12. Reduce 1110 quarters to Tons, etc.

    Ans. 13 Tons, 17 cwt. 2 qr. to oz.

13. Reduce 6563 drms. to quarters, etc.

    Ans. 1 qr. 10 oz. 3 dr. to dr.

14. Reduce 1128 in. to rods, etc.

    Ans. 5r. 3 yds. 2 ft. 6 in.

15. Reduce 878 in to rods, etc.

   Ans. 4r. 2 yds. 1 ft. 2 in. to in.

16. Reduce 14630 ft. to fur., etc.

   Ans. 22 fur. 6 r. 3 yds. 2 ft. to ft.

17. Reduce 101656 in. to miles, etc.

   Ans. 4 m. 6 fur. 20 r. 16 in. to in.

18. Reduce 85½ in. to yds., etc.

   Ans. 2 yds. 1 qr. 2 na. to inches.

19. Reduce 54 in. to ft., etc.

    Ans. 4 ft. 6 in. to inches.

20. Reduce 1013 qts. to Tuns, etc.

    Ans. 1 Tun, 1 gal. 1 qt. to qt.

21. Reduce 16184 gills to Tuns, etc.

   Ans. 2 Tun, 1 gall. 3 qts. to gills.

22. Reduce 24256 gills to Tuns, etc.

   Ans. 3 Tuns, 2 galls. to gills.

23. Reduce 10224 pints to bbls., etc.

   Ans. 40 bbls. 18 galls. to pints.

24. Reduce 506 quarts to bushels, etc.

Ans. 15 bu. 2 pks. to quarts.

25. Reduce 620730 ft. to Acres, etc.

Ans. 14 A. 1 R. to ft.

26. Reduce 68970 yds. to Acres, etc.

Ans. 14 A. 1 R. to yds.

27. Reduce 58 rods to acres, etc.

Ans. 14 A. 2 R. to rds.

28. Reduce 111417768 rods to acres, etc.

Ans. 17 A. 3 R. 2 r. to in.

29. Reduce 4014489602 in. to miles, etc.

Ans. 10 miles 2 in. to in.

30. Reduce 59 rods to A., etc.

Ans. 14 A. 3 R. to rods.

31. Reduce 2200 yds. to mi., etc.

Ans. 1 mile 2 fur. to yds.

32. Reduce 12540 ft. to mi., etc.

Ans. 2 miles 3 fur. to ft.

33. Reduce 106 in. to ft., etc.

Ans. 8 ft. 10 in. to in.

34. Reduce 60 in. to yds., etc.

Ans. 1 yd. 2 ft. to inches.

35. Reduce 144 to yds., etc.

Ans. 2 yds. 2 ft. 8 in. to in.

36. Reduce 6600 to miles, etc.

Ans. 1 m. 2 fur. to feet.

37. Reduce 15147 in. to mi. etc.

Ans. 2 m. 3 fur. 5 r. to inches.

38. Reduce 177 qts. to bu., etc.

Ans. 5 bu. 2 pks. 1 qt. to qts.

26

39. Reduce 49610 yds. to acres, etc.

Ans. 10 A. 1 R. to yards.

40. Reduce 842 rods to A., etc.

Ans. 5 A. 1 R. 2 r. to rods.

41. Reduce 43908624 in. to A., etc.

Ans. 7 A. 0 R. 1 ft. to inches.

42. Reduce 910800 sec. to S., etc.

Ans. 8 S. 13° to seconds.

43. Reduce 3780 mi. to S., etc.

Ans. 2 S. 3° to minutes.

44. Reduce 122523 sec. to S. etc.

Ans. 1 S. 4° 2′ 3″ to sec.

45. Reduce 18125 sec. to deg., etc.

Ans. 5° 2′ 3″ to sec.

46. Reduce 3180 mi. to da., etc.

Ans. 2 days, 5 hours, to min.

47. Reduce 518700 mi. to da., etc.

Ans. 360 days, 5 hrs. to min.

48. Reduce 18120 sec. to da., etc.

Ans. 5 hours, 2 min. to sec.

49. Reduce 43380 sec. to hrs., etc.

Ans. 12 h. 3 min. to sec.

50. Reduce 132480 mi. to yds., etc.

Ans. 1 yr. 1 m. 1 da. to min.

51. Reduce 2030 mills to dollars, etc.

Ans. $2, 3 cts. to mills.

52. Reduce 40116 in. to fur., etc.

Ans. 5 fur. 2 r. 10 ft. to in.

53. Reduce 16530 in. to fur., etc.

Ans. 2 fur. 3 r. 8 ft. to in.

54. Reduce 731 ft. to fur., etc.

> *Ans.* 1 fur. 4 r. 5 ft. to ft.

55. Reduce 2216½ yds. to miles, etc.

> *Ans.* 1 m. 2 fur. 3 r. to yds.

56. Reduce 243 rods to A., etc.

> *Ans.* 1 A. 2 R. 3 r. to r.

57. Reduce 120879 ft. to A., etc.

> *Ans.* 2 A. 3 R. 4 r. to ft.

58. Reduce 6720 grains to lbs., etc.

> *Ans.* 1 lb. 2 oz. to grains.

59. Reduce 563 dwt. to lbs., etc.

> *Ans.* 2 lb. 4 oz. 3 dwt. to dwt.

60. Reduce 19776 grs. to lbs., etc.

> *Ans.* 3 lb. 5 oz. 4 dwt. to gr.

61. Reduce 161 drams to oz., etc.

> *Ans.* 16 oz. 5 dr. to drams.

62. Reduce 18144 gills to hhd., etc.

> *Ans.* 2 tuns, 1hhd. to gills.

63. Reduce 1073 galls. to Tuns, etc.

> *Ans.* 4 tuns, 1 hhd. 4 gal. to gal.

64. Reduce 279 in. to yards, etc.

> *Ans.* 7 yds. 2 ft. 3 in. to in.

65. Reduce 66 in. to ft., etc.

> *Ans.* 5 ft. 6 in. to inches.

66. Reduce 4045852800 in. to sq. m., etc.

> *Ans.* 1 sq. m. 5 A. to in.

LESSON XXII.

334. REDUCTION OF DENOMINATE FRACTIONS TO COM-
POUND NUMBERS.

EXAMPLE.

Reduce $\frac{2}{3}$£ to a compound number. *Ans.* 13s. 4d.

Reduce $\frac{2}{3}$ of a pound to shillings by multiplying
by 20, because 20 shillings make 1£.

$\frac{2}{3} \times \frac{20}{1} = \frac{40}{3} = 13\frac{1}{3}$ shillings. Reduce $\frac{1}{3}$ of a shil-
ling to pence by multiplying by 12, because 12 pence
make 1 shilling. $\frac{1}{3} \times \frac{12}{1} = \frac{12}{3} = 4$ pence. The an-
swer is 13s. 4d.

R U L E .

*Reduce the given fraction to the next lower de-
nomination. If the product ends in a fraction, reduce
the fraction to the next lower denomination. If the
2d product ends in a fraction, reduce that fraction to
the next lower denomination. Proceed in this way
until the lowest unit of the table is reached. If there
be any remainder, place it in the form of a fraction.
The several integral parts of the product will be the
required compound number.*

1. Reduce $\frac{2}{3}$ £ to a compound number.
                                   Ans. 13s. 4d. to a denominate.
2. Reduce $\frac{5}{7}$ £ to a compound number.
                                   Ans. 14s. 3d. 7$\frac{5}{7}$ far.
3. Reduce $\frac{6}{7}$ £ to a compound number.
                                   Ans. 17s. 1d. 2$\frac{6}{7}$ far.

4. Reduce $\frac{2}{3}$ £ to a compound number.

<div align="right"><i>Ans.</i> 13s. 4d.</div>

5. Reduce $\frac{1}{9}$ £ to a compound number.

<div align="right"><i>Ans.</i> 2s. 2d. 2$\frac{2}{3}$ far.</div>

6. Reduce $\frac{1}{2}$ £ to a compound number. <i>Ans.</i> 10s.

7. Reduce $\frac{1}{6}$ s. to a compound number.

<div align="right"><i>Ans.</i> 3s. 4d.</div>

8. Reduce $\frac{1}{8}$ lb. Troy to a compound number.

<div align="right"><i>Ans.</i> 1 oz. 10 dwt.</div>

9. Reduce $\frac{2}{5}$ lb. Troy to a compound number.

<div align="right"><i>Ans.</i> 2 oz. 13 dwt. 8 grs.</div>

10. Reduce $\frac{5}{6}$ lb. Troy to a compound number.

<div align="right"><i>Ans.</i> 10 oz.</div>

11. Reduce $\frac{1}{8}$ oz. Troy to a compound number.

<div align="right"><i>Ans.</i> 2 dwt. 10 grs.</div>

12. Reduce $\frac{2}{9}$ oz. Troy to a compound number.

<div align="right"><i>Ans.</i> 4 dwt. 10$\frac{2}{3}$ grs.</div>

13. Reduce $\frac{6}{11}$ oz. Troy to a compound number.

<div align="right"><i>Ans.</i> 10 dwt. 21$\frac{9}{11}$ grs.</div>

14. Reduce $\frac{3}{10}$ Ton Av. to a compound number.

<div align="right"><i>Ans.</i> 6 cwt.</div>

15. Reduce $\frac{1}{14}$ Ton Av. to a compound number.

<div align="right"><i>Ans.</i> 1 cwt. 1 qr. 17 lbs. 13 oz. 11$\frac{3}{7}$ dr.</div>

16. Reduce $\frac{1}{3}$ cwt. Av. to a compound number.

<div align="right"><i>Ans.</i> 1 qr. 8 lbs. 5 oz. 5$\frac{1}{3}$ drs.</div>

17. Reduce $\frac{2}{3}$ cwt. Av. to a compound number.

<div align="right"><i>Ans.</i> 2 qr. 16 lbs. 10 oz. 10$\frac{2}{3}$ drs.</div>

18. Reduce $\frac{5}{6}$ qr. Av. to a compound number.

<div align="right"><i>Ans.</i> 20 lbs. 13 oz. 5$\frac{1}{3}$ drs.</div>

19. Reduce $\frac{5}{7}$ qr. Av. to a compound number.

<div align="right"><i>Ans.</i> 17 lbs. 13 oz. 11$\frac{2}{7}$ drs.</div>

26*

20. Reduce ⅟₈ lb. Av. to a compound number.

Ans. 2 oz.

21. Reduce ⅝ lb. Av. to a compound number.

Ans. 10 oz.

22. Reduce ⅓ 3 to a compound number.

Ans. 1 ℈ 10 grs.

23. Reduce ⅕ ℈ to a compound number.

Ans. 4 grs.

24. Reduce ⅓ mile to a compound number.

Ans. 2 fur. 26 r. 3 y. 2 ft.

25. Reduce ⅞ mile to a compound number.

Ans. 6 fur. 26 r. 3 y. 2 ft.

26. Reduce ½ fur. to a compound number.

Ans. 20 rods.

27. Reduce ⅚ fur. to a compound number.

Ans. 33 r. 1 yd. 2 ft. 6 in.

28. Reduce ⅓ rod to a compound number.

Ans. 1 yd. 2 ft. 6 in.

29. Reduce ½ yard to a compound number.

Ans. 1 ft. 6 in.

30. Reduce ⅝ foot to a compound number.

Ans. 7½ inches.

31. Reduce ₁/₂₀ league to a compound number.

Ans. 1 fur. 8 rods.

32. Reduce ½ foot to a compound number.

Ans. 6 inches.

33. Reduce ¼ E. F. to a compound number.

Ans. 1 qr.

34. Reduce ⅕ E. E. to a compound number.

Ans. 1 qr.

35. Reduce ⅓ E. Fl. to a compound number.

Ans. 1 qr.

36. Reduce ¾ yd. to a compound number.

Ans. 3 qr.

37. Reduce ⅚ A. to a compound number.

Ans. 3 R. 13 r. 10 y. 0 ft. 108 in.

38. Reduce ⅔ R. to a compound number.

Ans. 26 r. 20 yds. 1 ft. 72 in.

39. Reduce ⅚ S. rod to a compound number.

Ans. 25 yds. 1 ft. 126 in.

40. Reduce ½ sq. yd. to a compound number.

Ans. 4 ft. 72 inches.

41. Reduce ¼ sq. ft. to a compound number.

Ans. 36 sq. in.

42. Reduce ⅚ sq. ft. to a compound number.

Ans. 120 sq. in.

43. Reduce ⅞ sq. yd. to a compound number.

Ans. 7 sq. ft. 126 in.

44. Reduce ⅚ sq. rod to a compound number.

Ans. 16 sq. yds. 7 ft. 36 in.

45. Reduce ⅜ R. to a compound number.

Ans. 24 rods.

46. Reduce ⅓ A. to a compound number.

Ans. 17 r. 23 yds. 4 ft. 108 in.

47. Reduce $\frac{8}{12}$ sq. m. to a compound number.

Ans. 26 r. 20 y. 1 ft. 48 in.

48. Reduce ½ cu. ft. to a compound number.

Ans. 864 cubic inches.

49. Reduce ⅔ cu. ft. to a compound number.

Ans. 1152 cu. inches.

50. Reduce ⅘ tun to a compound number.

          Ans. 1 pi. 1 hhd. 12 gal. 2 qt. ⅘ pts.

51. Reduce ⅓ pi. to a compound number.

          Ans. 42 galls.

52. Reduce ⁵⁄₇ hhd. to a compound number.

          Ans. 45 gall.

53. Reduce ⅞ gill to a compound number.

          Ans. 1 qt. 1 pt. ⅞ gill.

54. Reduce ⅓ qrt. to a compound number.

          Ans. 2⅔ gills.

55. Reduce ⅔ pi. to a compound number.

          Ans. 1 hhd. 21 galls.

56. Reduce ½ bbl. to a compound number.

          Ans. 15 gall. 3 qt.

57. Reduce ¼ pi. to a compound number.

          Ans. 10 gal. 2 qt.

58. Reduce ⅓ qr. to a compound number.

          Ans. 2 bu. 5 qt. ⅔ pts.

59. Reduce ⅙ bu. to a compound number.

          Ans. 6 qts. ⅘ pt.

60. Reduce ¼ pk. to a compound number.

          Ans. 1 qt. ⅔ pts.

61. Reduce ⅓ qt. to a compound number.

          Ans. ⅔ pt.

62. Reduce ½ yr. to a compound number.

          Ans. 182 da. 12 hrs.

63. Reduce ⅔ yr. to a compound number.

          Ans. 243 da. 8 hours.

64. Reduce ⅕ da. to a compound number.

          Ans. 4 h. 48 min.

65. Reduce ⅕ S. to a compound number.

          Ans. 3° 20′.

LESSON XXIII.

335. REDUCTION OF DENOMINATE NUMBERS TO
COMPOUND FRACTIONS.

EXAMPLE.

1. Reduce 13s. 4d. to a compound fraction.

Reduce 13s. 4d. to pence. $13 \times 12 = 156$ pence, $156 + 4 = 160$ pence. Reduce 1 £. to pence, $20 \times 12 = 240$ pence. Make the given number (reduced to pence) the numerator and the £ reduced to pence the denominator. We now have $\frac{160}{240}$ for our fraction. Reduced to its lowest terms gives $\frac{2}{3}$ £ for the answer.

RULE.

Reduce the unit of the proposed fraction to the lowest denomination of the given denominate number. Make this the Denominator.

Reduce the given compound number to the same unit. Make this the Numerator.

Reduce the fraction to its lowest terms.

1. Reduce 13s. 4d. to the fraction of a £.
<div align="right">*Ans.* $\frac{2}{3}$£.</div>

2. Reduce 14s. 3d. 1½ far. to the fraction of a £.
<div align="right">*Ans.* $\frac{4}{7}$£.</div>

3. Reduce 17s. 1d. 2½ far. to the fraction of a £.
<div align="right">*Ans.* $\frac{4}{7}$£.</div>

4. Reduce 13s. 4d. to the fraction of a £.
<div align="right">*Ans.* $\frac{2}{3}$£.</div>

5. Reduce 2s. 2d. 2⅔ far. to the fraction of a £.

Ans. ⅛£.

6. Reduce 10s. to the fraction of a £.   *Ans.* ½£.

7. Reduce 3s. 4d. to the fraction of a £.

Ans. ⅙s.

8. Reduce 1 oz. 10 dwt. Troy to the fraction of a lb.   *Ans.* ⅛ lb.

9. Reduce 2 oz. 13 dwt. 8 grs. Troy to the fraction of a lb.   *Ans.* ⅔ lb.

10. Reduce 10 oz. Troy to the fraction of a lb.

Ans. ⅚ lb.

11. Reduce 2 dwt. 12 grs. Troy to the fraction of an oz.   *Ans.* ⅛ oz.

12. Reduce 4 dwt. 10⅔ grs. Troy to the fraction of an oz.   *Ans.* ⅞ oz.

13. Reduce 10 dwt. 21 9/11 grs. Troy to the fraction of an oz.   *Ans.* 6/11 oz.

14. Reduce 6 cwt. to the fraction of a ton.

Ans. ⅗ ton.

15. Reduce 1 cwt. 1 qr. 17 lbs. 13 oz. 11¾ dr. to the fraction of a ton.   *Ans.* 1/14 ton.

16. Reduce 1 qr. 8 lbs. 5 oz. 5⅓ dr. to the fraction of a cwt.   *Ans.* ⅓ cwt.

17. Reduce 2 qrs. 16 lbs. 10 oz. 10⅔ dr. to the fraction of a cwt.   *Ans.* ⅞ cwt.

18. Reduce 20 lbs. 13 oz. 5¼ drs. to the fraction of a qr.   *Ans.* ⅚ qr.

19. Reduce 17 lbs. 13 oz. 11¾ drms. to the fraction of a qr.   *Ans.* 5/7 qr.

20. Reduce 2 oz. to the fraction of a lb.

Ans. ⅛ lb.

21. Reduce 10 oz. to the fraction of a lb.

Ans. ⅝ lb.

22. Reduce 1 ℈ 10 grs. to the fraction of a ʒ.

Ans. ½ oz.

23. Reduce 4 grs. to the fraction of a ℈.

Ans. ⅕ ℈.

24. Reduce 2 fur. 26 r. 3 yds. 2 ft. to the fraction of a mile. *Ans.* ⅓ m.

25. Reduce 6 fur. 26 r. 3 y. 2 ft. to the fraction of a mile. *Ans.* ⅞ m.

26. Reduce 20 rods to the fraction of a fur.

Ans. ½ fur.

27. Reduce 33 r. 1 yd. 2 ft. 6 in. to the fraction of a fur. *Ans.* ⅚ fur.

28. Reduce 1 yd. 2 ft. 6 in. to the fraction of a rod. *Ans.* ⅓ rod.

29. Reduce 1 ft. 6 in. to the fraction of a yd.

Ans. ½ yd.

30. Reduce 7½ inches to the fraction of a foot.

Ans. ⅝ ft.

31. Reduce 1 fur. 8 rods to the fraction of a league. *Ans.* 1/20 lea.

32. Reduce 6 in. to the fraction of a foot.

Ans. ½ ft.

33. Reduce 1 qr. Cloth Measure to the fraction of a E. E. *Ans.* ¼ E. E.

34. Reduce 1 qr. to the fraction of a E. F.

Ans. ⅙ E. F.

35. Reduce 1 qr. to the fraction of a E. F.

Ans. ⅕ E. F.

36. Reduce 3 qrs. to the fraction of a yd.

Ans. $\frac{3}{4}$ yd.

37. Reduce 3 R. 13 r. 10 y. 0 ft. 108 in. to the fraction of an acre.   *Ans.* $\frac{5}{6}$ acre.

38. Reduce 26 r. 20 y. 1 ft. 72 in. to the fraction of a rood.   *Ans.* $\frac{2}{3}$ rood·

39. Reduce 25 y. 1 ft. 126 in. to the fraction of a rod.   *Ans.* $\frac{5}{6}$ rod.

40. Reduce 4 ft. 72 in. to the fraction of a yard.

Ans. $\frac{1}{2}$ sq. yd.

41. Reduce 36 sq. in. to the fraction of a foot.

Ans. $\frac{1}{4}$ sq. ft.

42. Reduce 120 sq. in. to the fraction of a foot.   .

Ans. $\frac{5}{6}$ sq. ft.

43. Reduce 7 sq. ft. 126 in. to the fraction of a yard.   *Ans.* $\frac{7}{8}$ sq. yd.

44. Reduce 16 sq yd. 7 ft. 36 in. to the fraction of a rod.   *Ans.* $\frac{5}{8}$ sq. rod.

45. Reduce 24 r. to the fraction of a rood.

Ans. $\frac{3}{5}$ rood.

46. Reduce 17 rods 23 yds. 4 ft. 108 in. to the fraction of an acre.   *Ans.* $\frac{1}{8}$ acre.

47. Reduce 426 A. 2 R. 26 r. 20 y. 1 ft. 48 in. to the fraction of a mile.   *Ans.* $\frac{8}{12}$ sq. mile.

48. Reduce 864 cubic inches to the fraction of a cu. ft.   *Ans.* $\frac{1}{2}$ cu. ft.

49. Reduce 1152 cubic in. to the fraction of a cu. ft.   *Ans.* $\frac{2}{3}$ cu. ft.

50. Reduce 1 pi. 1 hhd. 12 galls. 3 qts. $\frac{4}{5}$ pint to the fraction of a tun.   *Ans.* $\frac{1}{5}$ tun.

51. Reduce 42 gallons to the fraction of a hhd.

Ans. ⅓ pipe.

52. Reduce 45 galls. to the fraction of a hhd.

Ans. ⅝ hhd.

53. Reduce 1 qt. 1 pt. ⅘ gill to the fraction of a gall.

Ans. ⅖ gall.

54. Reduce 2⅔ gills to the fraction of a quart.

Ans. ⅓ quart.

55. Reduce 1 hhd. 21 galls. to the fraction of a pipe.

Ans. ⅜ pipe.

56. Reduce 15 galls. 3 qts. to the fraction of a bbl.

Ans. ½ bbl.

57. Reduce 10 galls. 2 qts. to the fraction of a tierce.

Ans. ¼ tierce.

58. Reduce 2 bu. 5 qts. ⅔ pt. to the fraction of a qr.

Ans. ⅓ qr.

59. Reduce 6 qts. ⅘ pt. to the fraction of a bu.

Ans. ⅛ bu.

60. Reduce 1 qt. ⅔ pt. to the fraction of a pk.

Ans. ⅙ pk.

61. Reduce ⅔ pt. to the fraction of a qt.

Ans. ⅙ qt.

62. Reduce 182 days 12 hrs. to the fraction of a yr.

Ans. ½ yr.

63. Reduce 243 days 8 hrs. to the fraction of a yr.

Ans. ⅔ yr.

64. Reduce 4 hrs. 48 mi. to the fraction of a da.

Ans. ⅕ da.

65. Reduce 3° 20′ to the fraction of a S.

Ans. ⅛ S.

27

19. Reduce 1 dr. to the decimal of a Ton.

Ans. .000001953125 Ton.

20. Reduce 1 in. to the decimal of a ft.

Ans. .0833⅓ ft.

21. Reduce 2 in. to the decimal of a yd.

Ans. .0555⅚ yd.

22. Reduce 3 in. to the decimal of a rod.

Ans. .0151₃₃¹⁷ rod.

23. Reduce 4 in. to the decimal of a fur.

Ans. .000505₁₃₃¹ fur.

24. Reduce 5 in. to the decimal of a mile.

Ans. .000078125 m.

25. Reduce 6 in. to the decimal of a Lea.

Ans. .0000315₆₆⁵⁵ Lea.

26. Reduce 1 Rood to the decimal of a sq. mile.

Ans. .0039₁₆¹ sq. m.

27. Reduce 9 sq. ft. to the decimal of a Rood.

Ans. .00082644₁₂₁⁷² R.

28. Reduce 2 pints to the decimal of a pipe.

Ans. .0019⅝⅜ pipe.

29. Reduce 2 galls. to the decimal of a Tun.

Ans. .0079⅔⅔ Tun.

30. Reduce 40 galls. to the decimal of a hhd.

Ans. 6349⅓⅓ hhd.

31. Reduce 1 gill to the decimal of a Tun.

Ans. .000124₁₂₆¹ Tun.

32. Reduce 2 qrts. to the decimal of a Tun.

Ans. .0198₁₂₆⁵¹ Tun.

33. Reduce 20£. 15s. 9d. 3 far. to the decimal of a £.

Ans. 20.790625£.

34. Reduce 1 hhd. 25 galls. 3 qts. to the decimal of a hhd.   *Ans.* 1.410714$\frac{2}{17}$ hhd.

35. Reduce 23 cwt. 2 qrs. 18 lbs. to the decimal of a Ton.   *Ans.* 1.184 Ton.

36. Reduce 9 cwt. 3 qrs. 23 lbs. to the decimal of a Ton.   *Ans.* .499 Ton.

37. Reduce 14£ 4s. 6d. 2 far. to the decimal of a £.   *Ans.* 14.22708$\frac{1}{4}$ £.

38. Reduce 1 lb. 9 oz. 13 dwt. to the decimal of a Ton.   *Ans.* .000806640625 Ton.

39. Reduce 19 cwt. 3 qrs. 23 lbs. to the decimal of a Ton.   *Ans.* .999 Ton.

40. Reduce 18 galls. 3 qts. 1 pt. to the decimal of a hhd.   *Ans.* .2996$\frac{2}{63}$ hhd.

41. Reduce 365 d. 4 hrs. 48 min. to the decimal of a year.   *Ans.* 1.000$\frac{14}{21}\frac{5}{9}$ yr.

42. Reduce 4 A. 3 R. 20 r. to the decimal of a mile.   *Ans.* .7617$\frac{3}{16}$ mile.

LESSON XXV.

CASE II.

337. Reduce 20 £ 15s. 9d. 3 farthings to the decimal of a £.

Find how many farthings in the given number, leaving out the 20 £ which will be a whole number.

27*

15s. × 12 = 180 pence, 180 + 9 = 189 pence.

189d. × 4 = 756 far. 756 + 3 = 759 farthings.

Now find how many farthings in a £ and make the result the denominator of a fraction having the first result for its numerator.

20 × 12 × 4 = 960 farthings in 1 £.

The fraction will be $\frac{759}{960}$, which reduced to a decimal gives 0.790625. Write the whole number (20 £) before the decimal point and you get the required answer.   *Ans.* 20.790625.

Or:

338. Divide the lowest denomination, farthings, by the number required to make one of a higher denomination. This gives 0.75 pence. Write the number of pence to the left of the decimal

OPERATION.

$$4)\overline{3}$$
$$12)\overline{9.75}$$
$$20)\overline{15.8125}$$
$$\overline{20.790625}$$

and divide by the number of pence required to make one of the next higher denomination. This gives 0.8125 shillings. Write the given number of shillings to the left of the decimal point and divide by the number of shillings required to make one pound sterling. This gives 0.790625. Write the number of £ to the left of the decimal and you have the required answer.

339. When reducing long measure or square measure, the pupil will be careful and remember that if there is a remainder after dividing by $5\frac{1}{2}$ or $30\frac{1}{4}$, that remainder is a fractional part of a rod and should be immediately reduced to yards.

43. Reduce 2 cwt. 1 qr. 6 lbs. to the decimal of a cwt.   *Ans.* 2.31 cwt.

44. Reduce 21 miles to the decimal of a degree.
                                    Ans. .3036$\frac{12}{13}$ deg.

45. Reduce 1 sq. rod 57 ft. to the decimal of a rood.   *Ans.* .0309 roods.

46. Reduce 13 £ 5s. to the decimal of a guinea.
                                    Ans. 12.619$\frac{1}{21}$ G.

47. Reduce 14s. 9d. to the decimal of a £.
                                    Ans. .7375 £.

48. Reduce 17 cwt. 3 qr. 10 lbs. to the decimal of a ton.   *Ans.* .8925 ton.

49. Reduce 10 £ 14s. 2 far. to the decimal of a £.
                                    Ans. 10.720$\frac{5}{8}$ £.

50. Reduce 1 mi. 4 fur. 4 rods to the decimal of a mile.   *Ans.* 1.5125 mile.

51. Reduce 5 A. 3 R. 20 r. to the decimal of an acre.   *Ans.* 5.875 acre.

52. Reduce 8 cwt. 3 qrs. 6 lbs. to the decimal of a ton.   *Ans.* .4405 ton.

53. Reduce 3 qrs. 6 lbs. to the decimal of a cwt.
$\qquad$ *Ans.* .81 cwt.

54. Reduce 8 oz. 2 dwt. 5 grs. to the decimal of a lb. $\qquad$ *Ans.* .67686$\frac{32}{36}$ lb.

55. Reduce 2 yds. 3 na. to the decimal of a yd.
$\qquad$ *Ans.* 2.1875 yds.

56. Reduce 2° 15′ 15″ to the decimal of a deg.
$\qquad$ *Ans.* .25416$\frac{2}{3}$ deg.

57. Reduce 20 lbs. 10 dwt. to the decimal of a lb.
$\qquad$ *Ans.* 20.416$\frac{2}{3}$ lb.

58. Reduce 255 da. 16 hrs. 2 mi. 10 sec. to the decimal of a yr. $\qquad$ *Ans.* .700$\frac{7266}{15768}$ yr.

59. Reduce 94 £ 7s. 8d. to the decimal of a £.
$\qquad$ *Ans.* 94.383$\frac{1}{3}$ £.

60. Reduce 2 £ 12s. 8d. to the decimal of a £.
$\qquad$ *Ans.* 2.63$\frac{1}{3}$ £.

61. Reduce 2 bu. 1 pk. 7 qts. to the decimal of a bushel. $\qquad$ *Ans.* 2.46875 bu.

62. Reduce 10 ton 5 cwt. 2 qrs. 18 lbs. to the decimal of a ton. $\qquad$ *Ans.* 10.284 ton.

63. Reduce 84° 18′ 15″ to the decimal of a deg.
$\qquad$ *Ans.* 84.304016$\frac{2}{3}$ deg.

64. Reduce 84° 18′ 15″ to the decimal of a circ.
$\qquad$ *Ans.* .23417$\frac{89}{108}$ cir.

65. Reduce 4° 18′ 15″ to the decimal of a circ.
$\qquad$ *Ans.* .0119$\frac{121}{216}$ circum.

LESSON XXVI.

340. REDUCTION OF DENOMINATE DECIMALS TO COM-
POUND NUMBERS.

EXAMPLES.

1. Reduce 0.790625 £ to a compound number.

	OPERATION.
Multiply by 20 thus, and cut off the required number of decimals. This will give shillings and a fraction.	.790625
	20
Multiply the fraction by 12 and cut off the required number of decimals. This will give pence and a fraction.	15.8125øø
	12
Multiply the fraction by 4 and cut off the required number of decimals. This gives 3 farthings and no remainder. The answer is then, 15s. 9d. 3 far.	16250
	8125
	9.75øø
	4
	3.00

RULE.

*Multiply the given decimal fraction by the number
representing the next lower denomination. Cut off
the required number of decimals. Multiply the deci-
mal part by the figure representing the next lower de-
nomination, cut off the required number of decimals
and continue till the lowest unit is reached. The inte-
grals of the products form the answer.*

EXAMPLES.

1. Reduce .00018$9\frac{13}{33}$ miles to a compound number. *Ans.* 1 foot.

2. Reduce .03125 gallon to a compound number. *Ans.* 1 gill.

3. Reduce .0000625 Ton to a compound number. *Ans.* 2 oz.

4. Reduce .006 Ton to a compound number. *Ans.* 12 lb.

5. Reduce .00104$\frac{1}{6}$ £ to a compound number. *Ans.* 1 far.

6. Reduce .003125 £ to a compound number. *Ans.* 3 far.

7. Reduce .00198$4\frac{8}{63}$ hhd. to a compound number. *Ans.* 1 pint.

8. Reduce .18 cwt. to a compound number. *Ans.* 18 lb.

9. Reduce 0.0115 Ton to a compound number. *Ans.* 23 lb.

10. Reduce .0375 £ to a compound number. *Ans.* 9d.

11. Reduce .75 £ to a compound number. *Ans.* 15s.

12. Reduce .0029$\frac{16}{21}$ Tun to a compound number. *Ans.* 3 qt.

13. Reduce .025 Ton to a compound number. *Ans.* 2 qrs.

14. Reduce .5 cwt. to a compound number. *Ans.* 2 qrs.

15. Reduce .0375 Ton to a compound number.

Ans. 3 qrs.

16. Reduce .0001875 Ton to a compound number.

Ans. 6 oz.

17. Reduce .2 Crown to a compound number.

Ans. 1s.

18. Reduce .000174$\frac{1}{36}$ lbs. to a compound number.

Ans. 1 qr.

19. Reduce .0000001953125 Ton to a compound number.

Ans. 1 dr.

20. Reduce .083$\frac{1}{3}$ ft. to a compound number.

Ans. 1 inch.

21. Reduce 0.0555$\frac{5}{9}$ yd. to a compound number.

Ans. 2 inches.

22. Reduce 0.0151$\frac{17}{33}$ rod to a compound number.

Ans. 3 inches.

23. Reduce .000505$\frac{1}{198}$ fur. to a compound number.

Ans. 4 inches.

24. Reduce .00078125 mile to a compound number.

Ans. 5 inches.

25. Reduce .0000315$\frac{55}{99}$ Lea. to a compound number.

Ans. 6 inches.

26. Reduce .0039$\frac{1}{16}$ sq. m. to a compound number.

Ans. 1 Rood.

27. Reduce .00082644$\frac{27}{121}$ R. to a compound number.

Ans. 9 sq. yds.

28. Reduce .0019$\frac{53}{63}$ pipe to a compound number.

Ans. 2 pints.

29. Reduce .0079$\frac{23}{63}$ Tun to a compound number.

Ans. 2 galls.

30. Reduce .6349$\frac{13}{68}$ hhd. to a compound number.

Ans. 40 galls.

31. Reduce .000124$\frac{1}{126}$ Tun to a compound number.

Ans. 1 gill.

32. Reduce .0198$\frac{51}{126}$ Tun to a compound number.

Ans. 2 qt.

33. Reduce 20.790625 £ to a compound number.

Ans. 20 £ 15s. 9d. 3 far.

34. Reduce 1.410714$\frac{2}{7}$ to a compound number.

Ans. 1 hhd. 25 gal. 3 qt. 1 pt.

35. Reduce 1.184 Ton to a compound number.

Ans. 23 cwt. 2 qr. 18 lbs.

36. Reduce .499 Ton to a compound number.

Ans. 9 cwt. 3 qr. 23 lbs.

37. Reduce 14.22708$\frac{1}{3}$ £ to a compound number.

Ans. 14£. 4s. 6d. 2 far.

38. Reduce .000806640625 Ton to a compound number.

Ans. 1 lb. 9 oz. 13 dwt.

39. Reduce .999 Ton to a compound number.

Ans. 19 cwt. 3 qr. 23 lb.

40. Reduce .2996$\frac{2}{63}$ hhd. to a compound number.

Ans. 18 gall. 3 qts. 1 pt.

41. Reduce 1.000$\frac{145}{219}$ yr. to a compound number.

Ans. 365 da. 5 h. 48 m.

42. Reduce .7617$\frac{3}{16}$ mile to a compound number.

Ans. 4 A. 3 R. 20 rods.

43. Reduce 2.31 cwt. to a compound number.

Ans. 2 cwt. 1 qr. 6 lb.

44. Reduce .3036$\frac{13}{63}$ deg. to a compound number.

Ans. 21 miles.

45. Reduce .0309 R. to a compound number.
$$Ans.\ \text{1 sq. rod 57 sq. ft.}$$

46. Reduce 12.619 Guineas to a compound number.
$$Ans.\ \text{13 £. 5s.}$$

47. Reduce .7375 £. to a compound number.
$$Ans.\ \text{14s. 9d.}$$

48. Reduce .8925 Ton to a compound number.
$$Ans.\ \text{17 cwt. 3 qrs. 10 lb.}$$

49. Reduce 10.720⅝ £. to a compound number.
$$Ans.\ \text{10 £. 14s. 0d. 2 far.}$$

50. Reduce 1.5125 mile to a compound number.
$$Ans.\ \text{1 m. 4 fur. 4 r.}$$

51. Reduce 5.875 Acre to a compound number.
$$Ans.\ \text{5 A. 3 R. 20 rods.}$$

52. Reduce .4405 Ton to a compound number.
$$Ans.\ \text{8 cwt. 3 qr. 6 lbs.}$$

53. Reduce .81 cwt. to a compound number.
$$Ans.\ \text{3 qr. 6 lbs.}$$

54. Reduce .67586⅔₆ lb. to a compound number.
$$Ans.\ \text{8 oz. 2 dwt. 5 grs.}$$

55. Reduce 2.1875 yds. to a compound number.
$$Ans.\ \text{2 yds. 3 na.}$$

56. Reduce .25416⅔ deg. to a compound number.
$$Ans.\ \text{2° 15' 15''.}$$

57. Reduce 20.416⅔ lb. to a compound number.
$$Ans.\ \text{20 lb. 10 dwt.}$$

58. Reduce 700 $\frac{7265}{15768}$ yr. to a compound number.
$$Ans.\ \text{255 da. 16 h. 2 m. 10 s.}$$

59. Reduce 94.383⅓ £ to a compound number.
$$Ans.\ \text{94 £ 7s. 8d.}$$

28

60. Reduce 2.63¼ £ to a compound number.

Ans. 2£ 12s. 8d.

61. Reduce 2.46875 bu. to a compound number.

Ans. 2 bu. 1 pk. 7 qts.

62. Reduce 10.284 Ton to a compound number.

Ans. 10 Ton 5 cwt. 2 qr. 18 lb.

63. Reduce 84.304016⅔ deg. to a compound number.

Ans. 84° 18′ 15″.

64. Reduce .23417$\frac{89}{108}$ circ. to a compound number.

Ans. 84° 18′ 15″.

65. Reduce .0119$\frac{121}{216}$ circ. to a compound number.

Ans. 4° 18′ 15″.

66. Reduce .05 £ to a compound number.

Ans. 1 shilling.

ADDITION OF DENOMINATE NUMBERS.

LESSON XXVII.

RULE I.

341. I. *Write the quantities so that the figures of the same denomination will be found in the same column.*

II. *Add the columns separately, beginning with the right-hand column.*

III. *After having found the sum of the first column, see how many units of the next column are contained in the sum of the first. Write the remainder under the first column and carry the quotient to the second column.*

IV. *Add the figures in the second column. See how many units of the next column are contained in the sum of the second column. Write the remainder under the second column, and carry the quotient to the next.*

V. *Continue in this way until the last column is reached. Add the figures in the last column, and write the sum under the last column.*

Note.—If fractional parts of denominate numbers are used, reduce the fractions to compound numbers, and add as before.

342. Note to the Pupil.—In adding sums in Long Measure and also in Square Measure the attention of the pupil is necessary to the following remarks:

"Inches and feet being of an even denomination, no difficulty is experienced in dividing the sum of the columns; but when the column of yards is added, the sum has to be divided by a mixed number, that is, by a whole number and a fraction. The remainder in this case is different from all others, because the remainder, if any, is not yards, but a fraction of a rod. This fraction of a rod must be multiplied by $5\frac{1}{2}$ (if Long Measure), or $30\frac{1}{4}$ (if Square Measure), to find the number of yards. For, since the remainder is a fraction of a rod, to reduce that fraction to a lower denomination, it is necessary to multiply it first by $5\frac{1}{2}$ (for Long Measure), or by $30\frac{1}{4}$ (for Square Measure), to find the number of yards. In many examples the answer results in yards and a fraction of a yard. This fraction of a yard must be reduced to feet, by multiplying it by 3 (for Long Measure), or by 9 (for Square Measure), and the number thus found must be added to the number found under the column of feet. If a fraction should occur with the feet, reduce it to inches by multiplying it by 12 (for Long Measure), or by 144 (for Square Measure), the result to be added to the number found under the column of inches. Having completed the several

operations as above indicated, examine the number under the column of inches and see if there are too many inches. If so, reduce them to feet, write the remainder in place of the figures under the column of feet, and add the quotient to the figures under the column of feet. See if there are more (feet) than would make a yard. If so, reduce the feet to yards, write the remainder, if any, in place of the number under the column of feet, and add the quotient to the number under the column of yards."

In adding the fractional parts of *Denominate numbers*, as $\frac{1}{3}$ mile and $\frac{1}{4}$ of a mile, add the fractions, thus, $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$ mile, if they are of the *same value* as above, and then find the value of the fraction thus found.

In the above example $\frac{1}{3}$ of a mile and $\frac{1}{4}$ of a mile are equal to $\frac{7}{12}$ of a mile, which, being reduced, gives 4 fur. 26 r. 3 yds. 2 ft. 0 in.

The same remarks apply to decimal fractions of Denominate Numbers.

If the fractions are of different denominate value, they may be reduced to the same value or denomination, thus $\frac{1}{3}$ pipe and $\frac{1}{4}$ Tun.

$\frac{1}{3}$ of a pipe is equal to $\frac{1}{6}$ of a Tun, because it requires 2 pipes to make a tun, and $\frac{1}{3}$ divided by 2 is the same as $\frac{1}{3} \times \frac{1}{2}$ or $\frac{1}{6}$ of·a Tun.

We now have $\frac{1}{6}$ Tun and $\frac{1}{4}$ of a Tun to be added together $=$ or $\dfrac{2+3}{12}=\frac{5}{12}$ Tun, which, when reduced, would give 1 hhd. and 42 galls.

Or:

Reduce the Tuns in the above example to pipes, thus $\frac{1}{3}$ pipe and $\frac{1}{4}$ Tun would be the same as $\frac{1}{3}$ pipe

and ½ of a pipe.   To reduce Tuns to pipes, we must multiply the fraction by 2, because 2 pipes make a tun.   By multiplying ¼ by 2, we get ²⁄₄ or ½.   Then adding the fractions ⅓+½=⅚ of a pipe, which reduced, gives 1 hhd. and 42 galls.

ADDITION OF DENOMINATE NUMBERS.

(1)					(2)				
£.	s.	d.	far.		Ton.	cwt.	qrs.	lbs.	oz.	dram.
1	5	2	2		1	0	3	5	8	1
	3	5	1		2	1	2	1	4	6
4	6	8	3		1	18	3	16	10	10
5	15	4	2		5	1	0	23	7	1

(3)					(4)				
lb.	oz.	dwt.	grs.		m.	fur.	r.	yds.	ft.	in.
1	3	12	21		1	3	3	2	1	4
	9	15	22		1	3	8	4	1	6
8	10	10	23			7	20	1	2	9
10	11	19	18		3 .	5	32	3	1	1

(5)					(6)					
£	s.	d.	far.	sq. m.	A.	R.	r.	yds.	ft.	in.
3	14	11	3	1	604	3	19	21	8	100
1	10	3	3		168	2	24	20	6	190
	15	10	3		60	3	32	10	5	36
6	1	2	1	2	194	1	36	23	1	2

28*

(7)

m.	fur.	r.	yds.	ft.	in.
2	2	20	4	2	5
1	4	25	5	2	6
	5	35	4	1	8
4	5	2	4	0	7

(8)

Ton.	cwt.	qr.	lb.	oz.	dr.
2	12	2	6	14	5
1	8	3	12	12	14
		2	20	12	2
4	2	0	15	7	5

(9)

lb.	oz.	dwt.	gr.
3	2	8	20
3	1	4	20
	8	10	12
7	0	4	4

(10)

sq. m.	A.	R.	r.	yds.	ft.	in.
1	64	2	30	20	8	120
2	136	3	15	6	7	20
	583	2	30	29	6	83
4	145	0	36	27	2	43

(11)

lbs.	oz.	dwt.	grs.
4	3	2	20
1	4	8	21
	2	6	4
		1	13

(12)

sq. m.	A.	R.	r.	y.	ft.	in.
1	64	3	31	21	7	120
	590	2	39	28	6	132
		3	28	15	5	72

ADDITION OF COMPOUND NUMBERS.

(1)

£.	s.	d.	far.
1	3	10	2
2	3	5	1
4	6	8	3
7	14	0	2

(2)

m.	fur.	r.	y.	ft.	in.
1	2	3	5	2	4
1	3	8	4	2	6
	7	20	2	1	9
3	4	33	2	0	7

(3)

Ton.	cwt.	qrs.	lbs.	oz.	dr.
1	2	3	5	2	1
2	1	3	1	4	6
1	18	3	16	12	10
5	3	1	23	3	1

(4)

£.	s.	d.	far.
3	4	11	3
1	16	3	3
	15	8	2
5	17	0	0

(5)

m.	A.	R.	r.	y.	ft.	in.
1	604	3	29	21	7	100
	62	2	24	20	6	190
	60	3	14	16	5	118
2	93	1	28	9	0	84

(6)

m.	fur.	r.	y.	ft.	in.
2	3	20	4	2	5
1	4	25	5	2	6
	5	30	4	1	11
4	5	37	4	0	10

(7)

Ton.	cwt.	qr.	lbs.	oz.	dr.
2	12	2	6	4	2
1	8	3	12	12	14
	8	2	20	14	2
4	10	0	13	15	2

(8)

lbs.	oz.	dwt.	grs.
3	2	8	20
3	10	14	22
	8	10	2
7	9	18	20

(9)

lb.	oz.	dwt.	grs.
1	3	12	20
2	9	15	21
8	10	19	22
13	0	8	15

(10)

m.	A.	R.	r.	y.	ft.	in.
1	64	2	30	20	8	121
2	100	3	25	16	7	120
	583	2	39	29	6	283
4	109	1	16	7	2	20

(11)

Tun.	pi.	hd.	gal.	qt.	pt.	gi.
1	1	1	60	2	1	2
1	1	1	20	3	1	2
1	1	1	30	2	1	2
1	1	1	20	2	1	2
6	0	0	6	1	0	2

(12)

Tun	pi.	hd.	gall.	qt.	pt.	gi.
1	1	1	60	2	0	2
1	0	1	30	0	1	0
1	0	0	40	2	2	2
1	1	0	0	2	2	2
7	1·	0	7	0	0	0

(13)

hhd.	bbl.	gall.	qts.	pts.
1	1	24	2	1
1	2	30	1	2
1	1	30	2	1
1	1	20	1	1
8	1	34	0	1

(14)

qr.	bu.	p.	qt.	pt.
1	4	2	4	1
1	5	1	5	1
	6	2	6	1
	1	1	2	1
4	2	0	3	0

(15)

qr.	bu.	p.	qt.	pt.
1	4	2	4	1
	5	3	5	1
		4	6	2
2	3	3	0	1

(16)

yr.	mo.	da.	hr.	sec.	mi.
1	2	3	2	15	20
	10	21	20	40	30
		20	18	40	40
2	1	15	17	36	30

(17)

yr.	da.	hrs.	mi.	sec.
1	200	3	2	1
	300	29	58	59
	20	30	30	22
		1	30	40
2	157	17	2	2

(18)

circ.	S.	°	′	″
1	6	15	30	30
	3	7	15	15
		20	40	50
2	10	13	26	35

circ.	S.	°	′	″
1	6	20	30	40
	9	30	18	50
		50	30	40
			12	22
2	6	11	32	·32

(19)

℔.	ʒ	Ӡ	☉	grs.
1	6	4	1	10
	9	8	1	15
		6	2	10
2	5	3	2	15

(20)

℔	ʒ	Ӡ	☉	grs.
1	9	7	2	18
	8	6	1	16
		5	2	18
			1	20
2	7	5	0	12

(21)

yd.	qrs.	na.	in.
1	2	3	2
	3	2	1
		1	2
2	3	0	½

(22)

(23)

yd.	qrs.	na.	in.
1	2	1	1
	3	3	2
		2	1
2	2	2	1¾

24. Add ⅓ mile and ¼ mile.

      Ans. 4 fur. 26 r. 3 yds. 2 ft. 0 in.

25. Add ½ rod and ⅓ rod. *Ans.* 3 yd. 2 ft. 6⅗ in.

26. Add ⅔ hhd. and ⅝ hhd.

      Ans. 57 galls. 1 quart 1 pint.

27. Add ⅓ pipe and ¼ Tun. *Ans.* 1 hhd. 42 galls.

28. Add ⅚ A. and ⅖ R.

     Ans. 3 R. 39 r. 30 yds. 2 ft. 36 in.

29. Add ⅙ Ton and ⅛ cwt.

     Ans. 3 cwt. 1 qr. 19 lbs. 7 oz. 1⅞ drs.

30. Add ⅚ Ton and ⅔ cwt.

     Ans. 17 cwt. 1 qr. 8 lbs. 5 oz. 5⅓ drs.

31. Add ½ qr. and ⅛ lb.

     Ans. 12 lbs. 13 oz. 5⅓ drs.

32. Add ⅛£ and $\frac{1}{12}$s.   *Ans.* 2s. 7d.
33. Add ⅚£ and ⅔d.   *Ans.* 16s. 8d. 2⅔ far.
34. Add ⅔£ and ⅓ far.   *Ans.* 13s. 4d. ½ far.
35. Add ½ sign and ⅔ degree.

     Ans. 15 degrees 40 mi.

36. Add ⅞ sq. yd. and ⅔ sq. ft.   *Ans.* 8 ft. 108 in.
37. Add ⅛ bu. and ¼ pk.   *Ans.* 7 qts. 1$\frac{7}{15}$ pints.
38. Add ⅓ yr. and ¼ day. *Ans.* 121 days 20 hours.
39. Add ⅛ century and ⅔ year.

     Ans. 11 yr. 9 mo. 10 da.

40. Add .025£. and .024 sh.   *Ans.* 6d. 1.152 far.
41. Add .35£ and .25 sh.   *Ans.* 12s. 3d.
42. Add .025 Ton and .02 lb.

     Ans. 2 qr. 0 lbs. 0 oz. 5.12 drams.

43. Add .05 cwt. and .033⅓ qr.

     Ans. 5 lbs. 13 oz. 5⅓ drams.

44. Add .0352 mile and .024 rod.

     Ans. 11 rods 1 yd. 1 foot 9.024 in.

45. Add .02 sq. m. and .55 R.

     Ans. 12 A. 3 R. 30 rods.

46. Add .186 sq. yd. and .5 sq. ft.

     Ans. 2 sq. ft. 25.056 in.

47. Add .025 day and .025 hr.

Ans. 37 mi. 30 sec.

48. Add .0252 yr. and .0252 da.

Ans. 9 da. 5 hrs. 21 m. 24.48 sec.

49. Add .05° and .005°.   *Ans.* 3 mi. 18 sec.

50. Add .005° and .025°.   *Ans.* 1 mi. 48 sec.

51. Add .025 hhd. and .03 gall.

Ans. 1 gall. 2 qrts. 0 pints 3.36 gills.

52. Add .066 Tun and .024 hhd.

Ans. 18 galls. 0 qts. 1 pint. 0.628 gill.

53. Add .4405 Ton and .81 cwt.

Ans. 8 cwt. 6 qrs. 12 lbs.

54. Add .875 A. and .0309 R.

Ans. 3 R. 21 r. 7 y. 1 ft. 36.144.

55. Add .999 Ton and .499 Ton.

Ans. 1 Ton 9 cwt. 3 qrs. 21 lbs.

56. Add .284 Ton and .8925 Ton.

Ans. 1 Ton 3 cwt. 2 qr. 3 lbs

SUBTRACTION OF COMPOUND NUMBERS.

LESSON XXIX.

342. Subtraction of compound numbers is the process of finding the difference between two denominate numbers of the same kind.

EXAMPLE IN ENGLISH MONEY.

	£	s.	d.	far.
From	78	11	5	2
Take	42	14	3	3
	35	17	1	3

Having placed the less number under the greater, farthings under farthings, etc., we begin and subtract, from farthings. 3 farthings from 2 farthings, we cannot; borrow 1 penny, which is equal to 4 farthings, and 2 farthings we have make 6 farthings. Now 3 far. from 6 far. leaves 3 far. Having borrowed 1 penny from the column of pence, we have to subtract 1 from the 5 in the minuend or add 1 to the 3 in the subtrahend. It is more convenient to add 1 to the figure in the subtrahend. Say, 3 and 1 I borrowed are 4 pence. 4 from 5 leaves 1. Write the 1 under the column of pence. As we did not require to borrow any thing, we have nothing to add. Say, 14 shillings from 11 shillings, we cannot; borrow 1 pound sterling, which is equal to 20 shillings, and 11 we have make 31 shillings. 14 from 31 leaves 17, which we write in the column of shillings, and carry 1 to the next higher number in the subtrahend. 42 and 1 are 43. 43£ from 78£ leaves 35£. The answer as found is 35£ 17s. 1d. 3 far.

Or:

Reduce the £, etc. of the minuend to farthings, and reduce the subtrahend to farthings. Subtract one

from the other.   Reduce the farthings thus found to the higher denominations for the answer.

RULE I.

Write the less denominate number under the greater, so that the units of the same denomination may stand under each other.

Subtract as in simple subtraction.

If any number in the subtrahend is greater than the corresponding number in the minuend, add to the number in the minuend as many units as makes one of the next higher denomination, and carry 1 to the next higher figure in the subtrahend after you subtract.

Proceed in this manner to the end of the sum.

RULE II.

Reduce the numbers to the lowest denomination. Subtract one from the other, and reduce the remainder to the higher denominations for the answer.

343. The pupil should be careful and make no mistakes in Long Measure, Square Measure, or any other measure having fractions.

344. To subtract denominate fractions, reduce them to the several denominations, and subtract according to Rule I. or II.

345. The same remark applies to denominate decimal fractions.

29

(1.)

	£	s.	d.	far.
From	78	10	4	1
Take	41	11	5	2
	36	18	10	3

(2.)

£	s.	d.	far.
65	6	9	1
13	7	10	2
51	18	10	3

(3.)

lbs.	oz.	dwt.	grs.
15	3	10	12
9	11	12	14
5	3	17	22

(4.)

Ton	cwt.	qr.	lbs.	oz.	drs.
17	16	2	4	0	12
7	17	3	5	1	13
9	18	2	23	14	15

(5.)

Ton	cwt.	qr.	lbs.	oz.	drs.
10	1	0	1	1	12
9	18	2	4	3	14
	2	1	21	13	14

(6.)

£	s.	d.	far.
8	10	3	2
4	11	5	3
3	18	9	3

(7.)

lbs.	oz.	dwt.	grs.
2	4	10	20
1	5	12	21
	10	17	23

(8.)

lbs.	oz.	dwt.	grs.
3	5	11	21
2	6	12	23
	10	18	22

(9.)

Ton	cwt.	qr.	lbs.	oz.	drs.
2	3	2	20	8	4
1	4	3	22	9	5
	18	2	22	14	15

(10.)

Ton	cwt.	qr.	lbs.	oz.	drs.
3	2	1	18	5	6
4	3	2	20	6	7
	18	2	22	14	15

(11.)

℔	℥	ʒ	℈	grs.
2	6	4	2	5
1	7	5	3	6
	10	6	1	19

(12.)

℔	℥	ʒ	℈	grs.
3	2	5	1	4
1	3	6	2	5
1	10	6	1	19

(13.)

m.	fur.	r.	y.	ft.	in.
2	4	10	2	1	4
1	5	11	3	2	5
	6	38	4	0	5

(14.)

m.	fur.	r.	y.	ft.	in.
4	2	5	3	2	6
3	3	6	4	3	7
	6	38	4	0	5

(15.)

L.	m.	fur.	r.	yds.	ft.	in.
1	3	4	20	2	1	8
1	2	5	21	3	2	9
		6	38	4	0	5

(16.)

sq. m.	A.	R.	r.	y.	ft.	in.
1	0	2	30	10	4	72
		3	40	20	5	100
	639	2	29	20	1	8

(17.)

sq. m.	A.	R.	r.	y.	ft.	in.
1	0	1	25	10	2	80
		1	30	5	3	100
	639	3	35	4	7	124

(18.)

sq. m.	A.	R.	r.	y.	ft.	in.
4	0	1	10	4	5	8
2	0	1	11	5	6	80
1	639	3	38	29	0	108

(19.)

Tun	pi.	hhd.	galls.	qts.	pt.	gills.
1	1	1	16	2	1	2
	1	0	30	3	0	3
1	0	0	48	3	0	2

(20.)

galls.	qrts.	pt.	gills.
16	2	1	2
5	3	0	3
10	3	0	3

(21.)

qr.	bu.	pks.	qrts.	pt.
1	4	2	4	1
	5	3	5	1
	6	2	7	0

(22.)

bu.	pk.	qt.	pt.
12	2	5	1
10	3	6	1
1	2	7	0

(23.)

yr.	mo.	da.	hrs.	mi.	sec.
4	5	4	10	20	20
3	6	5	11	30	35
10	28	22	49	45

(24.)

yrs.	mo.	da.	hrs.	mi.	sec.
4	2	12	5	10	10
2	3	15	6	11	12
1	10	26	22	58	58

(25.)

w.	da.	hr.	mi.
4	2	3	5
2	3	4	6
1	5	22	59

(26.)

circ.	S.	deg.	′	″
4	3	10	20	20
2	4	11	21	21
1	10	28	58	59

(27.)

S.	deg.	′	″
4	3	14	21
2	4	16	22
1	28	57	59

(28.)

£	s.	d.	far.
1	3	4	2
	12	5	3
	10	10	3

(29.)

circ.	S.	deg.	′	″
2	1	4	1	2
	2	5	1	3
1	10	28	59	59.

(30.)

m.	fur.	r.	yds.	ft.	in.
1	2	1	2	1	6
	4	0	3	0	7
	6	0	4	2	5

29*

LESSON XXX.

31. From $\frac{5}{6}$ mile subtract $\frac{1}{3}$ mile. *Ans.* 4 furlongs.

32. From $\frac{2}{3}$ mile subtract $\frac{1}{3}$ mile.

Ans. 2 fur. 26 r. 3 yds. 2 ft.

33. From $\frac{2}{3}$ £ subtract $\frac{1}{3}$ £.

Ans. 6 shillings 8 pence.

34. From $\frac{1}{2}$ £ subtract $\frac{1}{4}$ sh.

Ans. 9 shillings 9 pence.

35. From $\frac{1}{8}$ £ subtract $\frac{1}{8}$d.   *Ans.* 2s. 5d. 2$\frac{2}{3}$ far.

36. From $\frac{1}{4}$ £ subtract $\frac{1}{2}$d.   *Ans.* 4s. 11d. 2 far.

37. From $\frac{1}{3}$ £ subtract $\frac{1}{4}$ sh.   *Ans.* 6s. 6d.

38. From $\frac{1}{2}$ lb. subtract $\frac{1}{2}$ oz.   *Ans.* 7 oz. 8 drs.

39. From $\frac{1}{3}$ lb. subtract $\frac{1}{6}$ lb.   *Ans.* 2 oz. 10$\frac{1}{3}$ drs.

40. From $\frac{1}{4}$ lb. subtract $\frac{1}{8}$ lb.   *Ans.* 2 oz. 0 drs.

41. From $\frac{1}{5}$ lb. subtract $\frac{1}{2}$ oz. *Ans.* 2 oz. 13$\frac{11}{15}$ drs.

42. From $\frac{1}{6}$ lb. subtract $\frac{1}{2}$ oz.   *Ans.* 2 oz. 7$\frac{7}{15}$ drs.

43. From $\frac{1}{7}$ lb. subtract $\frac{1}{2}$ oz.   *Ans.* 2 oz. 2$\frac{4}{7}$ drs.

44. From $\frac{1}{4}$ ton subtract $\frac{1}{2}$ cwt. *Ans.* 4 cwt. 2 qrs.

45. From $\frac{1}{2}$ cwt. subtract $\frac{1}{2}$ lb.

Ans. 1 qr. 24 lbs. 8 oz.

46. From $\frac{1}{4}$ qr. subtract $\frac{1}{5}$ lb.

Ans. 5 lb. 14 oz. 10$\frac{2}{5}$ drs.

47. From $\frac{1}{3}$ lb. subtract $\frac{1}{6}$ oz.   *Ans.* 5 oz. 3$\frac{1}{3}$ drs.

48. From $\frac{1}{2}$ oz. subtract $\frac{1}{4}$ oz.   *Ans.* 4 drams.

49. From $\frac{1}{3}$ league subtract $\frac{1}{8}$ mile.

Ans. 7 furlongs.

50. From $\frac{1}{4}$ mile subtract $\frac{1}{2}$ yard.

Ans. 1 fur. 39 rods 5 yds.

51. From $\frac{1}{6}$ fur. subtract $\frac{1}{3}$ rod.

> *Ans.* 6 rods 1 yd. 2 ft. 6 in.

52. From $\frac{1}{8}$ rod subtract $\frac{1}{2}$ yard.   *Ans.* 6$\frac{3}{4}$.

53. From $\frac{1}{3}$ sq. mile subtract $\frac{1}{4}$ sq. mile.

> *Ans.* 106 A. 2 R. 26 r. 20 yds. 1 ft. 72 in.

54. From $\frac{1}{6}$ rood subtract $\frac{1}{2}$ rod.

> *Ans.* 7 rods 15 yds. 1 ft. 18 in.

55. From $\frac{1}{2}$ sq. yd. subtract $\frac{1}{3}$ foot.

> *Ans.* 4 feet 24 inches.

56. From $\frac{1}{3}$ sq. foot subtract $\frac{1}{4}$ inch. *Ans.* 47$\frac{3}{4}$ in.

57. From $\frac{1}{8}$ tun subtract $\frac{1}{3}$ hhd.

> *Ans.* 10 galls. 2 qts.

58. From $\frac{1}{4}$ gall subtract $\frac{1}{6}$ gill.

> *Ans.* 1 pt. 3$\frac{5}{6}$ gills.

59. From $\frac{1}{2}$ barrel subtract $\frac{1}{4}$ barrel.

> *Ans.* 7 galls. 3 qts. 1 pint.

60. From $\frac{1}{3}$ bushel subtract $\frac{1}{6}$ peck.

> *Ans.* 1 pk. 1 qr. $\frac{2}{3}$ pint.

61. From $\frac{1}{8}$ bushel subtract $\frac{1}{10}$ bushel.

> *Ans.* 3 qrts. $\frac{2}{5}$ pint.

62. From $\frac{1}{2}$ year subtract $\frac{1}{2}$ day.   *Ans.* 182 days.

63. From $\frac{1}{4}$ day subtract $\frac{1}{8}$ hour.

> *Ans.* 5 hrs. 30 mi.

LESSON XXXI.

64. From .05 £ take .005 £.

> *Ans.* 10 pence 3.2 far.

65. From .25 mile take .025 mile.

> *Ans.* 1 furlong 32 rods.

66. From .75 £ take .075 £.

> *Ans.* 13 sh. 6 pence.

67. From .05 £ take .05 sh.

> *Ans.* 11 pence 1.6 far.

68. From .525 £ take .52 sh.

> *Ans.* 9 sh. 11 pence 3.04 far.

69. From .05 lb. take .05 oz.   *Ans.* 12 drams.

70. From .025 lb. take .025 oz.   *Ans.* 6 drams.

71. From .02 lb. take 002 lb.   *Ans.* 4.608 drams.

72. From .0025 lb. take 00025 lb.

> *Ans.* 0.576 drams.

73. From .06 lb take .003 lb. *Ans.* 14.592 drams.

74. From .02 ton take .02 qr.

> *Ans.* 1 qr. 14 lbs. 8 oz.

75. From .05 cwt. take .05 lb.

> *Ans.* 4 lbs. 15 oz. 3.2 drams.

76. From .5 qr. take .05 lb.

> *Ans.* 12 lbs. 7 oz. 3.2 drams.

77. From .5 lb. take .05 drams.

> *Ans.* 7 oz. 15.95 drams.

78. From .5 oz. take .05 dram.   *Ans.* 7.95 drams.

79. From .025 mile take .025 fur.   *Ans.* 7 rods.

80. From .025 fur. take .25 yard.

> *Ans.* 5 yds. 0 ft. 9 in.

81. From .025 rod take .025 foot.   *Ans.* 4.65 in.

82. From .25 yard take .05 foot.   *Ans.* 8.4 in.

83. From .033⅓ sq. m. take .024 R.

> *Ans.* 21 A. 1 R. 13 r. 10 yd. 0 ft. 31.104 in.

84. From .024 R. take .033⅓ yd.

> *Ans.* 3.552 inches.

85. From .03⅓ yd. sq. take 033⅓ ft.

<div align="right"><i>Ans.</i> 38.4 inches.</div>

86. From .086 tun take .025 gall.

<div align="right"><i>Ans.</i> 21 galls. 1 qt. 1 pt. 3.704 gill.</div>

87. From .025 gall. take .05 barrel.   <i>Ans.</i> 0.4 gill.

88. From .05 barrel take .052 gall.

<div align="right"><i>Ans.</i> 1 gall. 1 qt. 1 pt. 4 gill .736.-</div>

89. From .02 league take .02 mile.

<div align="right"><i>Ans.</i> 12 rods 4 yds. 1 ft. 2.4 inches.</div>

90. From .02 mile take .02 fur.

<div align="right"><i>Ans.</i> 5 rods 3 yds. 0 ft. 10.8 inches.</div>

91. From .02 fur. take .02 rod.

<div align="right"><i>Ans.</i> 4 yd. 0 ft. 10.44 in.</div>

92. From .02 rod take .02 yard. <i>Ans.</i> 3.24 inches.

93. From .02 yard take .02 foot.   <i>Ans.</i> 0.48 inch.

94. From .02 foot take .02 inch.   <i>Ans.</i> 0.22 inch.

95. From .02 in. take .002 inch.   <i>Ans.</i> .018 inch.

96. From .5864 gall. take .26 gall.

<div align="right"><i>Ans.</i> 1 qt. 0 pi. 2.4448 gill.</div>

LESSON XXXII.

347. To find the difference of dates.

1. What is the difference of time between Nov. 13th, 1864 and Aug. 11th, 1869.

RULE.

Place the earlier date under the later.

Consider 30 *days as one month and* 360 *days as one year.*

Subtract one from the other by the rule.

OPERATION.		
year.	month.	day.
1869	7	11
1864	10	13

Write the year of the later date, 1869. Begin and count the number of months from January to August. Thus, January, 1; February, 2; March, 3; April, 4; May, 5; June, 6; July, 7. We find 7 months to have elapsed. Write down the figure 7 a little to the right of the year, as above, for months. Write the figure 13 a little to the right of the figure 7, as above, for the days. Find the time for the minuend, and subtract one from the other.

EXAMPLES FOR PRACTICE.

2. A note dated March 3d, 1860, became due Oct. 3d, 1868. How long was it on interest?

3. Washington was born Feb. 22d, 1732, and died Dec. 14th, 1799. How old was he when he died?

4. America was discovered by Columbus, Oct. 12th, 1492. What time elapsed from that time to the birth of Geo. Washington? To his death?

5. The Declaration of Independence was signed July 4th, 1776. What time elapsed between that event and the death of Washington?

6. How old was Washington when the colonies declared themselves free and independent?

MULTIPLICATION OF COMPOUND NUMBERS.

LESSON XXXIII.

348. Multiplication of Compound Numbers is the process of taking a given compound number any given number of times.

There are two methods. The pupil may choose that which he understands best.

As one method is a proof of the other, it would be well to make each and every sum by the two rules.

EXAMPLE.

Multiply 2£ 4s. 3d. 2 far. by 8.

OPERATION.

£	s.	d.	far.
2	4	3	2
			8
17	14	4	0

Multiply as in multiplication of abstract numbers. 8 times 2 far. are 16 far. 4 far. make a penny. 4 into 16, 4 times and 0 remaining. Write the 0 under the farthings, and carry the 4 to be added to the next result. 8 times 3d. are 24d. and 4 are 28d. 12 pence make a shilling. 12 into 28, 2 times and 4d. over. Write the 4d. under the pence, and carry

the 2 to be added to the next result.   8 times 4s. are
32s. and 2 are 34s.   20 shillings make 1 £.   20 into
34 goes 1 time and 14 shillings over.   Write the 14
under the shillings, and carry the 1 to be added to
the next result.   8 times 2£ are 16£, and 1 are 17£,
which is written under the column of £.   The an-
swer is 17£ 12s. 4d. 0 far.

349. Or, reduce the compound number to its
lowest denomination, multiply, and reduce the result
to the higher denominations.

In the above example, 2£ 4s. 3d. 2 far. are equal
to 2126 farthings, which multiplied by 8 gives 17008
farthings.   Reducing 17008 farthings, we get 17£
14s. 4d. 0 far. for the answer.

RULE.

I. *Multiply each denomination of the compound
number as in multiplication of abstract numbers.*

II. *Find how many of the next higher denomina-
tion is contained in each result, and add it to the
result found for the next higher denomination.*

Or :

*Reduce the compound number to its lowest denom-
ination, multiply the result thus found by the multi-
plier, after which reduce the result to the several higher
denominations.*

EXAMPLES FOR PRACTICE.

(1.)			(2.)			(3.)		
£	s.	d.	£	s.	d.	£	s.	d.
2	4	8	9	11	7	10	7	4
		3			4			5
6	14	0	38	6	4	51	16	8

(4.)				(5.)				
£	s.	d.	far.	Ton.	cwt.	qr.	lbs.	oz.
8	5	4	2		8	2	7	10
			6					7
49	12	3	0	3	0	0	3	6

(6.)

Ton.	cwt.	qr.	lbs.	oz.
4	15	3	8	5
				8
38	6	2	16	8

(7.)

cwt.	qr.	lbs.	oz.	dr.
9	1	8	6	4
				9
4 Ton. 4 cwt.	0 qr.	0 lbs.	8 oz.	4 dr.

(8.)

cwt.	qr.	lbs.	oz.	dr.
10	3	2	8	5
				10
5 Ton. 7 cwt.	3 qr.	0 lbs.	3 oz.	2 dr.

30

	(9.)					(10.)					
m.	fur.	r.	yd.	ft.	in.	m.	fur.	r.	y.	ft.	in.
7	7	14	1	2	1	2	3	4	2	1	8
					11						12
87	0	37	2	0	5	29	5	13	3	0	6

(11.)

L.	m.	fur.	r.	y.	ft.	in.
3	2	5	20	2	1	0
						14
54	1	5	5	5	0	6

	(12.)					(13.)		
m.	fur.	r.	y.	ft.	lb.	oz.	dwt.	gr.
8	3	30	5	2	1	10	14	20
				15				16
127	0	25	2	1	6	30	3	17	8

	(14.)				(15.)		
lbs.	oz.	dwt.	grs.	lbs.	oz.	dwt.	grs.
2	11	12	15	3	10	11	10
			17				18
39	5	14	15	69	10	5	12

	(16.)				(17.)			
lbs.	oz.	dwt.	grs.	sq. m.	A.	R.	r.	yds.
4	5	10	5	1	40	2	30	20
			19					20
84	8	13	23	21	173	3	13	6¾

(18.)

sq. m.	A.	R.	r.
2	30	3	20
			21
43	8	1	20

(19.)

sq. m.	A.	R.	r.	yds.
3	140	0	0	30
				22
70	520	0	21	24¾

(20.)

R.	r.	y.	ft.	in.
1	20	20	8	72
				23
34	35	27	8	108

(21.)

Tun.	pi.	hhd.	galls.	qt.
1	1	1	60	2
				24
47	1	1	3	0

(22.)

pi.	hhd.	galls.	qt.	pt.
1	1	30	2	1
				25
45	1	9	2	1

(23.)

hhd.	galls.	qt.	pt.	gills.
1	20	2	1	1
				26
34	33	0	0	3

(24.)

galls.	qrts.	pt.	gills.
40	2	1	2
			27
1098	2	0	2

(25.)

day.	hr.	mi.	sec.
1	3	20	40
			28
31	21	38	40

(26.)

hr.	mi.	sec.
3	20	40
		29
96	59	20

(27.)

hr.	mi.	sec.
2	20	30
		30
70	15	0

(28.)

hr.	mi.	sec.
1	20	40
		31
41	40	40

LESSON XXXIV.

350. EXAMPLES.

29. If a family consume 20 galls. 3 qts. of wine in one week, what quantity will they require for 52 weeks?   *Ans.* 17 hhd. 8 galls.

30. I have 2 dozen silver spoons, and each spoon weighs 13 oz. 7 dwt. 4 grs.; what is the weight of all?   *Ans.* 26 lbs. 8 oz. 12 dwt. 0 grs.

31. Sam. Jones bought 7 squares of ground, each square measured 21 rods, 5 yds. 2 ft. 8 in.; how much land did he buy?

Ans. 3 fur. 34 r. 2 yds. 2 ft. 2 in.

32. If the sun moves 14 S. 0° 11′ 30″ in 1 day, how far will it move in a year?

Ans. 426 C. 0 S. 9° 57′ 30″

33. If 1 dollar will buy 2 A. 1 R. 1 r. 1 yd. 1 ft. 1 in. of ground, how much land can I buy for 250 dollars?   *Ans.* 564 A. 0 R. 19 r. 5 yds. 6 ft. 70 in.

34. If a man can travel 1 mi. 7 fur. 8 rd. in one hour, how far can he travel in 24 hours?

Ans. 7 Lea. 0 mi. 4 fur. 32 rods.

35. A load of hay weighs 3 Tons, 1 cwt. 1 qr. 1 lb., how much will 25 loads weigh?

Ans. 76 Ton. 11 cwt. 2 qr. 0 lbs.

36. If it requires 12 yards, 2 qrs. to make 1 dress, how many yards will make 20 dresses? *Ans.* 250 yds.

37. If 1 boarder consumes 2 lbs. 1 oz. 3 drs. of provisions, how much will 26 boarders consume?

Ans. 2 qrs. 3 lbs. 14 oz. 14 drs.

38. Multiply 2 yds. 1 qr. 1 na. by 26.

Ans. 60 yds. 2 nails.

39. If 1 dollar will buy 7 lbs. 1 oz. 12 drs. of meat, how much can I buy for 28 dollars?   *Ans.* 87 lbs.

LESSON XXXV.

351. FRACTIONAL PARTS.

40. Multiply $\frac{1}{3}$ mile by 2.

Ans. 5 fur. 13 r. 1 yd. 2 ft. 6 in.

41. Multiply $\frac{1}{4}$ mile by 3.   *Ans.* 6 fur.

42. Multiply $\frac{1}{8}$ mile by 3.   *Ans.* 3 fur.

43. Multiply $\frac{1}{9}$ fur. by 5. *Ans.* 22 r. 1 yd. 0 ft. 8 in.

44. Multiply $\frac{1}{12}$ £ by 3.   *Ans.* 5s.

45. Multiply $\frac{1}{8}$ £ by 5.   *Ans.* 12s. 6d.

46. Multiply $\frac{1}{3}$ Tun by 2.   *Ans.* 1 pipe 42 galls.

47. Multiply $\frac{1}{4}$ Tun by 3.   *Ans.* 1 pipe 1 hhd.

48. Multiply .002 £ by 25.   *Ans.* 1s.

49. Multiply .02 £ by 20.   *Ans.* 8s.

50. Multiply .2s. by 13.   *Ans.* 2s. 7d. $\frac{8}{10}$ far.

51. Multiply .025 mile by 13.   *Ans.* 2 fur. 24 rds.

52. Multiply .025 fur. by 13.   *Ans.* 13 rods.

53. Multiply .025 rod by 15.

Ans. 2 yds. 0 ft. 2.25 in.

54. Multiply .03 gall. by 12.

Ans. 1 qt. 0 pt. 3.52 gill.

55. Multiply .003 hhd. by 2.

Ans. 1 qt. 1 pt. .096 gill.

56. Multiply .08 League by 20.

Ans. 1 L. 1 mi. 6 fur. 16 r.

57. Multiply .008 League by 25.

Ans. 4 fur. 32 rods.

58. Multiply .018 Ton by 12.

Ans. 4 cwt. 1 qr. 7 lbs.

59. Multiply .025 qr. by 3.   *Ans.* 1 lb. 14 oz.

60. Multiply .086 cwt. by 4.

Ans. 1 qr. 9 lbs. 6 oz. 6 drs. $\frac{4}{16}$.

61. Multiply .08 lbs. by 2.   *Ans.* 2 oz. 8.96 drs.

62. Multiply .025 £ by 3.   *Ans.* 1s. 6d.

63. Multiply .025 mile by 30.   *Ans.* 6 furlongs.

64. Multiply .02 fur. by 36.

Ans. 28 r. 4 yds. 1 ft. 24 in.

65. Multiply .025 rod by 38.

Ans. 5 yds. 0 ft. 8.1 in.

66. Multiply .0625 League by 3.

Ans. 4 fur. 20 rods.

67. Multiply .08 League by 200. *Ans.* 16 Leagues.

DIVISION OF COMPOUND NUMBERS.

LESSON XXXVI.

352. Divide 2 £ 3s. 4d. and 3 far. by 5.

5)2 £ 3s. 4d. 3 far.(8s. 8d. 0 far. $\frac{3}{5}$.

$2 \times 20 = 40$

5)43

40

$3 \times 12 = 36$

5)40

40

5)3 far.

5 into 2 £ cannot be divided, so that we must reduce the pounds to shillings.   2 pounds is equal to 40 shillings, and 3 shillings, are 43 shillings ; 5 into 43 shillings, 8 times, or 8 shillings, and 3 shillings remaining.   Reduce the 3 shillings to pence, and add the pence thus found to the pence in the dividend—3 shillings=36 pence and 4 pence=40 pence. 5 into 40 pence, 8 times or 8 pence, and 0 remaining. 5 into 3 far., 0 farthings, and $\frac{3}{5}$ remaining.   The answer is 8s. 8d. 0$\frac{3}{5}$ far.

Or :

Reduce the compound number to the lowest denomination.   Divide the number found by the

given divisor, and reduce the quotient to the several higher denominations.

$$2 \text{ £ } 3s. \text{ } 4d. \text{ } 3 \text{ far.} = 2083 \text{ farthings.}$$

$$2083 \text{ far.} \div 5 = 416\tfrac{2}{3} \text{ farthings.}$$

$$
\begin{array}{l}
4)416\tfrac{2}{3} \\
\hline
12)\overline{104} \quad - \quad 0\tfrac{2}{3} \text{ far.} \\
\hline
\quad 8s. \text{ } 8d.
\end{array}
$$

R U L E.

Divide each denomination in its order, as in simple numbers, beginning with the highest. If there is a remainder, reduce the remainder to the next lowest denomination, and add in the number, if any, of this denomination found in the dividend. Divide as before.

Continue until the several denominations of the dividend are divided. The quotient thus found will be the answer.

Or :

353. *Reduce the dividend to its lowest denomination. Divide this result by the divisor, and reduce the quotient to the several higher denominations.*

(I would again caution the pupil in regard to Long Measure and Square Measure, when dividing by fractional parts.)

1. Divide 3£ 4s. 6d. 2 far. by 3.

Ans. 1£ 1s. 6d. ⅔ far.

2. Divide 2£ 5s. 8d. 3 far. by 4.

Ans. 11s. 5d. ¾ far.

3. Divide 1£ 6s. 4d. 2 far. by 5.

Ans. 5s. 3d. 1⅕ far.

4. Divide 3£ 9s. 7d. 3 far. by 6.

Ans. 11s. 7d. 1⅙ far.

5. Divide 4£ 8s. 10d. 2 far. by 7.

Ans. 12s. 8d. 1¾ far.

6. Divide 6£ 2s. 1d. 3 far. by 8.

Ans. 15s. 3d. ⅞ far.

7. Divide 4£ 8s. 6d. 3 far. by 9.

Ans. 9s. 10d. ⅓ far.

8. Divide 2£ 7s. 6d. 3 far. by 10.

Ans. 4s. 9d. ³⁄₁₀ far.

9. Divide 1£ 2s. 3d. 2 far. by 11.

Ans. 2s. 0d. 1³⁄₁₁ far.

10. Divide 2£ 3s. 4d. 2 far. by 12.

Ans. 3s. 7d. 1½ far.

11. Divide 17 hhds. 8 galls. by 52.

Ans. 20 galls. 3 quarts.

12. Divide 26 lbs. 8 oz. 12 dwt. 0 grs. by 24.

Ans. 1 lb. 1 oz. 7 dwt. 4 grs.

13. Divide 3 fur. 34 r. 2 y. 2 ft. 2 in. by 7.

Ans. 21 rods 5 yds. 2 ft. 8 in.

14. Divide 426 circ. 0 S. 9° 57′ 30″ by 365.

Ans. 14 S. 0° 11′ 30″.

15. Divide 564 A. 0 R. 19 r. 5 y. 6 ft. 70 in. by 250.

Ans. 2 A. 1 R. 1 rod 1 yd. 1 ft. 1 in.

16. Divide 7 lea. 0 m. 4 fur. 32 rods by 24.

Ans. 7 fur. 8 rods.

17. Divide 76 Ton 11 cwt. 2 qrs. by 25.

Ans. 3 Tons 1 cwt. 1 qr. 1 lb.

18. Divide 250 yards by 20. *Ans.* 12 yards 2 qrs.

19. Divide 2 qrs. 3 lbs. 14 oz. 14 drs. by 26.

Ans. 2 lbs. 1 oz. 3 drams.

20. Divide 60 yards 2 nails by 26.

Ans. 2 yds. 1 qr. 1 na.

21. Divide 17£ 14s. 4d. by 8.

Ans. 2£ 4s. 3d. 2 far.

22. Divide 3 Tons 3 lbs. 6 oz. by 7.

Ans. 8 cwt. 2 qrs. 7 lbs. 10 oz.

23. Divide 38 Ton 6 cwt. 2 qr. 16 lbs. 8 oz. by 8.

Ans. 4 Ton 15 cwt. 3 qrs. 8 lbs. 5 oz.

24. Divide 4 Ton 4 cwt. 8 oz. 4 drs. by 9.

Ans. 9 cwt. 1 qr. 8 lbs. 6 oz. 4 drs.

25. Divide 5 Ton 7 cwt. 3 qrs. 3 oz. 2 drs. by 10.

Ans. 10 cwt. 3 qr. 2 lb. 8 oz. 5 drms.

26. Divide 87 mi. 37 rods 2 yds. 5 in. by 11.

Ans. 7 mi. 7 fur. 14 r. 1 yd. 2 ft. 1 in.

27. Divide 29 mi. 5 fur. 13 r. 3 y. 6 in. by 12.

Ans. 2 mi. 3 fur. 31 rods 0 y. 2 ft. 2 in.

28. Divide 54 Lea. 1 mi. 5 fur. 5 r. 5° yd. 6 in. by 14. *Ans.* 3 Lea. 2 mi. 5 fur. 20 rods 2 yd. 1 ft. 0 in.

29. Divide 127 mi. 25 r. 2 yd. 1 ft. 6 in. by 15.

Ans. 8 mi. 3 fur. 31 r. 0 yd. 0 ft. 6 in.

30. Divide 30 lbs. 3 oz. 17 dwt. 8 grs. by 16.

Ans. 1 lb. 10 oz. 14 dwt. 20 grs.

31. Divide 39 lbs. 5 oz. 14 dwt. 15 grs. by 17.

Ans. 2 lbs. 3 oz. 17 dwt. $7\frac{11}{17}$ grs.

32. Divide 69 lbs. 10 oz. 5 dwt. 12 grs. by 18.

Ans. 3 lbs. 10 oz. 11 dwt. 10 grs.

33. Divide 84 lbs. 8 oz. 13 dwt. 23 grs. by 19.

Ans. 4 lbs. 5 oz. 10 dwt. 5 grs.

34. Divide 34 R. 35 r. 27 y. 8 ft. 108 in. by 23.

Ans. 1 R. 20 rods 20 yds. 8 ft. 72 in.

35. Divide 1098 galls. 2 qts. 2 gills by 27.

Ans. 40 galls. 2 qts. 1 pt. 2 gills.

36. Divide 41 hrs. 40 mi. 40 sec. by 31.

Ans. 1 hr. 20 mi. 40 sec.

37. Divide 70 mi. 520 A. 21 r. 24¾ yds. by 22.

Ans. 3 mi. 140 A. 0 R. 0 r. 30 yd.

38. Divide 34 hhd. 33 galls. 3 gills. by 26.

Ans. 1 hhd. 20 gall. 2 qts. 1 pt. $1\frac{1}{26}$ gi.

39. Divide 70 hrs. 15 mi. by 30.

Ans. 2 hrs. 20 mi. 30 sec.

40. Divide 43 mi. 8 A. 1 R. 20 rd. by 21.

Ans. 2 mi. 30 A. 3 R. 20 rods.

41. Divide 45 pi. 1 hhd. 9 gall. 2 qt. 1 pt. by 25.

Ans. 1 pi. 1 hhd. 40 gall. 2 qt. $1\frac{8}{25}$ pt.

42. Divide 96 hrs. 59 mi. 20 sec. by 29.

Ans. 3 hrs. 20 mi. 40 sec.

43. Divide 21 mi. 173 A. 3 R. 13 r. 6¾ y. by 20.

Ans. 1 mi. 40 A. 2 R. 30 r. 20 yd.

44. Divide 47 Tun 1 pi. 1 hd. 3 gall. by 24.

Ans. 1 Tun 1 pi. 1 hhd 60 gall. 2 qt.

45. Divide 31 day 21 hrs. 38 mi. 40 sec. by 28.

Ans. 1 da. 3 hrs. 20 mi. 40 sec.

LESSON XXXVII.

354. EXAMPLES IN THE FOREGOING RULES.

1. What cost 39 A. 2 R. 15 r. of land at $13.98 per acre.   *Ans.* $553.52+

2. What cost 176 A. 3 R. 25 r. of land at $75. per acre?   *Ans.* $13267.96+

3. What cost 20 A. 2 R. 24 rods of land at $3 per acre?   *Ans.* $61.95.

4. What cost 10 yd. 3 qr. 2 na. of silk at 80 cents per yard?   *Ans.* $8.70.

5. What cost 15 yd. 2 qr. 3 na. of cloth at $2.50 per acre?   *Ans.* $39.21+.

6. What cost 25 yd. 1 qr. 3 na. silk @ 50 cents per yard?   *Ans.* $12.71+.

7. What cost 67 bu. 3 pk. 7 qt. potatoes at $2.50 per bushel?   *Ans.* $169.92.

8. What cost 125 bu. 3 pk. 1 qt. of wheat at 87 cents per bushel?   *Ans.* $109.42.

9. What cost 25 bushels 1 pk. 3 qt. of onions at $2.50 per bushel?   *Ans.* $63.36.

10. What cost 503 bu. 4 qts. corn at 43 cents per bushel?   *Ans.* $216.34.

11. What cost 76 bu. 1 qt. of beans at 66⅔ cents per bushel?   *Ans.* $50.68.

12. What cost 10 bu. 3 pk. of pears at $1.50 per bushel?   *Ans.* $16.125.

13. What cost 25 bu. 1 pk. of corn at $1.35 per bushel?   *Ans.* $34.08¾.

14. What cost 1 bu. 1 pk. 1 qt. peas at $1.50 per bushel? *Ans.* $1.92 +.

15. What cost 17 cwt. 3 qrs. 23 lbs. hay at $13.50 per Ton? *Ans.* $1.11 +.

16. What cost 3 Ton 10 cwt. 3 qr. of iron at $30 per Ton? *Ans.* $106.12½.

17. What cost 16 boxes sugar, each box containing 4 cwt. 3 qr. 18 lbs. at $6 per cwt.? *Ans.* $473.28.

18. What will 9 boxes sugar cost, each weighing 8 cwt. 2 qr. 12 lbs., at $12.50 per cwt.? *Ans.* $969.75.

19. If I buy 16 cwt. 3 qr. 21 lbs. 6 oz. sugar, at $7.50 per cwt., what will my bill amount to?
Ans. $127.22.

20. I sold 5 cwt. 2 qr. hay, at $27.50 per Ton. What was the amount of my sale? *Ans.* $7.36½.

21. If I can ride 5 fur. 3 r. 10 ft. 6 in. in a steam-car for 22 cents per mile, what ought to be my fare?
Ans. 14 cents.

22. To every one of my 10 children I gave 26£ 14s. 8d. 3 far. How much did all of them receive?
Ans. 2£ 13s. 5d. 2$\frac{7}{10}$ far.

23. If I can travel 1 mile 1 fur. 1 rod 1 ft. 1 in. in one day, how far can I travel in 20 days?
Ans. 22 mi. 4 fur. 21 rods 1 yd. 1 ft. 2 in.

24. Add together 4 feet 3 in. *Ans.* 51 inches.

25. Reduce each of the following to a compound number: 3.250£, 32.5£, 325.2£. *Last Ans.* 325£ 4s.

26. Reduce 1 hour to the decimal of a day.
Ans. 0.0416⅔.

27. Reduce 1 farthing to the decimal of a £.
Ans. .001041 +.

31

28. Reduce 1 foot to the decimal of a yard.

Ans. 0.33⅓.

29. Reduce 1 cent to the decimal of a dollar.

Ans. .01.

30. Reduce 1 shilling to the decimal of a £.

Ans. .05£.

31. How many inches in 1 mile ?   *Ans.* 63360.

32. How many inches in 1 square mile ?

Ans. 4014489600.

33. How many miles in 2534400 inches ?

Ans. 40 miles.

CHAPTER XV.

DUODECIMALS.

LESSON I.

355. DUODECIMALS increase and decrease in a twelvefold ratio.   The table is uniform, it requiring 12 of a lower denomination to make 1 of a higher. The highest denomination is the foot, then comes primes, seconds, thirds, fourths, etc., etc.

356. The number of commas above the figure shows its denomination.   Thus, 4′ is read 4 primes, 3″ is read 3 seconds, 8‴ is read 8 thirds, 9⁗ is read 9 fourths, etc.

TABLE.

12 thirds (''') make 1 second.

12 seconds   "   1 prime.

12 primes   "   1 foot.   ft.

357. In multiplication, the indices are added, and written over the product. Thus, $6'' \times 3' = 18'''$. That is, 6 seconds multiplied by 3 primes is equal to 18 thirds.

358. In division, the index of the divisor is subtracted from the index of the dividend. Thus, $18''' \div 6'' = 3'$. That is, 18 thirds divided by 6 seconds is equal to 3 primes.

359. NOTE.—The above table may be carried out as far as the pupil may in his judgment see proper. The name given to the other denominations is determined by the number of strokes or commas written over it. Thus, $6''''''$ is read 6 sevenths, $8'''''''''$ is read 8 tenths, etc. In calculating, it will be correct enough to bring the answer to thirds. One third is $\frac{1}{1728}$ part of a foot, which is small enough for any answer.

LESSON II.

360. ADDITION OF DUODECIMALS.

For addition of Duodecimals the same rule applies as for addition of denominate numbers. The scale is uniform, every higher denomination requiring 12 of a lower one up to feet. The table then corresponds to Long measure.

EXAMPLE.

Add	2 ft.	3′	4″
	6 ft.	8	9
	12 ft.	3	8
	21	4	1

Adding the first column of seconds we find 25. 12 seconds make 1 prime. 12 into 25, 2 times and 1 sec. remaining. Write the 1 sec. under the column of seconds, and carry the 2 primes to the column of primes. Adding the second column, gives 14, and 2 we had to carry, makes 16. 12 primes make one foot. 12 into 16, once and 4 over. Write the 4 remaining primes under the column of primes, and carry 1 to the next column. Adding the column of feet gives 20 feet, and 1 foot makes 21 feet. Write the sum under the column of feet, and the required answer will be 21 ft. 4′ 1″.

361. Masons and artists use this measure, and calculate their work by the foot, prime and second. The yard is very seldom make use of, except in cases where work is taken by the yard, such as paving streets, building stone bridges, etc., etc.

EXAMPLES.

	(1)				(2)				(3)	
ft.	′	″		ft.	′	″		ft.	′	″
2	4	6		1	1	1		4	4	4
4	5	9		2	2	2		2	6	8
8	7	6		3	3	3		3	7	9
4	3	2		4	4	4		4	8	7
9	8	7		5	5	5		5	9	6
8	6	4		6	6	6		21	0	10
37	11	10		7	7	7				
				30	6	4				

	(4)				(5)				(6)	
ft.	′	″		ft.	′	″		ft.	′	″
3	2	8		4	9	3		6	7	8
1	4	3		3	9	4		9	4	3
9	7	6		9	4	3		2	8	7
8	2	4		3	4	9		4	3	2
6	4	8		2	1	2		9	8	4
28	9	5		23	4	9		32	8	0

	(7)				(8)				(9)	
ft.	′	″		ft.	′	″		ft.	′	″
5	8	3		7	2	4		1	4	6
2	7	9		2	9	9		2	8	4
6	4	3		8	7	6		5	6	7
9	8	7		3	8	4		8	9	4
4	3	2		9	8	7		3	8	2
28	8	0		32	0	6		21	7	11

	(10)			(11)			(12)	
ft.	′	″	ft.	′	″	ft.	′	″
5	3	8	4	6	2	17	3	8
4	6	9	2	4	6	9	2	4
2	8	7	8	9	7	4	3	1
9	5	4	3	4	1	8	7	6
13	12	6	9	8	2	2	4	8
						9	3	1
36	0	10						

	(13)			(14)			(15)	
ft.	′	″	ft.	′	″	ft.	′	″
7	4	3	1	3	6	2	6	8
2	9	8	2	8	9	9	4	9
5	3	8	7	4	3	2	1	8
7	2	9	9	8	7	9	7	8
8	4	3	6	8	9	8	1	9

	(16)			(17)			(18)	
ft.	′	″	ft.	′	″	ft.	′	″
2	8	7	4	2	3	7	2	6
3	4	6	3	6	9	2	9	8
9	8	7	1	2	3	3	4	7
2	4	9	2	4	8	2	8	4
8	7	3	7	6	4	7	6	3

	(19)			(20)			(21)	
ft.	′	″	ft.	′	″	ft.	′	″
7	4	2	17	2	9	8	7	6
3	2	1	3	8	7	1	2	3
2	8	7	2	9	4	4	5	6
9	4	3	1	2	3	7	8	9
8	10	11	4	5	6	10	11	10

	(22)			(23)			(24)	
ft.	′	″	ft.	′	″	ft.	′	″
1	2	3	7	6	4	7	4	6
4	5	6	2	4	6	2	8	9
7	8	9	2	8	6	9	4	3
10	11	10	4	9	7	3	2	8
9	8	7	4	3	9	9	7	6

	(25)			(26)			(27)	
ft.	′	″	ft.	′	″	ft.	′	″
4	8	7	6	8	7	7	9	10
6	4	3	2	9	8	1	2	3
9	8	2	1	9	9	4	5	6
4	6	1	3	4	6	7	8	9
3	1	8	7	2	8	10	11	11

	(28)			(29)			(30)	
ft.	′	″	ft.	′	″	ft.	′	″
2	4	8	1	2	3	9	8	7
1	2	4	4	5	6	6	5	4
4	2	1	7	8	9	1	2	3
6	7	8	10	11	12	3	2	1
7	8	6	1	2	3	9	8	9

	(31)			(32)			(33)	
ft.	′	″	ft.	′	″	ft.	′	″
7	6	3	8	9	3	9	8	7
2	8	7	2	4	6	2	1	2
2	4	9	3	6	9	3	1	4
12	8	7	1	2	3	4	1	5
6	4	3	1	3	2	5	1	6

	(34)			(35)			(36)	
ft.	′	″	ft.	′	″	ft.	′	″
2	6	8	8	7	6	9	8	7
9	8	12	2	4	6	2	9	3
12	1	3	8	9	4	8	7	9
4	2	6	2	6	3	12	2	8
6	8	9	1	4	5	4	1	9

(37)		
ft.	′	″
7	2	3
1	2	9
1	8	6
1	4	3
1	6	3

(38)		
ft.	′	″
3	8	7
2	1	4
1	9	7
6	2	1
8	9	7

(39)		
ft.	′	″
1	1	1
2	2	2
3	3	3
5	5	5
4	4	4
6	6	6
7	7	7
8	8	8

(40)		
ft.	′	″
2	3	6
2	3	6
2	3	6
2	3	6
2	3	6
2	3	6
2	3	6
2	3	6

(41)		
ft.	′	″
7	6	9
9	6	7
6	7	9
9	7	6
2	4.	8
8	2	4
1	2	3

(42)		
ft.	′	″
1	2	3
4	5	6
7	8	9
10	11	1
2	3	4
5	6	7
8	9	10
11	1	2

(43)		
ft.	′	″
1	2	6
1	2	3
6	5	4
7	8	9
3	2	1
4	5	6
9	8	7

LESSON III.

362. SUBTRACTION OF DUODECIMALS.

In subtraction of duodecimals the same rule applies as in subtraction of denominate numbers.

EXAMPLE.

From 24 ft. 8′ 2″ take 8 ft. 9′ 4″.

OPERATION.

$$
\begin{array}{rrr}
24 \text{ ft.} & 8' & 2'' \\
8 & 9 & 4 \\
\hline
15 & 10 & 10
\end{array}
$$

Begin at the right hand column and subtract. 4″ from 2″, you cannot subtract, borrow 1′, which is equal to 12″. Add the 12″ to the 2″, making 14″. 4″ from 14″ leaves 10″. Write the 10″ under the column, and return the 1′ you borrowed. Add it to the subtrahend, which now becomes (9′ + 1′) 10′. Subtract the second column. 10′ from 8′ you cannot subtract. Borrow 1 foot, which is equal to 12′. Add the 12′ and the 8′. 12′ + 8′ = 20. Now subtract. 10′ from 20′ leaves 10′. Write the 10′ under the column of primes, and add 1 ft. to the subtrahend in the column of feet. 1 ft. and 8 ft. are 9 ft. 9 ft. from 24 ft. leaves 15 ft. The required answer is 15 ft. 10′ 10″.

EXAMPLES.

1. From	14 ft.	8′	6″
Take	10 ft.	3′	4″
	4	5	2

2. From	28 ft.	9′	8″
Take	14 ft.	8′	7″
	13	9	8

3. From	16 ft.	9′	8″
Take	3 ft.	8′	9″
	13	0	11

4. From	86 ft.	4′	9″
Take	78 ft.	6′	10″
	7	9	11

5. From	16 ft.	3′	8″
Take	2 ft.	4′	9″
	13	10	11

6. From	20 ft.	4′	6″
Take	18 ft.	5′	8″
	1	10	10

7. From	22 ft.	6′	8″
Take	21 ft.	7′	9″
		10	11

8. From	34 ft.	6′	2″
Take	33 ft.	7′	3″
		10	11

9. From	18 ft.	2′	1″
Take	4 ft.	3′	2″
	13	10	11

		ft.	'	''
10.	From	18 ft.	9'	7''
	Take	12 ft.	10'	8''
		5	10	11
11.	From	26 ft.	4'	6''
	Take	18 ft.	3'	9''
12.	From	16 ft.	10'	8''
	Take	14 ft.	9'	8''
13.	From	22 ft.	6'	8''
	Take	18 ft.	5'	9''
14.	From	18 ft.	4'	3''
	Take	12 ft.	9'	4''
15.	From	27 ft.	5'	8''
	Take	26 ft.	6'	9''
16.	From	16 ft.	4'	8''
	Take	10 ft.	3'	9''
17.	From	23 ft.	4'	8''
	Take	22 ft.	5'	9''
18.	From	28 ft.	7'	9''
	Take	18 ft.	9'	8''
19.	From	16 ft.	3'	8''
	Take	15 ft.	4'	7''

		ft.		′		″		‴		⁗		⁗′

20.	From	291 ft.		8′		6″						
	Take	17 ft.		9′		8″						
21.	From	19 ft.		8′		3″						
	Take	18 ft.		9′		2″						
22.	From	18 ft.		8′		9″		10‴				
	Take	14 ft.		8′		10″		9‴				
23.	From	16 ft.		2′		4″						
	Take	14 ft.		8′		9″						
24.	From	2 ft.	3′		8″		7‴		6⁗		7⁗′	
	Take	1 ft.	4′		6″		9‴		7⁗		8⁗′	
25.	From	16 ft.		4′		3″						
	Take	12 ft.		6′		8″						
26.	From	6 ft.		4′		7″		0‴		0⁗		
	Take	2 ft.		3′		2″		8‴		9⁗		
27.	From	24 ft.		8′		7″						
	Take	13 ft.		4′		8″						
28.	From	17 ft.		2′		6″		7‴		8⁗		
	Take	3 ft.		4′		8″		9‴		8⁗		
29.	From	16 ft.		3′		8″						
	Take	12 ft.		4′		9″						
30.	From	16 ft.		3′		8″		7‴		9⁗		
	Take	4 ft.		5′		6″		9‴		10⁗		

LESSON IV.

363. MULTIPLICATION OF DUODECIMALS.

The same rule applies as for multiplication of denominate numbers.

EXAMPLE.

Multiply 3 ft. 4′ 6″ by 8.

Multiply the seconds. 8 × 6″ = 48″. 12″ = 1′. 12 into 48, 4 times and 0″ remaining. Write the 0″ under the seconds and carry the 4 primes.   Multiply   the   primes. 8 × 4′ = 32′ and 4′ are 36′.   12′

OPERATION.
3 ft.   4′   6″
8
27 ft.   0′   0″

make 1 foot.   12 into 36, 3 times and 0′ remaining. Write the 0′ under the primes and carry the 3 feet. Multiply the feet. 8 × 3 = 24, and 3 are 27 ft. Write 27 under the column of feet. The answer is 27 feet 0 primes 0 seconds.

EXERCISES.

1. Multiply 2 ft. 4′ 6″ by 3.   *Ans.* 7 ft. 1′ 6″.
2. Multiply 3 ft. 5′ 3″ by 4.   *Ans.* 13 ft. 9′ 0″.
3. Multiply 4 ft. 6′ 4″ by 5.   *Ans.* 22 ft. 7′ 8″.
4. Multiply 5 ft. 7′ 5″ by 6.   *Ans.* 33 ft. 8′ 6″.
5. Multiply 6 ft. 8′ 6″ by 7.   *Ans.* 46 ft. 11′ 6″.
6. Multiply 7 ft. 9′ 7″ by 8.   *Ans.* 62 ft. 4′ 8″.
7. Multiply 8 ft. 10′ 8″ by 9.   *Ans.* 80 ft. 0′ 0″.
8. Multiply 9 ft. 11′ 9″ by 3.   *Ans.* 29 ft. 11′ 3″.
9. Multiply 10 ft. 1′ 10″ by 4. *Ans.* 40 ft. 7′ 4″.

10. Multiply 11 ft. 2' 11'' by 5.   *Ans.* 56 ft. 2' 7''.
11. Multiply 12 ft. 3' 2'' by 6.   *Ans.* 73 ft. 7' 0''.
12. Multiply 13 ft. 4' 3'' by 7.   *Ans.* 93 ft. 5' 9''.
13. Multiply 14 ft. 5' 4'' by 8.   *Ans.* 115 ft. 6' 8''.
14. Multiply 15 ft. 6' 5'' by 9.   *Ans.* 139 ft. 9' 9''.
15. Multiply 16 ft. 7' 6'' by 10. *Ans.* 166 ft. 3' 0''.
16. Multiply 17 ft. 8' 7'' by 11.

Ans. 194 ft. 10' 5''.

17. Multiply 18 ft. 9' 8'' by 12. *Ans.* 225 ft. 8' 0''.

LESSON V.

CASE II.

364. TO FIND THE AREA OF ANY PARALLELOGRAM.

RULE.

Multiply the length by the breadth.

EXAMPLE.

1. Find the area of a room 18 ft. 4' 3'' long and 12 ft. 6' 4'' wide.

OPERATION.

		18 ft.	4'	3''	
		12 ft.	6'	4''	
		72''	16'''	12''''	
	108'	24''	18'''.		
216	48'	36''			
229	11'	2''	10'''	0''''.	

Analyse as follows: Seconds by seconds gives ('''') fourths. $4 \times 3 = 12''''$. Seconds by primes gives (''') thirds. $4 \times 4 = 16$. Seconds by feet give seconds. $4'' \times 18$ ft.$= 72$. Multiply by the 6'. Primes by seconds gives (''') thirds. $6' \times 3'' = 18'''$. Write the 18''' under the 16'''. 6' by $4' = 24''$ seconds. Write the 24'' under the 72''. $6' \times 18$ ft.$= 108'$. Multiply by the 12 ft. Feet by seconds gives seconds. 12 ft.$\times 3'' = 36''$. Write the 36 under the column of seconds. Feet by primes gives primes. $12 \times 4' = 48'$. Write the 48' under the column of primes. Feet by feet give square feet $12 \times 18 = 216$ feet. Add the several products beginning at the 12''''. 12'''' make 1'''. 12 into 12 once and 0'''' remaining. Add the thirds. $16 + 18 = 34'''$. $12 = 1$. 12 into 34 2 times and 10''', over. Write the 10''' under the column of thirds. Add the column of seconds. $72 + 24 + 36 = 132$ and 2 to carry makes 134''. $12 = 1$. 12 into 134 11 times and 2'' over. Write the 2'' under the column of seconds. Add the column of primes. $108 + 48 = 156$ and $11 = 167$. 12' make 1 foot. 12 into 167 13 times and 11' remaining. Write the 11' under the column of primes and carry the 13 to the next column. $216 + 13 = 229$ square feet. The answer is 229 sq. ft. 11' 2'' 10'''.

1. A school-room is 28 ft. 4' in length, and 18 ft. 3' in breadth. How many square feet of flooring is required for the room? *Ans.* 517 ft. 1'.

2. If the above flooring is worth 12½ cents a foot, how much would it cost?   *Ans.* $64.33 +.

3. How much would it cost to plaster the ceiling of that room at 25 cents per foot?   *Ans.* $129.26 +.

4. A ball-room is 108 ft. 3′ in length, and 84 ft. 3′ in breadth. How many square feet of plastering would be required to cover the ceiling?

Ans. 9120 ft. 0′ 9″.

5. How much would the above plastering cost, at 30 cents a foot?   *Ans.* $2736.002 +.

6. If the room was 40 feet high, how many ft. of plastering would be required to cover the 4 sides, providing there were no openings?   *Ans.* 15400 ft.

7. How much would the plastering of the ceiling and the 4 sides cost, at $1 per square yard?

Ans. $2724.44 +.

8. Suppose there were 6 openings on each side of the ball-room, 4 feet wide and 9 ft. 3′ high; how many yards of plastering would be required to cover the ball-room?   *Ans.* 2625 yds. 7 ft. 6′ 9″.

9. How much would the plastering of the room cost, less the openings, @ $1.25 per square yard?

Ans. $3282.22⅓ +.

10. A yard is 200 feet long, by 48 feet wide; how many square feet in that yard?   *Ans.* 9600 ft.

11. How many bricks would cover that yard, allowing 40 bricks to the square yard? How much would the bricks cost at $12½ per thousand?

1st *Ans.* 42566⅔ bricks.   2d *Ans.* $532.08.

12. My fence is 200 feet long by 12 feet high. How many square feet of lumber in my fence, and what is it worth at $13 per thousand feet?

Ans. $31.20

13. A plantation has a front of 28 acres by a depth of 200 acres. How many acres in that plantation? *Ans.* 5600 acres.

14. A mason undertakes to build the walls of a fort for $33⅓ per square yard. The fort is 690 ft. 8′ square, and 10 ft. high. How much is his contract?

Ans. $102320.987+.

15. A brick is 8 inches long and 4 inches wide. How many bricks do I require to pave a sidewalk 300 ft. long by 12 ft. wide? *Ans.* 16200 bricks.

16. How many bricks do I require to pave a yard 90 feet long and 40 feet wide? *Ans.*

17. A city railroad is 2¼ miles long. It is required to pave 12 feet in width across the track. How much would the paving cost at 45 cents a yard? If the stringers of each track were 8 inches wide, how many yards should be deducted from the superficial measurement?

$7040, 1st *Ans.*   1955⅚. 2d *Ans.*

LESSON VI.

365. DIVISION OF DUODECIMALS.

1. Divide 34 ft. 5′ 6″ by 3.   *Ans.*

Reduce feet and primes to seconds:

$12 \times 34 = 408$, $408 + 5 = 413′$
$12 \times 413′ = 4956$, $4956 + 6 = 4962″$.

$4962 \div 3 = 1654''$, reduced gives 11 ft. 5' 10''

Or:

Divide as follows:

```
3)34 ft. 5' 6''(11 ft. 5' 10''
  33
  ──
   1
  12
  ──
  12
   5
  ──
  17
  15
  ──
   2
  12
  ──
  24
   6
  ──
  30
  30
  ──
   ×
```

Analysis of the above sum:

3 into 34 ft., 11 times, 11 times 3 are 33, 33 from 34 leaves 1 foot. 1 foot reduced to primes is equal to $12 \times 1 = 12'$, and 5', are 17 primes.   3 into 17 primes, 5 times, 5 times 3 are 15, 15 from 17 leaves

2. 2 primes reduced to seconds gives 2×12=24 seconds, and 6 seconds make 30 seconds. 3 into 30, 10 times, 10 times 3 are 30, 30 from 30 leaves 0.

The answer is 11 ft. 5' 10''.

EXERCISES.

1.	Divide 14 ft. 6' 8'' by 2.	*Ans.* 7 ft. 3' 4''.
2.	Divide 21 ft. 9' 6'' by 3.	*Ans.* 7 ft. 3' 2''.
3.	Divide 16 ft. 4' 8'' by 4.	*Ans.* 4 ft. 1' 2''.
4.	Divide 20 ft. 6' 8'' by 5.	*Ans.* 4 ft. 1' 4''.
5.	Divide 36 ft. 6' 6'' by 6.	*Ans.* 6 ft. 1' 1''.
6.	Divide 42 ft. 7' 8'' by 7.	*Ans.* 6 ft. 1' 1¼''.
7.	Divide 16 ft. 8' 8'' by 8.	*Ans.* 2 ft. 1' 1''.
8.	Divide 27 ft. 9' 9'' by 9.	*Ans.* 3 ft. 1' 1''.
9.	Divide 20 ft. 10' 10'' by 10.	*Ans.* 2 ft. 1' 1''.
10.	Divide 25 ft. 11' 11'' by 11.	*Ans.*
11.	Divide 24 ft. 8' 8'' by 12.	*Ans.*
12.	Divide 26 ft. 3' 3'' by 13.	*Ans.*
13.	Divide 14 ft. 6' 6'' by 14.	*Ans.*
14.	Divide 15 ft. 4' 4'' by 15.	*Ans.*
15.	Divide 48 ft. 3' 3'' by 16.	*Ans.*
16.	Divide 34 ft. 6' 6'' by 17.	*Ans.*
17.	Divide 36 ft. 4' 4'' by 18.	*Ans.*
18.	Divide 38 ft. 5' 6'' by 19.	*Ans.*
19.	Divide 44 ft. 6' 6'' by 20.	*Ans.*
20.	Divide 38 ft. 6' 6'' by 21.	*Ans.*
21.	Divide 44 ft. 8' 8'' by 22.	*Ans.*
22.	Divide 46 ft. 6' 6'' by 23.	*Ans.*
23.	Divide 69 ft. 3' 3'' by 24.	*Ans.*
24.	Divide 75 ft. 4' 4'' by 25.	*Ans.*
25.	Divide 98 ft. 3' 3'' by 26.	*Ans.*

26. Divide 54 ft. 6′ 6″ by 27.   *Ans.*
27. Divide 56 ft. 8′ 8″ by 28.   *Ans.*
28. Divide 68 ft. 4′ 4″ by 29.   *Ans.*
29. Divide 90 ft. 6′ 8″ by 30.   *Ans.*
30. Divide 93 ft. 6′ 9″ by 31.   *Ans.*
31. Divide 24 ft. 8′ 8″ by 32.   *Ans.*
32. Divide 20 ft. 6′ 6″ by 33.   *Ans.*
33. Divide 2 ft. 1′ 2″ by 34.   *Ans.*

LESSON VII.

366. The area and one of the sides of a square figure being given, to find the other side.

RULE.

Divide the area by the given side.

EXAMPLE.

1. The area of my farm is 64000 square feet, and the length of one of its sides is 450 ft. 2 primes 6 seconds. What is the length of the other side ?

Arrange the numbers as for division of denominate numbers, and divide the highest term of the dividend by the highest term of the divisor. Then multiply all of the terms of the divisor by the quotient thus obtained, subtract the result from the corresponding terms of the dividend, and take down the next denomination of the dividend to form a new divisor. It may happen that the highest term of the divisor may be too great for the highest term of the dividend. If so, reduce the highest term of the div-

idend to the next lower denomination. Divide as before, care being taken to place the proper sign to the quotient figure.

Note.—In division, the indices of the divisor are subtracted from those of the dividend, and the remainder will show what index belongs to the quotient figure.

367. Thus, primes divided by primes give primes, seconds divided by seconds give seconds, seconds divided by primes give primes, thirds divided by primes give seconds, etc., etc., etc., etc., etc., etc.

<div align="center">OPERATION.</div>

```
450 ft. 2' 6")6400 ft. 0' 0"(14 ft. 0' 4" 4''' 9''''
              6302   11  0
              _____
               97 ft. 1' 0"
                 165' 0" 0'''
                1980" 0''' 0''''
                1800" 10''' 0''''
                _____
                 180" 2''' 0''''
                 2162''' 0'''' 0'''''
                 1800''' 10'''' 0'''''
                 _____
                  361''' 2'''' 0'''''
                  4334'''' 0''''' 0''''''
                  4051'''' 10''''' 6''''''
                  _____
                   272'''' 1''''' 6''''''
                      etc., etc.
```

<div align="center">ANALYSIS.</div>

450 ft. 2' 6" into 6400 ft. 0' 0" 14 times (14 feet). 14 × 450 ft. 2' 6"=6302 ft. 11' 0".   Subtracting 6302 ft. 11' 0" from the dividend leaves a remainder of 97

ft. 1′ 0″. Reducing 97 ft. 1′ 10″ to primes gives 165 primes 0′ 0̇″. 450 ft. 2′ 6̇″ into 165 primes, we cannot divide. Reduce 165 primes to seconds and write 0 primes in the quotient. 165 primes is equal to 1980″ 450 ft. 2′ 6″ into 1980″, 4″ times. 4 × 450 ft. 2′ 6″= 1800″ 10‴ 0⁗. Subtracting 1800″ 10‴ 0⁗ from 1980″ leaves 180″ 2‴ 0⁗. Reducing 180″ 2‴ to thirds gives 2162‴ 450 ft. 2′ 6″ into 2162‴, 4‴ times. 4 × 450 ft. 2′ 6″=1800‴ 10⁗ 0⁗′. Subtracting leaves 361‴ 2⁗ 0⁗′. Reducing 361‴ 2⁗ to fourths gives 4334⁗ 0⁗′. Dividing 4334⁗ by 450 ft. 2′ 6″, gives 9⁗ for the quotient. 9⁗ × 450 ft. 2′ 6″=4051⁗ 10⁗′ 6⁗″. Subtracting, leaves 272⁗ 1⁗′ 6⁗″.

This operation may be carried out as far as the pupil sees fit.

EXAMPLES.

1. A piece of carpet contains 517 ft. 1′. The length of one of its sides is 18 ft. 3′. What is the length of the other side? *Ans.* 28 ft. 4′

2. The flooring of a room contains 9120 ft. 0′ 9″, and it is 84 ft. 3′ in width. How long is the room? *Ans.* 108 ft. 3′.

3. My gallery is 28 ft. 4′ long and has an area of 517 ft. 1′. How wide is it? *Ans.* 18 ft. 3′

4. The brick wall of a house is 108 ft. 3′ high and it has an area of 9120 ft. 0′ 9″. How long is it? *Ans.* 84 ft. 3′

5. If the area of a flower garden was 7 ft. 1′ 6″ and its length was 3 ft., how wide is it?

Ans. 2 ft. 4′ 6″.

6. If my parlor is 12 ft. long and it has an area of 225 ft. 8′, how wide is it?   *Ans.* 18 ft. 9′ 8″.

7. The area of the ceiling of a schoolroom is 229 ft. 11′ 2″ 10‴, and the room is 12 ft. 6′ 4‴ wide. How long is it?   *Ans.* 18 ft. 4′ 3″.

CHAPTER XVI.

PERCENTAGE.

LESSON I.

366. PERCENTAGE or PER CENT. means by the hundred.

It may be applied to any thing; thus, 6 per cent. of 100 dollars, 4 per cent. of 25 bales of cotton, 10 per cent. of 100 kegs of lard, 5 per cent. of 250 scholars, 3 per cent. of 1000 books, etc., etc.

6 per cent. of any thing is 6 one hundredths of it. It may be multiplied two ways. 1st. As if it were a common fraction. Thus:

6 per cent. of 100 dollars.

OPERATION.

$$\frac{6}{\cancel{100}} \times \frac{\cancel{100}}{1} = \$6$$

The above is read

6 one hundredths of 100 dollars.

Cancelling leaves $6 for the answer.

Or:

By the decimal method, which is most commonly used. 6 per cent. is 6 one hundredths, and is written .06.

By multiplying 100 dollars by .06, we get:

$$100 \times .06 = 600$$

Cutting off the number of decimals from the product, gives $6.00 for the answer.

(100 per cent. of any thing is the number itself. 100 per cent. is equal to 1 whole one.)

367. From what we have just learned, we have the following rule for finding the percentage of any number:

RULE

Multiply the number by the rate per cent., written as a decimal, and from the product point off as many figures as there are decimal figures in the multiplier and multiplicand.

EXAMPLE.

1. What is 3 per cent. of $15.25 ?

$$15.25 \times .03 = 0.4575$$

In the above example there are 2 decimal figures in the given number and 2 in the rate per cent.,

33

making 4 in both. Counting from right to left, cut off 4 figures from the product. In the above example there are but 4 figures in the product. Place the period or decimal point before the left-hand figure, as above, and to the left of thr decimal point write the figure 0. This shows that there are no whole ones. The above answer is read : 45 cents 7 mills and 5 tenths of a mill ; or, 45 cents 7 mills and one half of a mill.

<div align="center">EXAMPLES.</div>

1. Write decimally the following 3 per cent., 6 per cent.,   4 per cent.,   7 per cent.,   9 per cent., 8 per cent.

2. What is 3 per cent. of 2000 dollars ?   6 per cent. ?   4 per cent. ?   7 per cent. ?   9 per cent. ? 8 per cent ?   1st. *Ans.* $60.

3. Express in figures 2½ per cent., 4½ per cent., 6⅛ per cent., 2¼ per cent., 4½ per cent.

Note.—In the above examples reduce the fractional part to a decimal, and write the result to the right of the given per cent. Thus 4½ per cent. is written .042. ½ reduced to a decimal, gives 2 tenths. Write the 2 to the right of the figure 4.

4. What is 2½ per cent. of 3000 dollars ?   4½ per cent. ?   6⅛ per cent. ?   2¼ per cent. ?   4½ per cent.   *Last Ans.* $135.

5. Write decimally 2¼ per cent., ½ per cent., ⅜ per cent., ¼ per cent.

Note.—In the three last examples the required answer is less than one cent, therefore we must write two 0's before it. Thus, .00⅛. Finding the value of ⅛ of a cent, and writing it after the 0's, gives .00125. (⅛ reduced to a decimal gives 125.)

6. What is 2⅛ per cent. of 3500 dollars ?   ⅛ per cent. ?   ⅜ per cent. ?   ¼ per cent.?   *Last Ans.* $8.75.

This sign % is the sign of per cent.   6% is read 6 per cent.

7. A bought 2500 dollars' worth of molasses, and lost ⅛ % of it. How much did he lose ? *Ans.* $3.12½.

8. John Edwards bought 450 head of cattle for 20 dollars a head, and lost 2¾% on his way to market. How many did he lose ? How many had he left ? How much money did he lose ?   *Last Ans.* $247.50.

9. A collector collected $25000 in a month, on which he was allowed ¼%. How much did he make ?
Ans. $62.50.

10. A gentleman spends 8% for clothing, 25% for boarding, 2% for washing, 30% for spending-money. How much per cent. of his salary does he save ?
Ans. 35%.

11. Suppose his salary to be $1500 a year, how much does he save ?   *Ans.* $525.

CHAPTER XVII.

INTEREST.

LESSON I.

368. Interest is money paid by the borrower to the lender for the use of the money lent.

A lends B $1000 for 1 month, B gives to A 25 dollars for the use of the money, when the time expires.

The 25 dollars in the above case is the Interest.

The $1000 is the Principal.

The 1 month is the Time.

The $1000 + $25 = $1025 is the Amount.

If a per cent. was expressed it would be the Rate.

Interest is reckoned by the year, months being fractional parts of a year, and days fractional parts of a month.

What is the interest on $200 for 2 yrs. at 6%.

$$200 \times 2 \times .06 = \$24.00 \; Ans.$$

Or:

$$200 \times .06 = \$12.00, \text{ Int. for 1 year.}$$
$$12.00 \times 2 = \$24.00, \text{ Int. for 2 years.}$$

RULE.

Multiply the Principal by the Rate for the Time. The rate must be always expressed decimally.

1. What is the interest on $200 for 1 year, at 3 %? *Ans.* $6.

2. What is the interest on $200 for 2 years, at 4 %? *Ans.* $16.

3. What is interest on $3000 for 2 years, @ 4½ %? *Ans.* $270.

4. What is interest on $1000 for 3 years, @ 2½ %? *Ans.* $75.

5. What is the interest on $3500 for 4 years, @ 2¼ %? *Ans.* $315.

6. What is the interest on $4000 for 5 years, @ 3⅛ %? *Ans.* $625.

7. What is the interest on $6000 for 6 years, @ 2 %? *Ans.* $720.

8. What is the interest on $8000 for 7 years, @ 2½ %? *Ans.* $1400.

9. What is the interest on $9000 for 8 years, @ 3⅓ %? *Ans.* $2400.

10. What is the interest on $10000 for 12 years, @ 4⅓ %? *Ans.* $5200.

369. RATES OF FOREIGN MONEY (gold basis).

Ounce of Sicily $2.40
£ of Jamaica 4.84
Pound Sterling 4.84
Real Plata of Spain (one bit) 10
Prussian Dollar 69
Bremen Dollar . . . . .78 +
Roman Dollar 1.05
Russian Dollar (Rouble) 75
Dollar of Denmark, Norway and Sweden . 1.05

370. PAPER.

Folio Post . . . . 16 by 21 inches.
Foolscap . . . . 14 by 17 "
Crown . . . . . 15 by 20 "

371. A sheet folded in 2 leaves is called a folio.

A sheet	"	4	"	"	quarto.
A sheet	"	8	"	"	octavo.
A sheet	"	12	"	"	12mo.

THE END.